
\$THE LEARNER FIRST'

## Resources to activate and assign competence Adapted from Swafford \& Kilpatrick



## Multiplying fractions - Smith \& Stein 1998



## THE LEARNER FIRST

## Try this with Level 3 or 4 - Swafford and Kilpatrick

## Estimate and explain why this is right or wrong $9.83 \times 7.65=7519.95$

## Procedural focused students may

- withdraw from doing it without a calculator
- revert to pen and paper methods (not understanding estimate)
- if calculating this have a $50 \%$ chance of a procedural error

Students with a conceptual understanding of place value concepts and operations immediately knew it was not right. "This is $10 \times 8$ so I think they have just put decimal in wrong place. I am thinking its meant to be 75.1995"




## What resources are we using?



## The Units of work are proving popular

## Planning space

Manage and create teaching plans. Long-term plans.

## Long-term plans

These long-term plans provide a starting point for planning a mathematics teaching programme for a year.

|  | Full-year plans | Plans, by term, in the Planning Space |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Term 1 | Term 2 | Term 3 | Term 4 |
| Early level 1 | + | 7 | 7 | 7 | 7 |
| Late level 1 | - | 7 | 7 | 7 | 7 |
| Early level 2 | $\pm$ | 7 | 7 | 7 | 7 |
| Late level 2 | - | 7 | 7 | 7 | 7 |
| Early level 3 | - | 7 | 7 | 7 | 7 |
| Late level 3 | + | 7 | 7 | 7 | 7 |
| Early level/4 | ¢ | 7 | 7 | 7 | 7 |
| Late lewel 4 | $\pm$ | 7 | 7 | 7 | 7 |

## Units offer sequenced activities suited to 2 week blocks

| Term One | Term Two | Term Three | Term Four |
| :---: | :---: | :---: | :---: |
| Houses (Thematic unit) | Fill It Up - Flat Space <br> (Measuring area) | Fraction Benchmarks <br> (Ordering fractional numbers) | Slosh, Dribble and Plop <br> (Measuring with capacity, metric units for capacity) |
| Place value with whole numbers <br> (Whole numbers) | Starting from Scratch <br> (Properties of 2D shapes, polygons) | Breakaway Bars <br> (Decimal fractions) | Which graph with Excel? <br> (Interpreting, choosing, and making graphs) |
| Street Maps <br> (Co-ordinates, compass directions, simple scale maps) | Eggs and a little bacon <br> (Multiplication and division of whole numbers) | Building patterns <br> (Patterns and relationships) | Cups and Cubes <br> (Equations and expressions) |
| Equality with multiplication and division <br> (Properties of multiplication and division) | What's in the bag? <br> (Probability) | Making Benchmarks: Volume or Party Volumes (Measuring volume) | Getting the point <br> (Decimal fractions) |
| Carrots or Fridge Pickers <br> (Statistical inquiry cycle, graphing and representing data) | Matariki Level 3 <br> (Thematic unit) | Dividing Fractions <br> (Division with fraction answers (quotients)) | Polyhedra (3D Shapes) <br> (Properties of 3D shapes) |

This plan is a starting point for planning a mathematics and statistics teaching programme for a year. The resources listed cover about $50 \%$ of your teaching time.
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## Teaching Points can support formative benchmarks

## Breakaway Bars

## Description of Mathematics

This unit builds on the idea that the need for fractions and decimals arises from division situations in which ones (wholes) do not give an adequate degree of precision. Lack of closure of whole number under the operation of division creates the need for rational numbers. The division situations can be either partitive (sharing) or quotative (measuring). In this unit sharing of a decimat model is used to connect fractions and decimals.

## Specific Teaching Points

A key idea is that decimals are a restricted form of equivalent fractions. For example, three quarters has decimal representation of 0.75 because $3 / 4=75 / 100$. As with whole numbers the place values in decimals are connected although separate columns are used to write numbers. For example, 0.75 has can be expressed in different decimal forms, such as 7 tenths and 5 hundredths, 75 hundredths, 750 thousandths, 7.5 tenths, etc. Flexibility in the way students think about decimal place value supports their fluency with calculation. Central to fluency is students' understanding of how decimal place value units can by partitioned and combined. For example, 2.3-0.7 requires a one in 2.3 to be partitioned into 10 tenths if subtraction is used, or 10 tenths to be combined to form a one if adding on from 0.7 is used.

## We can use consolidation tasks to practice these

## Prior experience highlights pre-requisite skills

## Activity

## Prior Experience

Students would benefit from previous experience with fractions, particularly using partitioning of areas to form equal parts and the naming of collections of those as non-unit fractions. This unit uses an area model, the decimat, that is based on tens frames used for whole number place value. It is expected that students will understand whole number place value in a flexible way. They are expected to see place value as a nested system, in that, place value units are nested in others. For example, 230 can be renamed as 23 tens. It is also expected that students will have a range of strategies for solving addition and subtraction problems with whole numbers, that include using standard place value (hundreds, tens and ones, etc.) and tidy numbers (rounding and compensating).

## Session One

1. Show the students a copy of the Breakaway Chocolate bar (Copymaster 1). You may like to house paper copies in the coloured sleeve provided so the chocolate bar looks more authentic. Ask the students why the bar might be suitable to share between two or ten people as is claimed on the wrapper. Students might say that the bar is about the right size or it divides easily into two or ten equal pieces. Slide out the paper 'bar' and ask the students what they notice about the snap lines. Discuss how the lines might be used to share a bar into either halves or tenths.
So imagine we share the bar equally among ten people, how much bar does each person get? How could we record this mathematically?
2. Students may need to connect to story shells such as "Bindi has 24 comic books. She shares the comics equally among 6 friends. How many comics does each friend get?" Through likeness they may see that the chocolate sharing can be expressed as division.
Ways to express the division answer quotient are: $1 \div 10=1 / 10$ or $1 \div 10=0.1$
(A scientific calculator provides both decimal and fraction answers)
3. Ask the students to express the sharing between two people as division: $1 \div 2=1 / 2$. "What is the decimal for one half?" Most students know that 0.5 is the decimal for one half though many are unaware of why that is true. "If two people share the bar, how many tenths do they get each?" (five tenths). So the five in 0.5 means, 5 tenths. So, $1 \div 2=5 / 10=0.5$. You may need to use pattern of dividing by ten to extend the decimal places to the right beyond what your students are used to.

## Rich learning activities

## Rich learning activities

Differentiated activities at Levels 1 to 5 of the NZC.

Activities have been developed at Levels 1 to 5 of the NZC.

- Level 1 rich learning activities
- Level 2 rich learning activities
- Level 3 rich learning activities
- Level 4 rich learning activities
- Level 5 rich learning activities
- Counting Collections (number sense activities for levels 1 to 5 )
- Differentiated units (level 4 and 5 units with cross curricular links)


## Geometry and Measurement

- How long is a piece of string? (GM3-1)
- Standing order (GM3-1, NA3-1)
- Sugar rush (GM3-1, NA3-1)
- Parking cars (GM3-1, GM3-4)
- Where is the epicentre? (GM3-1, GM3-5)
- Across Lake Taupo (GM3-1, NA3-1)
- Noah's mystery_parcel (GM3-1, GM3-2)
- Folding Boxes (GM3-2)
- Platonic crackers (GM3-3)
- Polygon puzzle (GM3-3, GM3-4)
- Banana cake (GM3-5)
- A case for a new phone (GM3-6)


## Statistics

- Big Feet (S3-1)
- Books vs Bean Bags? Part i (S3-1)
- Books vs Bean Bags? Part ii (S3-1)
- Books vs Bean Bags? Part iii (\$3-1)
- Listening to music (S3-2)
- What are we eating? (S3-2)
- Penalty shoot-out (S3-3)


## Number and Algebra

- Carbon offset (NA3-1, NA3-2)
- Standing order (NA3-1, GM3-1)
- Sugar rush (NA3-1, GM3-1)
- Bill's dollars (NA3-1, NA3-2, NA3-6)
- Cricket with no ticket (NA3-1, NA3-6)
- Wifi units (NA3-1, NA3-2, NA3-6)
- Loads of sugar (NA3-1, NA3-4, NA3-6, GM3-1)
- A share of the spoils (NA3-1, NA3-5)
- Fraction circles (NA3-1, NA3-5)
- Domino donuts (NA3-1, NA3-6)
- A close game (NA3-1, NA3-7)
- Across Lake Taupo (NA3-1, GM3-1)
- Camping.groups (NA3-2, NA3-6)
- Vege rows (NA3-3, NA3-8)
- The seventh wave (NA3-3, NA3-8)
- sports tops (NA3-3, NA3-7, NA3-8)
- Broken Sparkles (NA3-4)
- Lunchtime Activities (NA3-5)


## This supports our procedural and conceptual skills

## The procedural approach (hide)

- The student is able to use appropriate strategies, including imaging and skip counting to solve a problem involving sequences.

Prompts from the teacher could be:

1. How many waves are there in each set?
2. Could you make a table or a sequence of images to represent of each set of waves?
3. Use your table or images to mark out which of the waves will be Sam's.
4. Find how many of the waves that Sam rides, are the seventh (biggest) wave.

Click on the image to enlarge it. Click again to close.


## Problem Solving activities



## Supports to develop pedagogical content knowledge

## Solution

This can be done by using equipment, by drawing, by algebra (see Toothpick Squares problem), or by using a table such as this.

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Freda | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 |
| Fred | 1 | 4 | 7 | 10 | 13 | 16 | 19 | 22 | 25 | 28 | 31 |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| Freda | 52 | 54 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 | 72 |
| Fred | 34 | 37 | 40 | 43 | 46 | 49 | 52 | 55 | 58 | 61 | 64 |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 |
| Freda | 74 | 76 | 78 | 80 | 82 | 84 | 86 | 88 | 90 | 92 | 94 |
| Fred | 67 | 70 | 73 | 76 | 79 | 82 | 85 | 88 | 91 | 94 | 97 |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | 33 | 34 | 35 |  |  |  |  |  |  |  |  |
| Freda | 96 | 98 | 100 |  |  |  |  |  |  |  |  |
| Fred | 100 |  |  |  |  |  |  |  |  |  |  |

[^0]
## E-Ako - PLD360 for kaiako

## PLD360

maths ad
Basic facts are presented in a number of ways and the understanding of some number properties is developed. This will underpin a student's success in learning and knowing 'basic facts'.

Subject
Matter Knowledge

Pedagogical Content Knowledge

Match the left and right hand columns.


## Start unknown

Change unknown
Inverse relationship (+/-)
Inverse relationship $(\times / \div)$
Commutative property (+)
Result unknown
Commutative property ( $\times$ )

How might you work with your students to explore, and understand these ideas?

## E-Ako - Maths adventures for äkonga

## e-ako

## PLD360

maths adventures
-(l)) Rosa and Robbie talk about the data they have collected.

Complete what they are saying by writing in each box.

| Year level | Main way of travelling to and from school |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Walk | Cycle | Car/van | Bus | Skateboard/ <br> scooter | Total |
| Year 1 | 10 | 0 | 12 | 1 | 0 | 23 |
| Year 2 | 14 | 0 | 12 | 0 | 0 | 26 |
| Year 3 | 16 | 0 | 12 | 0 | 0 | 28 |
| Year 4 | 14 | 2 | 11 | 1 | 1 | 29 |
| Year 5 | 4 | 4 | 6 | 4 | 3 | 21 |
| Year 6 | 9 | 8 | 6 | 1 | 2 | 26 |
| Total | 67 | 14 | 59 | 7 | 6 | 153 |

- ${ }^{(j))}$ The data from our school questionnaire tell us that, of the total 153 students at Kiwi School, 67 of them more often walk to and from school than travel any other way.

We also found from another question, that 25 students sometimes walk.
That means there can be up to 92 students walking to and from Kiwi School!
So we want to know,
How safe are the students who walk to and from our school?

## Building a Just-in-Time Community



Group by Rob Proffit-White

Connect to maths through rich learning experiences.
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## What sub strands are causing issues?

"If they cannot apply, it could be due to misconceptions"(Carpenter \& Lehrer, 1999) In your experiences which of the following sub strands do you encounter a growing need to revisit, reteach or remediate?:



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## A deep dive into the descriptors

|  | Number Strategies - Key ideas | Number Knowledge - Key ideas |
| :---: | :---: | :---: |
| Level 1 | Counting can be used to solve number problems. Students see numbers as made up of ones, and to operate with numbers need to count the individual items. There are two main counting strategies: Counting from one. Counting on | Objects in a set can be counted. Students are learning to count with understanding and identify "how many' in sets of objects. There are two elements involved in counting the objects in a set: <br> Number word sequence, One-to-one, This moves to 'skip counting' eg 2s, 5s, 10s |
| Level 2 | Numbers can be partitioned and combined to solve simple addition and subtraction problems. Students recognise part-whole thinking and apply it to derive results from known facts. Strategies include <br> Compensation eg $7+6 ; 6+6=12$, so $7+6=13$ <br> $P V$ partitioning eg $23+13 ;(20+10)+(3+3)=30+6=36$ | Our number system is based on groupings of the number ten. Students develop an understanding of place value. "Houses' can be used to show columns <br> eg 7 in tens represents 70 ones. |
| Level 3 | Numbers can be partitioned and combined to solve more complex (multi step) addition and subtraction and simple multiplication and division problems eg $\begin{aligned} & 53-28 ; 53-30=23,23+2=25 \\ & 43-38 ; \text { solve as } 38+[]=43 \end{aligned}$ <br> If $i$ know $5 \times 5=25$ then $I$ know $6 \times 5$ | Numbers can be represented in a variety of ways including fractions, decimals and percentages for representing small numbers. <br> The fraction $3 / 4,4$ is division of equal parts, 3 is number of the parts Decimals extend the PV system. Each column to the right of point is worth ten times less (a tenth of) <br> Percentages thought of as fractions (out of 100 parts) |
| Level 4 | Rational numbers can be represented and operated on in a variety of Rational numbers include; natural ( $1,2,3,4$ ), whole numbers ( $0,1,2,3,4$ ), form <br> Strategies that can be used include reversibility, doubling and halving | ys to solve problems. egers ( $-2,-1,0,1,2$ ) represented by number lines, exponents, expanded + standard mpensation, place value partitioning, using the distributive law. |

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## Choosing resources to target objectives

NA4-1 Use range of multiplicative strategies when operating on whole numbers.
A practical understanding of these when solving problems (including the ability to make initial estimations)

```
37+41+40+38 as 4 x 40-4
24 x 35
13\times6
14\times9
9 x 78
276\div12
12 x 33
216\div12
354\div6
as
(20 x 35) + (4 x 35)
as
10\times6 + 3 x 6
2x(7x9)
9\times80-9 x 2
240\div12+36\div12
4 x 99
216\div2\div2\div3
6x[ ] = 354
```

Why is their a difference between $(20 \div 10) \div 2$ and $20 \div(10 \div 2)$

# Why this is currently an important area 

## TIMSS Year 5

Maths item analysis

## The report looked at Year 5



## 90\% of students reported they had been taught this

$|$| MoE Data Report: TIMMS | International <br> Average $\%$ | New Zealand <br> $\%$ |
| :--- | :---: | :---: |
| $27 \times 43=$ | 53 | 16 |
| Add 385 to 5876 | 67 | 26 |
| Number added to 73 with sum of 1068 | 49 | 25 |
| $3126+845+72=$ | 72 | 39 |
| $6 \times 312=$ | 65 | 30 |
| 927 divided by 3 = | 46 | 19 |
| $0.2+0.02+2.2$ | 55 | 21 |

## moveNprove <br> before unit

Which of these has the largest value?
0.2
0.02
0.22
0.202


Ideas from Unit for explicit teaching groups

| 0.2 |  | 0.02 |  |  |  | 0.22 |  |  | 0.202 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18\% |  | 3\% |  |  |  | 41\% |  |  | 38\% |
| 7\% |  | 1\% |  |  |  | 14\% |  | 78\% |  |
|  |  |  | $\downarrow^{\text {PARTS OF ONE }}$ |  |  |  | thousandths |  |  |
| H |  | T | 0 |  | T | H | o | T | H |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  | 0 | - | 2 | 0 | 0 |  |  |
|  |  |  | 0 | - | 0 | 2 | 0 |  |  |
|  |  |  | 0 | - |  | 2 | 0 |  |  |
|  |  |  | 0 |  |  | 0 | 2 |  |  |

## A great find for visual problems

## Traci Jackson

@traciteacher
Playing with math• life-long learner• educator •math TOSA• NBCT•PCMI alum• @MoMath Rosenthal Prize •/mathwalk• "thinkingciassroom•\|teamPUSD ( ) San Diego $\mathcal{O}$ bitly/mathwalks2021 囲 Joined April 2009


Which One Docsn't Eclong


Submitted by Kay McHeffey


Open Middle Problem


Rewelun Spriin


Splat


Submitted by Kay McHeffey

## Many sources we use including...


©

## Any recommendations?

## We still use all problem types eg Application

Sione's luggage has a mass of 7.5 kg . He had to remove his tablet that had a mass of 750 g . What is the mass of his luggage now?

The Kuhui Ako is organizing a kapa haka competition with all the schools. If there are 534 people going and a bus can carry 48 , how many buses will be needed?

My daily runs on FitBit so far this week have been $8.6 \mathrm{~km}, 9.4 \mathrm{~km}, 10.9 \mathrm{~km}$, 7.75 km and 8.1 km . If my target is 50 km by this evening, how much do I need to run today?

- Reactivate or consolidate procedural fluency.
- Connect inquiry process to explicit teaching of method

- Diagnose what knowledge students have, pre teaching
*Learning can become stuck in "methods of calculating"


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[^0]:    The table shows that Fred gets to the $100^{\text {th }}$ square and has to wait two jumps for Freda to catch up.

