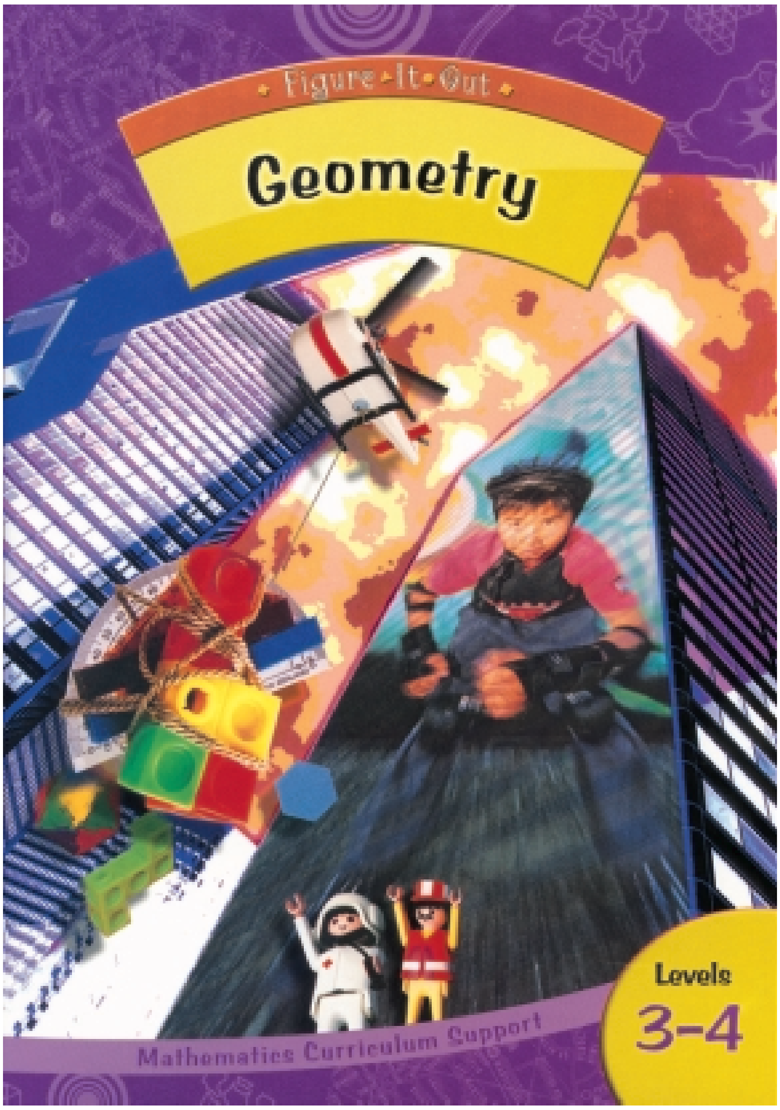
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

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Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum.* The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students. The teachers' notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

There are eight booklets for levels 3–4: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes.* The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 3–4) are suitable for most students in year 6. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 3–4.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask their students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

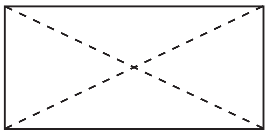
Geometry

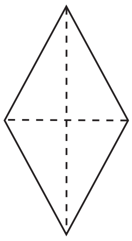
Answers

Page 1: Cornered!

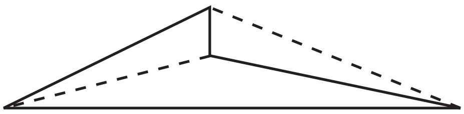
Activity One

1. Answers will vary, but the shapes can be any shape with four vertices. For example:



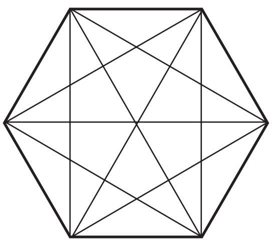


2. Shapes with four vertices (four-sided polygons)
3. Two. (A diagonal does not have to be within the shape.)

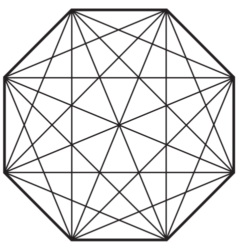


Activity Two

1. Yes, C to E
2.



hexagon
9 diagonals



octagon
20 diagonals

3. a.

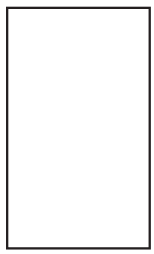
	Number of vertices	Number of diagonals
Triangle	3	0
Quadrilateral	4	2
Pentagon	5	5
Hexagon	6	9
Septagon	7	14
Octagon	8	20
Nonagon	9	27
Decagon	10	35

b. The difference between each number of diagonals increases by 1 for each extra vertex (+ 2, + 3, + 4, + 5, and so on).

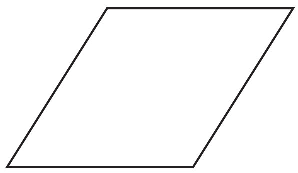
Pages 2–3: Getting in Shape

Activity One

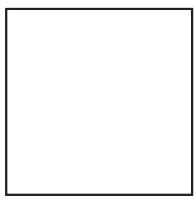
1. a.



(Rectangle locker)



(Rhombus locker)



(Square locker)

b. The only locker that the rectangle can use is the rectangle locker. The only locker that the rhombus can use is the rhombus locker. The square could use any of the lockers because it is also a rectangle and a rhombus, but two lockers are already taken, so it has to use the one labelled square locker.

2. a. Rhombus should avoid the diagonal equaliser because it would make him a square. He should also avoid the right-angled corner shaper because he hasn't got any right angles.

   b. Rectangle should avoid the side trimmer because it would make him a square.

   c. The diagonal equaliser and the side trimmer won't help Square because his diagonals and squares are already the same length.

3. a. He could go into any shower because he has equal diagonals, equal angles, and equal sides.

   b. Rectangle could go into the equal diagonals and the equal angles showers. Rhombus can only go into the equal sides shower.

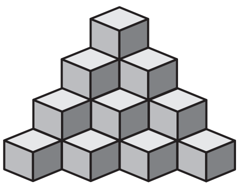
Page 4: Crazy Cubes

Activity One

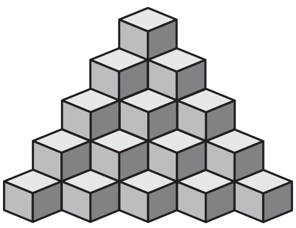
1. Practical activity. A table is a good way to organise the information:

Hour	Number of cubes in crystal	Difference
1	1	
2	4	3
3	10	6

2. a. Practical activity. The crystal should look like this:



hour 4



hour 5

   b. The crystal has 20 cubes at hour 4 and 35 cubes at hour 5.

3. When the crystal is made with cubes, the difference between the number of cubes in each stage is a triangular number.
   So a note to Cecily could read:

   Dear Cecily

   The crystal pattern is growing a little more each hour. In the first hour, the crystal grew by three cubes. In the next hour, the crystal grew by six more cubes, then by 10, and then by 15. I think the pattern is triangular.

   Yours sincerely

   signed

Activity Two

1. Practical activity
2. 63

Page 5: Inside Out

Activity One

1. The pieces of a triangle fit together to form a semicircle. The sum of the angles is 180°.
2. They always add up to 180°.

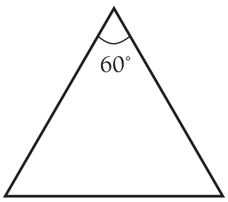
Investigation

The interior angles add up to 360°.

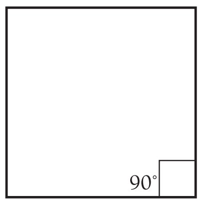
Activity Two

Shape	Vertices (corners)	Sum of interior angles	Each interior angle
Equilateral triangle	3	180° ($\frac{1}{2}$ turn)	60°
Square	4	360° (full turn)	90°
Regular pentagon	5	540° ($1\frac{1}{2}$ turns)	108°
Regular hexagon	6	720° (2 turns)	120°

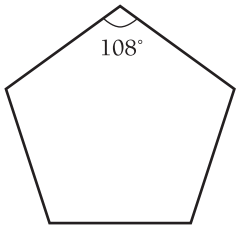
The shapes are:



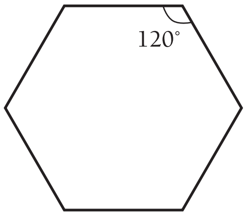
equilateral triangle



square



regular pentagon



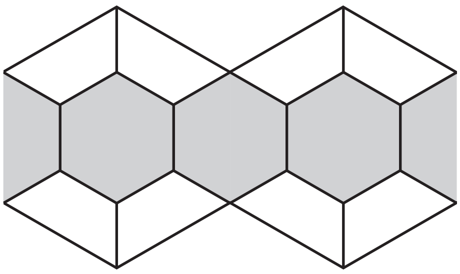
regular hexagon

Page 6: Tiling

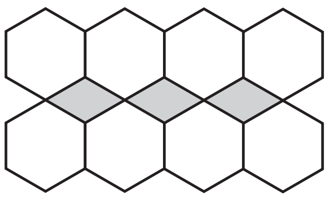
Activity

1. Drawings will vary, but all sets of pattern blocks can be made into tessellations.
2. a. No, it is not possible to make a tessellation with hexagons and squares. The inside (interior) angles of the polygons that meet at each point do not add up to 360°.

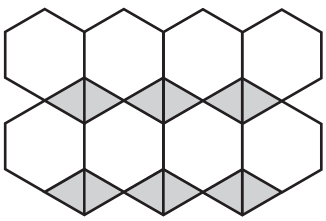
   b. You could use trapezia, rhombuses, or triangles with hexagons to make a tessellation. For example:



trapezia and hexagons

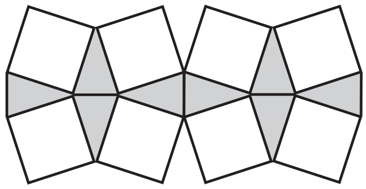


rhombuses and hexagons



triangles and hexagons

3. A tessellation with triangles and squares could look like this:



4. Side lengths that meet must be the same and the interior angles of the shapes about a point must add up to 360°. If the shapes do not meet both of these requirements, they will not tessellate.

Page 7: Changing Shapes

Activity

1. Practical activity
2. The shape you start with must be a polygon that has opposite sides parallel and congruent (for example, a rectangle or a regular hexagon). Your new shape must have the same area as the original shape.

Pages 8–9: Triangle Tricks

Activity

1. Practical activity
2. a. Practical activity

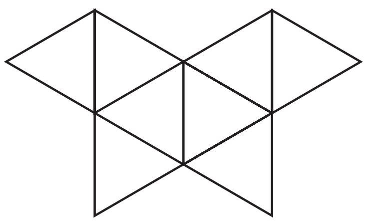
   b. The interior angles of a square add up to 360°, and the interior angles of a hexagon add up to 720°, which is two times 360°. Although these shapes have been altered, the interior angles will still add up to 360° (for the shape based on a square) and 720° (for the shape based on a hexagon). Each new shape will tessellate because the interior angles around the vertex will always add up to 360°.

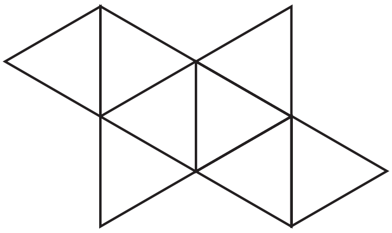
Pages 10–11: Caught in the Nets

Activity

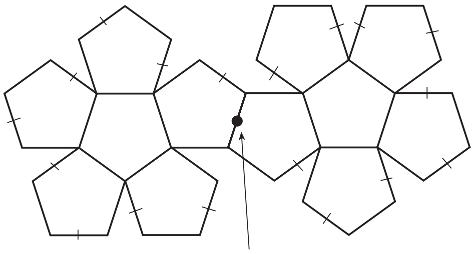
1. a. Yes, because there are four triangles at each vertex.

   b. Two examples of octahedron nets are:

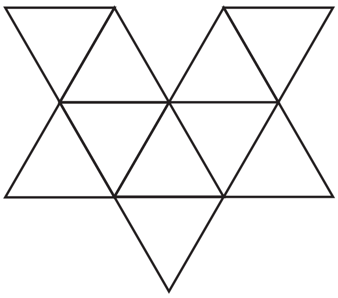




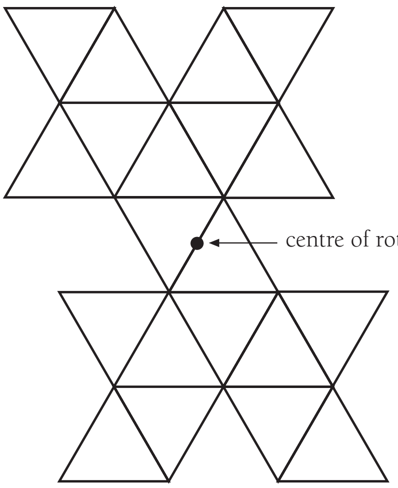
2. **a.** Yes, because the net has six faces or one-half of the dodecahedron and there are three pentagons at each vertex.
   b. Practical activity.
   The whole net for the dodecahedron is:



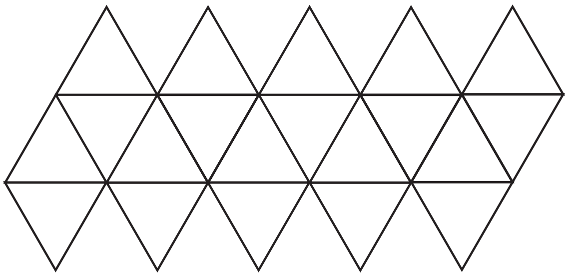
3. **a.** One way of drawing a half-net for the icosahedron is:



You can expand this to a full net:



Another net for the icosahedron is:

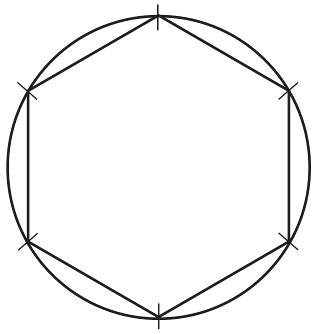


   b. Rotational symmetry
4. Practical activity

Page 12: Compass Shapes

Activity

1. **a.** Six times
   b. Your diagram should look like this:



2. Make six equally spaced marks on the circumference using the radius as the length of each space. Join every second point to draw an equilateral triangle.
3. Practical activity
4. Practical activity

Page 13: Cube Creations

Activity

1. **a.–b.** Practical activity
2. **a.–b.** Practical activity
3. **a.** Practical activity
   b. Strategies will vary. You could:
   - decide on the size of the cube and make sure none of the pieces are longer than the sides of the cube
   - arrange the multilink cubes into a bigger cube without sticking them together. Then take groups of cubes off the big cube and stick them together in the same shape they had in the cube.

Page 14: Building Boldly

Activity

1. Using the top view:

a.

1	2
3	1

The smallest number is 7.

b.

2	2
3	2

The largest number is 9.

2. Using the top view:

2	3	1
	1	2
1	1	

The smallest number is 11.

2	3	2
	2	2
1	1	

The largest number is 13.

Page 15: Blocked Plans

Activity One

1. Plan A goes with Perspective 2.
   Plan B goes with Perspective 3.
   Plan C goes with Perspective 1.

Activity Two

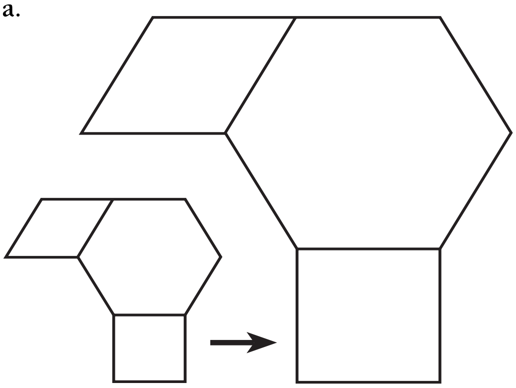
1. Practical activity. Teacher to check
2. Answers will vary.

Page 16: Growing Up and Down!

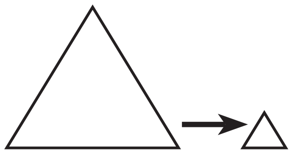
Activity

1. The knob setting says what to multiply each side of the shape by. Every side of the first shape is enlarged by 2 and every side of the second shape is reduced by half.

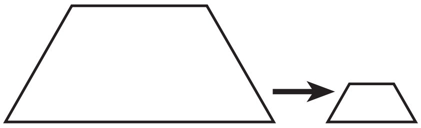
2. a.



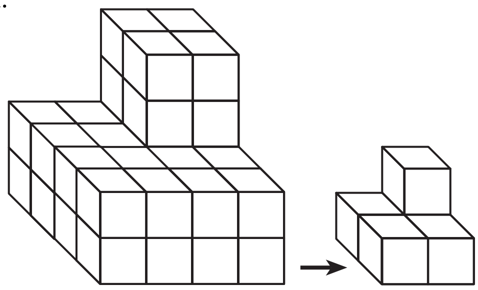
b.



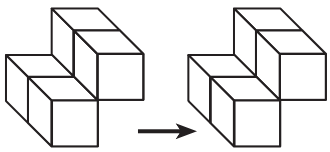
c.



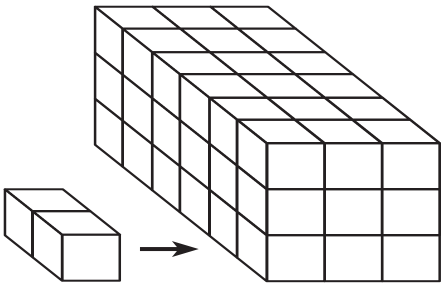
d.



e.



f.



3. Answers will vary. Two-dimensional models can only change in area. Three-dimensional models can change in both area and volume.

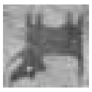
Page 17: Cut It Out!

Activity One

1. Practical activity
2. Both strips will have reflectional symmetry, but strip A will have twice as many lines of reflectional symmetry as strip B.

Activity Two

1. **a.** Practical activity
   b. Answers may vary, but they should be similar to the answer for **c** below.
   c. The motif has a diagonal line of reflectional symmetry, and the pattern has repeated vertical lines of reflectional symmetry.
2. **a.** By repeatedly rotating a printing block and pressing it down next to the previous print
   b. If you consider the four dogs as one region, then that region has been translated.
   c.



   d. The first and fifth segments are the same. The first segment has been translated to the fifth segment. The symmetry is repeated after every four segments.
   No matter where you start in the pattern, counting four segments will bring you to the next point of symmetry.
   e. Practical activity

Pages 18–19: Sharp Corners

Activity One

1. **a.** Grade one corners are the fastest.
   b. Grade four corners are the slowest.
2. Yes. The table at the top of the next column shows the corners and their grades.

Corner	Grade
a	2
b	2
c	1
d	1
e	2
f	3
g	2
h	3
i	4
j	1
k	2
l	1
m	3
n	4
o	3
p	2

Activity Two

Practical activity. Answers will vary.

Pages 20–21: Who Lives Where?

Activity

1. A – Sam
   B – Matiu
   C – Joe
   D – Paora
   E – Lisa
   F – Mae Ling
   G – Lachlan
   H – Miriama
2. Approximately 820 m
3. Sam. (Mae Ling walks approximately 500 m, Miriama walks approximately 940 m, Sam walks approximately 420 m, and Paora walks approximately 560 m.)
4. Practical activity

Pages 22–23: Ringing the Road

Activity

1. Practical activity. Teacher to check map:
   - Clue 1 takes them to Elstow.
   - Clue 2 takes them to the start of the Kaimai Rail Tunnel and on to a point 1 km from Okauia.

- Clue 3 takes them directly west to Waterworks Rd.
- Clue 4 takes them to a point that is 1 km from Monvale.
- Clue 5 takes them to Lake Ruatuna and on to Ngutuna.
- Clue 6 takes them to Te Puia Springs.

2. The return trip to Percival Rd is 58.25 kilometres in a northeasterly direction.

Page 24: Around School

Activity

1. Answers will vary.
2. Answers will vary, depending on what you expected. After a lot of throws, you will probably end up in about the same place as you started because all the directions and distances on the dice cancel each other out.

   Considering both dice together, there are three sides with north on them, which have a total of 9 m, and three sides with south on them, which have a total of 9 m, so these distances and directions will cancel each other out.
   The same applies for east and west distances.

Figure It Out

Geometry

Teachers' Notes

Overview: Geometry

Title	Content	Page in students' book	Page in teachers' notes
Cornered!	Investigating shapes	1	11
Getting in Shape	Classifying quadrilaterals	2–3	12
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Inside Out	Investigating internal angles of polygons	5	17
Tiling	Investigating tessellations	6	18
Changing Shapes	Exploring symmetry and tessellations	7	20
Triangle Tricks	Exploring symmetry	8–9	20
Caught in the Nets	Drawing nets for polyhedra	10–11	21
Compass Shapes	Constructing shapes and patterns using a compass	12	23
Cube Creations	Making three-dimensional shapes	13	23
Building Boldly	Interpreting plan views of buildings made with cubes	14	24
Blocked Plans	Interpreting plan views of buildings	15	26
Growing Up and Down!	Exploring enlargement	16	27
Cut It Out!	Exploring symmetry and transformations	17	28
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Who Lives Where?	Interpreting scale maps	20–21	30
Ringing the Road	Applying compass directions and scale	22–23	31
Around School	Following instructions using compass directions	24	31

Page 1: Cornered!

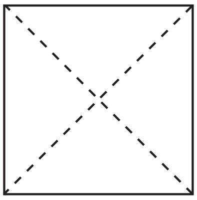
Achievement Objective

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)

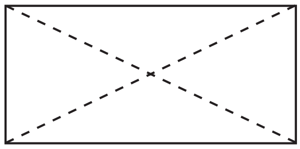
Activity One

This activity is useful for reinforcing students' understanding of the geometric terms *vertex* (plural: vertices) and *diagonal*. A vertex is the corner point of a polygon or polyhedron. It is also the point where two or more rays or line segments meet. A diagonal is a line segment that joins two vertices of a polygon but is not a side.

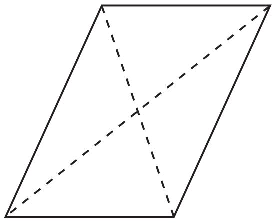
The students are asked in question **2** to find shapes with two diagonals. They can experiment with different shapes and should realise that all quadrilaterals have two diagonals.



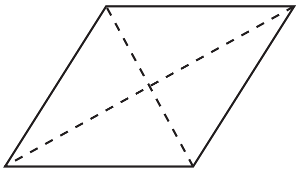
square



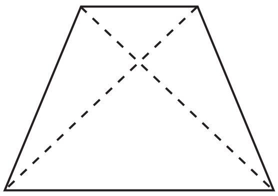
rectangle



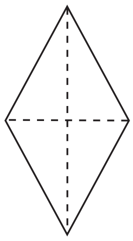
parallelogram



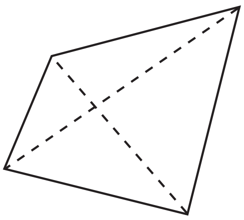
rhombus



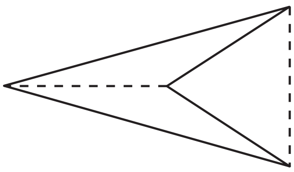
trapezium



kite



convex quadrilateral



concave quadrilateral

Activity Two

A systematic way of recording the diagonals for the octagon in question **2** is:

AC	AD	AE	AF	AG
BD	BE	BF	BG	BH
CE	CF	CG	CH	
DF	DG	DH		
EG	EH			
FH				

Finding the diagonals of regular shapes can lead students to create patterns such as the star patterns created by the regular pentagon and hexagon.

In question **3**, the students examine their tables for patterns. One pattern is described in the Answers, but there is a rule for this that you may like to explore with the students. For example, the octagon has eight vertices. Five diagonals can be drawn from each vertex. This would seem to give a total of 40 diagonals, but 20 of these have already been drawn from other vertices, so there are only 20 diagonals (40 ÷ 2). (Another way to look at it is that each diagonal has been counted from both ends.)

There is a general rule that you can use to calculate the number of diagonals a polygon has. First, you need to find out how many diagonals radiate from any one vertex of the polygon.

1. Each diagonal connects two vertices. So, start by dividing the number of vertices by 2. (In the case of the octagon, 8 ÷ 2 = 4.)

2. Multiply your answer from step 1 by the *number of vertices – 3*. (For the octagon, 4 x (8 – 3) = 4 x 5 = 20.)

Why subtract 3?

Firstly, when you join a vertex of a polygon to *every* other vertex in the polygon (that includes the two adjacent vertices), you will always have one line less than there are sides to the polygon. Check this!

Secondly, the lines joining adjacent vertices are not diagonals. There are two of these lines to every vertex. So the number of diagonals radiating out from any vertex is *the number of sides that the polygon has – 1 – 2*, which equals *n – 3*.

As an algebraic formula, this rule is $^1/_2 n \times (n - 3)$ where n is the number of vertices the polygon has.

Students who are comfortable with algebraic notation could try out this rule on the shapes in their chart. (This activity connects to the Algebra strand.) See also Problem Four, page 11, in *Problem Solving*, Figure It Out, Levels 3–4.

Pages 2–3: Getting in Shape

Achievement Objective

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)

Activity

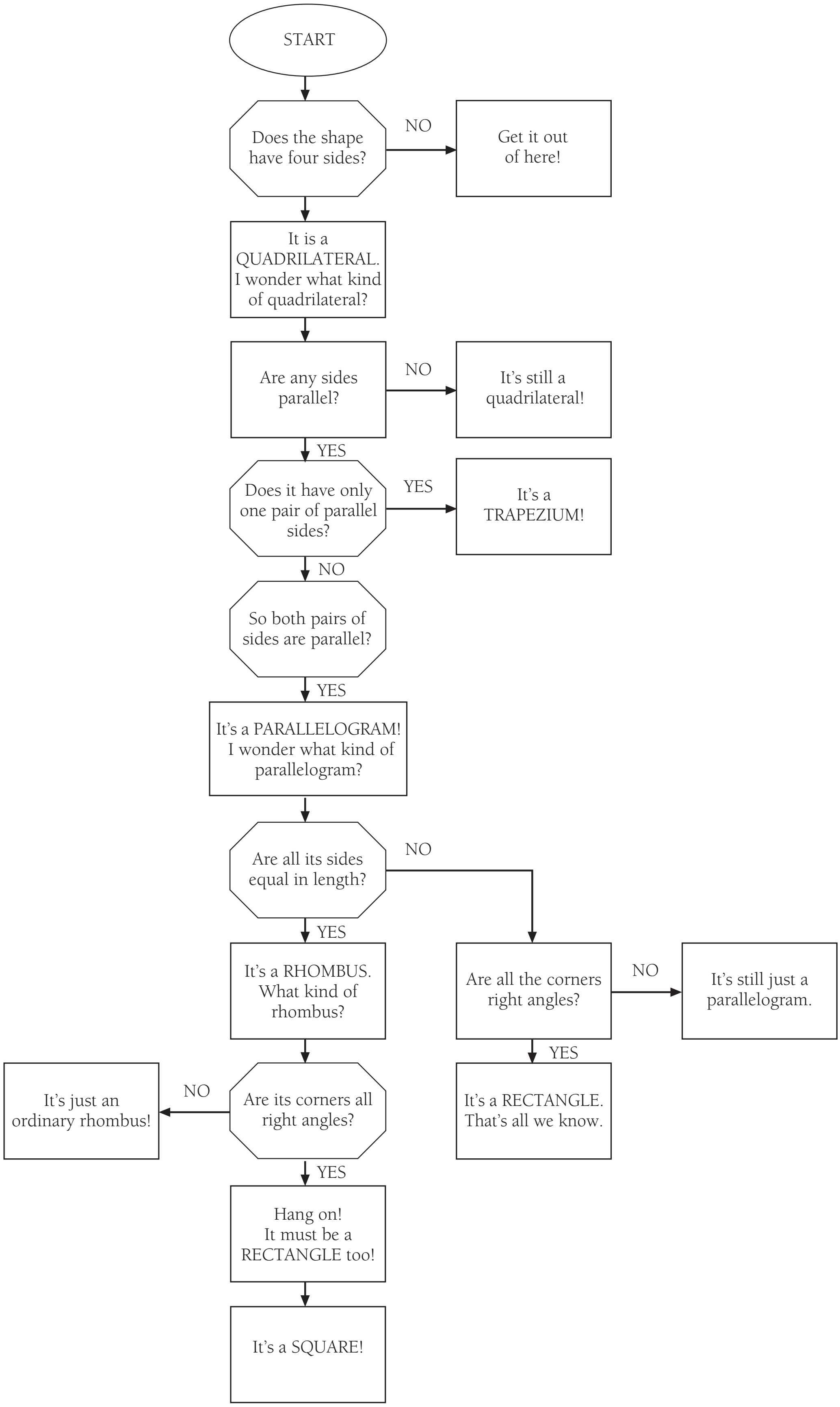
This activity is designed to reinforce students' understanding about shape properties of three quadrilaterals – the rectangle, the square, and the rhombus. Students must learn to use the properties to define the shapes. Key words that students should be using with confidence to describe shape properties are: sides, equal, parallel, diagonals, and angles. The students need to study a variety of quadrilaterals so that they can identify properties. The students who don't understand these properties will have difficulty classifying shapes. For example, they may not understand that a square is a type of rectangle and that a rectangle is a type of parallelogram. Many activities imply that shapes are distinct and unrelated. The students need to realise that a shape such as a square is also a rectangle, parallelogram, kite, rhombus, and quadrilateral.

To help the students sort quadrilaterals correctly, give them a set of quadrilaterals – squares, rectangles, parallelograms, rhombuses, kites, trapezia, and other quadrilaterals – and get them to sort the shapes according to the following properties:

- at least one right angle
- four right angles
- both pairs of opposite sides equal
- both pairs of opposite angles equal
- at least one line of symmetry

- more than one line of symmetry
- diagonals equal
- diagonals that bisect each other.

Alternatively, you could make an enlarged copy of the following flow chart and have the students sort and name the quadrilaterals using this process.



Scale 1 mm : 0.25 km
1 cm : 2.5 km


Mangapiko Valley
Waiti
Tahuna
Hoe-O Tainui
Otway
Waitoki
Waitawheta
Mangaiti
Springdale
Elstow
TE AROHA
Mt Te Aroha 952m
KAIMAI RANGE
KAIMAI MAMAKU FOREST PARK
Waiorongomai
Te Puninga
Mangateparu
Waihou
Tauhei
Whitikahu
Te Aroha West
Wairakau
Waitoa
Tatuanui
Shaftesbury
Manawaru
Gordonton
Kuranui
MORRINSVILLE
Hungahunga
Motumaoho
Kereone
Gordon
Puketaha
Eureka
Kiwitahi
Walton
Kutia
Wardville
WAHAROA
Turangaomoana
Newstead
Tauwhare
Scotsman Valley
Matangi
Te Miro
Richmond Downs
Peria
Okauia
Te Ohaki Pa
Okauia Pa
Pukemoremore
MATAMATA
Matai
Taihoa
Bruntwood
Hautapu
Fencourt
Kaipaki
Pukeroro
CAMBRIDGE
French Pass
Whitehall
Buckland
Hinuera
Te Poi
Leamington
Pukerimu
Karapiro
Selwyn
Taotaoroa
Pukekura
Monvale
Piarere
Waiomou
Maungatautari
Okoroire
Rotoorangi
Te Rahu
Horahora
TIRAU
Tapapa
Kairangi
Rangiaowhia
Hairini
Drakau
Puahue
Paepaerahi
Pukeatua
Pohara Marae
PUTARURU
Pinedale
Parawera
Arapuni
Puketurua
Owairaka
Ngatira

Page 4: Crazy Cubes

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- draw diagrams of solid objects made from cubes (Geometry, level 4)

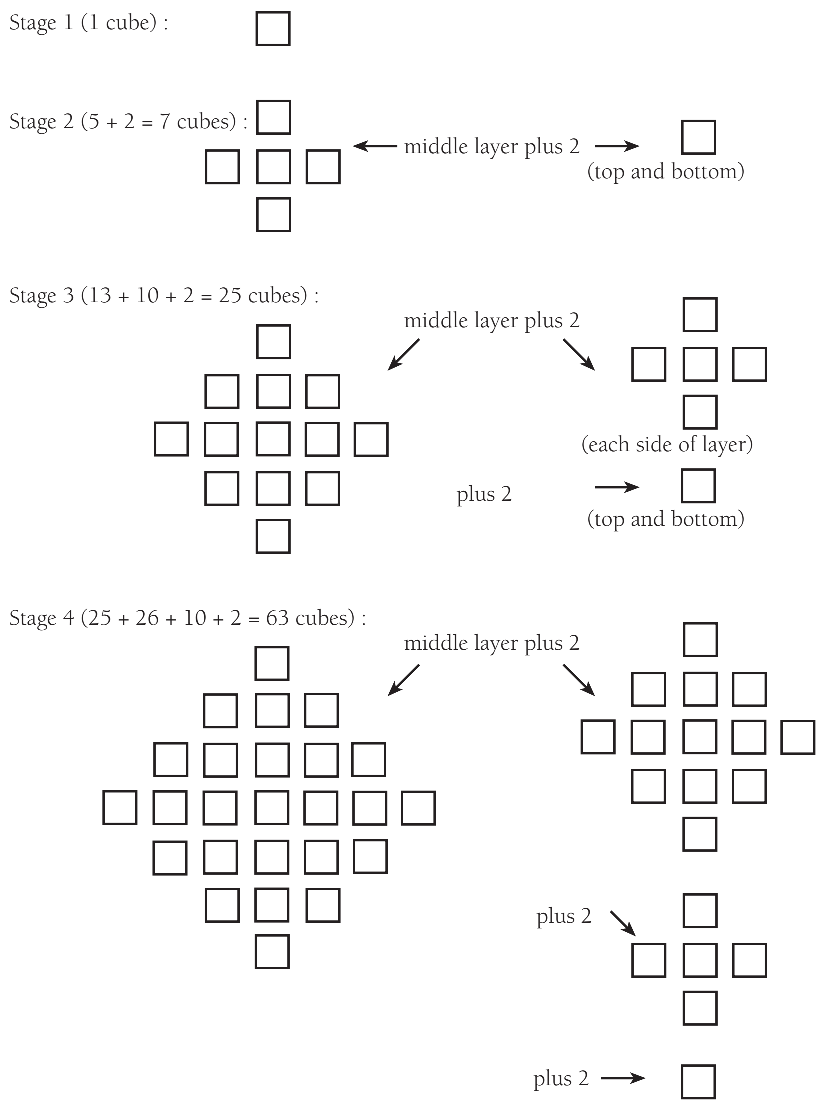
Activity One

This activity requires students to view 3-dimensional models made from multilink cubes, make the models, and practise drawing models on isometric paper (see the copymaster at the end of these notes). Drawing the models on isometric paper is quite a challenging task, but it follows on from the activities on pages 6 and 10 in *Geometry,* Figure It Out, Level 3.

If the students record their findings for question **1** on a chart as shown in the Answers, it makes it easier for them to look for patterns and to predict the next model in the sequence. As noted in the answer for question **3**, the difference generates triangular numbers. (This activity connects to the Algebra strand.)

Activity Two

Some students may have difficulty working out the various layers of the isometric drawing in this activity. The following diagram of the layers may help.



Stage 5 will be 41 + 50 + 26 + 10 + 2, and so on.

As an extension, the students could also use the models in both activities to draw views from different perspectives such as the front, top, and side. This task can help them to visualise and draw 3-dimensional models. It also makes them realise that cubes can be part of a model but cannot always be seen in a drawing. Alternatively, given a drawing of different views, the students could create the 3-dimensional model.

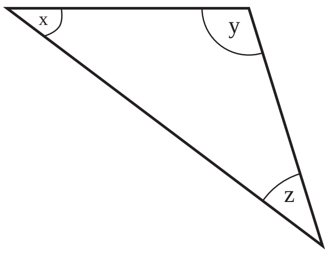
Page 5: Inside Out

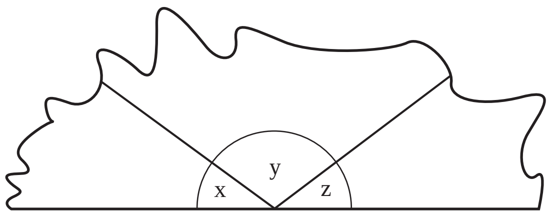
Achievement Objective

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)

Activity One

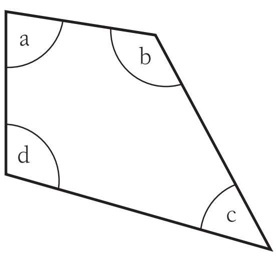
For this activity, you may need to remind the students that the sum of the angles in a straight line is 180° and the sum of the angles in a circle is 360°. This practical task is a very useful way for the students to discover that the sum of the interior angles of a triangle always equals 180°. The students could tear off the corners so that they can clearly recognise the angle originally taken from the triangle.

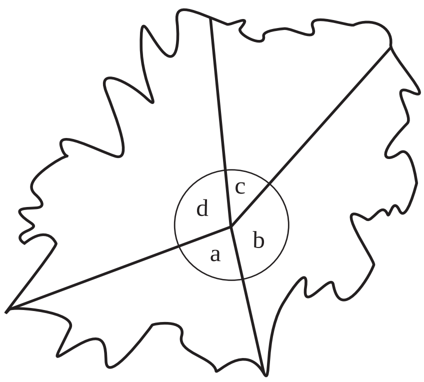




Investigation

Through this investigation, students should discover a similar proof for the sum of the angles of a quadrilateral. They should draw a range of different quadrilaterals (square, rhombus, trapezium, and so on), label and tear off each of the corners, and place them together to show that they form a revolution (360°).





Activity Two

In this activity, students use the sum of the interior angles of a triangle and of a square to find the size of each interior angle of a pentagon and a hexagon. The pattern is that each additional vertex adds another 180° to the sum of the interior angles. Students then use this information to draw the shapes using protractors. After the students have drawn the pentagon and hexagon, they could check the sum of the interior angles by using the same method they used for the triangle and quadrilateral. Notice that the angles of the pentagon add up to 540° (360° + 180°). This is a full circle plus a semicircle. However, there is no way of combining the individual pieces to form a full circle plus a semicircle. The hexagon yields 720°, which is two full circles.

This activity will help the students when they create tessellations (see page **6** of the student booklet) and explore why certain regular shapes tessellate and others do not.

Page 6: Tiling

Achievement Objectives

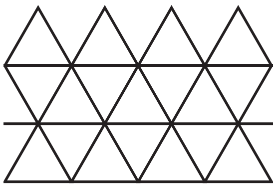
- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)
- apply the symmetries of regular polygons (Geometry, level 4)

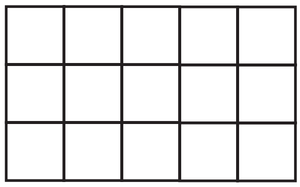
Activity

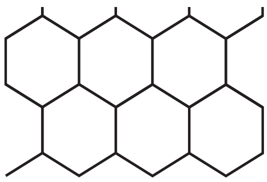
In this activity, students create a tessellation, which is a design made up of repeated tiles with no overlap or gaps. Tessellations are also known as plane tilings because they continue in every direction and so cover the plane. The students should be able to easily tessellate regular shapes.

The first example on the student page, a tessellation made from hexagonal pattern blocks, is a regular tessellation because the polygon is regular and identical.

There are three regular tessellations. They are:

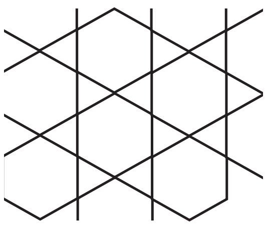

equilateral triangles


squares

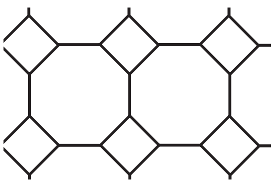

regular hexagons

A second group of tessellations is classified as semi-regular tessellations. All the polygons are regular, but there are two or more different polygons in the tessellation. The numbers under each tessellation below list in order the number of sides in the polygons that meet at a vertex. For example, the (3, 6, 3, 6) under the first example indicates that the tessellation is formed by placing a triangle, a hexagon, a triangle, and a hexagon together at a vertex.

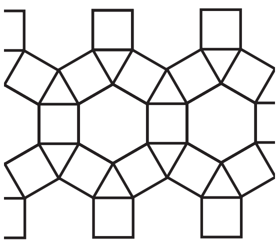
There are eight semi-regular tessellations. They are:

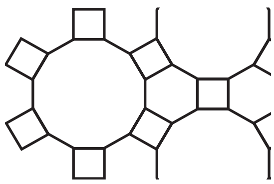

hexagons and triangles

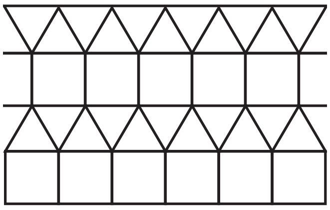
(3, 6, 3, 6)


octagons and squares

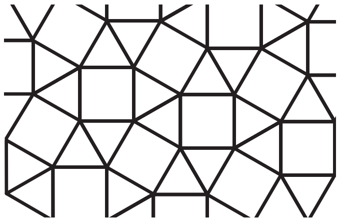
(4, 8, 8)


triangles, squares, and hexagons
(3, 4, 6, 4)

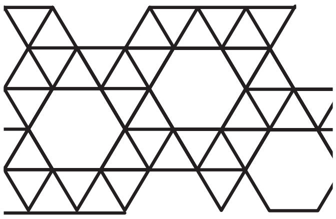

squares, hexagons, and dodecagons
(4, 6, 12)



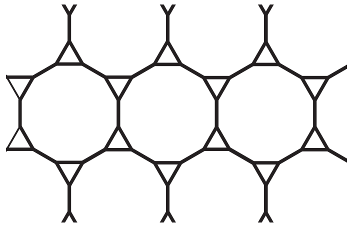
triangles and squares
(3, 3, 3, 4, 4)



triangles and squares
(3, 3, 4, 3, 4)



triangles and hexagons
(3, 3, 3, 3, 6)

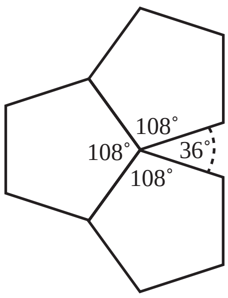


triangles and dodecagons
(3, 12, 12)

In questions **2** and **3**, the students need to experiment with combinations of shapes and then look carefully at their patterns to find reasons why the chosen shapes do or do not tessellate. In question **4**, they discuss this with a classmate.

To understand why some shapes tessellate and others do not, the students need to have examined the interior angles of polygons. The activity on page **5** covers this. They need to remember that the interior angles of a triangle add up to 180° and each extra vertex in a shape adds another 180° to the total.

To explain why some shapes tessellate and others do not, the students will need to understand what interior angles are and to know that the sum of the interior angles meeting at a vertex must equal 360°. If they understand this, they will be able to explain why the three regular tessellations (equilateral triangles, squares, and hexagons) work, whereas pentagons do not tessellate. For example, the pentagon has five sides and its interior angles add up to 540°. So each interior angle is 108°. The interior angles of three pentagons placed together do not add up to 360° at the vertex, so they will not tessellate.



The students should use similar reasoning to explain why some combinations of shapes tessellate and others do not.

Page 7: Changing Shapes

Achievement Objective

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)

Activity

This activity gives students a simple method for creating an original tessellating shape. This method is sometimes called the "bite" or "nibble" method. The main rule to follow when changing shapes to create a tessellating tile is that the shape must retain the same area as the original, so each piece that is cut off must be rejoined. Students need to be careful to just translate pieces and not to inadvertently flip them over.

When they have completed the tile, they translate the tile by sliding it along the plane – up, down, left or right – to create a tessellating pattern. The shape that the students start with must be a polygon in which opposite sides are parallel and congruent because an operation on one side will always affect the opposite side. The square is the easiest shape to start with, but encourage the students to experiment with other shapes using this bite method. The more sides the polygon has, the more sides that can be altered. Therefore, starting with a shape such as the regular hexagon can lead to an interesting design.

This activity gives students an opportunity to explore their own artistic creativity. They can add detail to the shapes to give the tessellation added appeal. However, it is important to use this activity not simply as an artistic activity, but as one that gives the students an opportunity to develop an understanding of translation and tessellation and to talk about their creations using the language of geometry.

Under the Sea, Figure It Out, Levels 2–3, page 11 and its accompanying teachers' notes discuss adapting a triangle to make a whole shape that tessellates.

Pages 8–9: Triangle Tricks

Achievement Objectives

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)
- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)

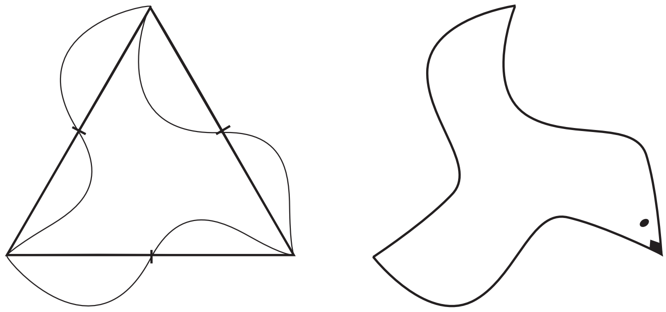
Activity

This method for creating tessellations is restricted to equilateral triangles, squares, and regular hexagons.

The students can alter all the sides of the polygon differently. If they have difficulty imagining an interesting animal or object, they could discuss their shapes with classmates to get ideas. By adding lines, marks, and colours, they can develop the artistic aspect of their tessellation and add an individual flavour to their patterns. They can experiment with tessellating their shape and using colour to complete the work. Using contrasting colours for adjacent tiles can be an effective technique.

In question **2b**, the students discuss with a classmate why their shapes tessellate. It is important that they do this because it may be only through discussion that they find that they have created the tessellation by using rotations, reflections, or translations.

Encourage the students to examine their tessellations for symmetries. The example on the following page is a good starting point. Get the students to start with a regular shape, such as an equilateral triangle, and cut and paste it as shown in the illustrations.



This gives a shape that has rotational symmetry of order 3, that is, the shape matches onto itself exactly three times during one complete turn. If the shape is then used to tessellate the plane, the pattern created has rotational symmetry of order 6. Why does the equilateral triangle tessellate to a pattern with rotational symmetry of order 6? The internal angle of the equilateral triangle is 60°. 6 x 60 = 360, so it takes six equilateral triangles to make up 360° at the point where the vertices meet. Therefore the pattern will have rotational symmetry of order 6.

The students may find that the way in which they have coloured their patterns alters the order of rotational symmetry.

The best known creator of such tessellations is M. C. Escher. He designed an amazing collection of interesting shapes that fit together to cover a plane. Compiling a collection of Escher designs for students to examine would be worthwhile. These are available in a number of books, such as *The Graphic Work of M. C. Escher* by M. C. Escher (London: Pan/Ballantine, 1961), and in posters and calendars. Another useful resource book for teachers is *Creating Escher-type Drawings* by E. R. Ranucci and J. L. Teeters (Palo Alto, CA: Creative Publications, 1977).

Pages 10–11: Caught in the Nets

Achievement Objectives

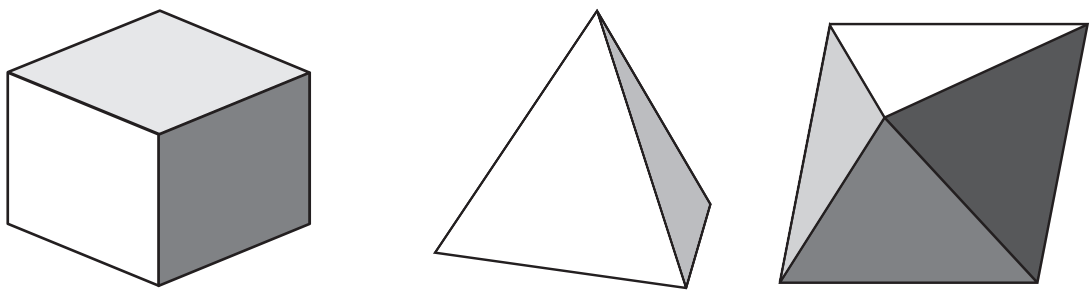
- design the net and make a simple polyhedron to specified dimensions (Geometry, level 4)
- apply the symmetries of regular polygons (Geometry, level 4)

Activity

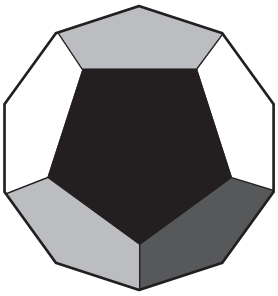
Before they construct the polyhedra in this activity, the students should have completed activities from *Geometry*, Figure It Out, Level 3, such as those on pages 5 and 11.

The polyhedra in questions **1**, **2**, and **3** are all Platonic solids. The Platonic solids are the regular polyhedra and derive their name from the Greek philosopher Plato (428–347 BC). They are called regular polyhedra because each one is made of just one type of regular polygon. The Platonic solids are the cube, the tetrahedron, the octahedron, the dodecahedron, and the icosahedron.

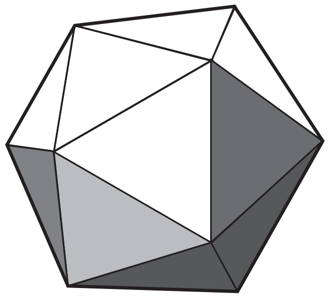
The five Platonic solids:



cube (hexahedron) tetrahedron octahedron



dodecahedron



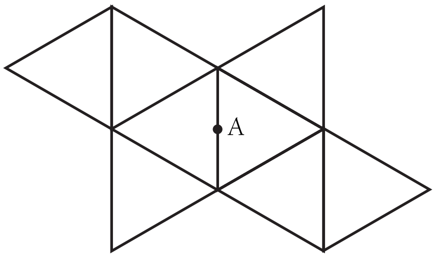
icosahedron

If possible, have models of the Platonic solids, made from regular polygon shapes, available for the students to examine. They can also make up the polyhedra themselves from the polygon shapes.

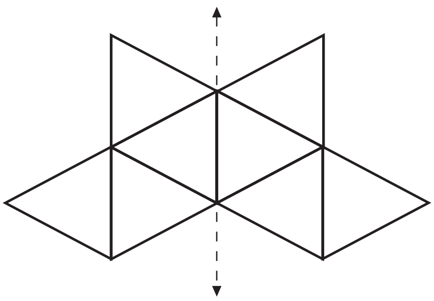
When the students are drawing nets, they should focus on one vertex to see what and how many shapes are around it. They then need to make sure that each vertex in their net has the same configuration of shapes around it.

Polyhedron	Polygons at each vertex
Cube	3 squares
Tetrahedron	3 triangles
Octahedron	4 triangles
Dodecahedron	3 pentagons
Icosahedron	5 triangles

Encourage students to look for symmetries in the nets they have drawn. They should be able to identify rotational and reflective symmetries in the nets.



This net for the octahedron has rotational symmetry about point A.



This same net for the octahedron also has reflective symmetry (“m” is the mirror line).

Students at this level should not be asked to design nets for more complex polyhedra, such as the one shown in question **4**. The icosidodecahedron has 32 sides and is one of the 13 Archimedean solids. They are made only from regular polygons but are called semi-regular polyhedra because they are made from more than one type of polygon.

As an extension, there is a relationship involving polyhedra that students can explore. Euler’s (1707–1783) Theorem says that for any polyhedron, the number of faces (F), vertices (V), and edges (E) satisfies the equation F + V = E + 2 (see *Answers and Teachers’ Notes: Geometry*, Figure It Out, Level 3, page 20).

Page 12: Compass Shapes

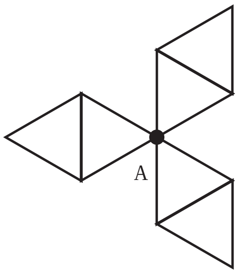
Achievement Objective

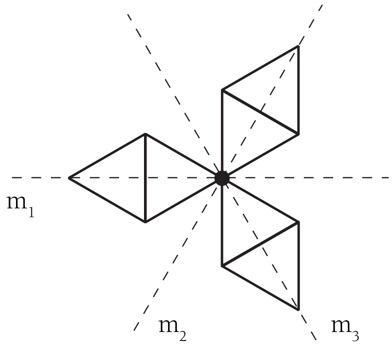
- construct triangles and circles using appropriate drawing instruments (Geometry, level 4)

Activity

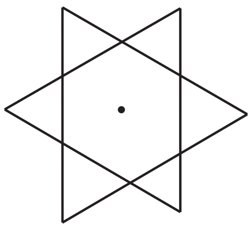
This activity initially helps students to recognise that the circumference is approximately six times the length of the radius. However, the objective of the activity is to give them practice in using the compass to make geometric designs beginning with a circle. This becomes a satisfying artistic activity, but you could also encourage them to use their design or logo to explore symmetry.

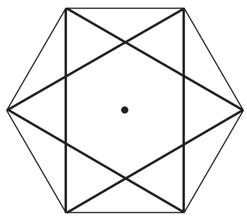
For example, the logo shown in the book has rotational symmetry of order 3 and three lines of reflective symmetry (m_1, m_2, m_3). (Point A on the diagram below is the centre of rotation.)

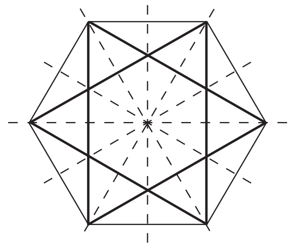




Another design made from the original inscribed hexagon could be:







This design has rotational symmetry of order 6 and six lines of reflective symmetry. Examining designs for reflective or rotational symmetry follows on from the activity on page 21 of *Geometry,* Figure It Out, Level 3.

Page 13: Cube Creations

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- draw diagrams of solid objects made from cubes (Geometry, level 4)

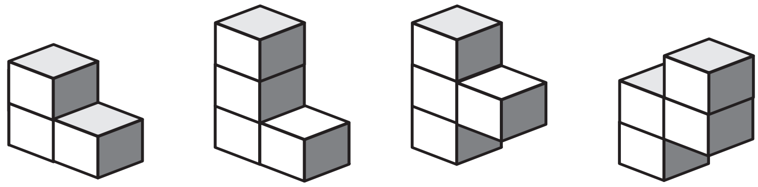
Activity

This practical activity helps students to develop visualisation skills. It builds on the activity on pages 6–7 of *Geometry,* Figure It Out, Level 3.

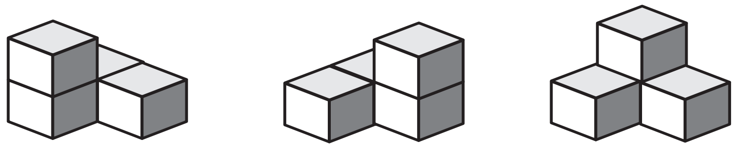
Before the students put the puzzle together, encourage them to spend time examining the pieces, visualising which pieces should join together rather than repeatedly trying different combinations. Using different colours for each layer, as shown in the students' booklet, will help the students to visualise how the pieces go together.

In question **2b**, the students can draw the pieces on isometric dot paper (see the back of these teachers' notes for a copymaster of isometric dot paper). These can be kept as a class resource for other students to use.

A very well-known 3 x 3 x 3 cube is the soma cube. It was invented by a Dane called Piet Hein. The cube is made up of all the irregular shapes formed by combining no more than four cubes. These shapes fit together to make the 3 x 3 x 3 cube. There are more than 230 different ways of constructing the cube. A good strategy for making the cube (and other target shapes) is to use the most complicated piece first. The seven soma pieces are shown below.

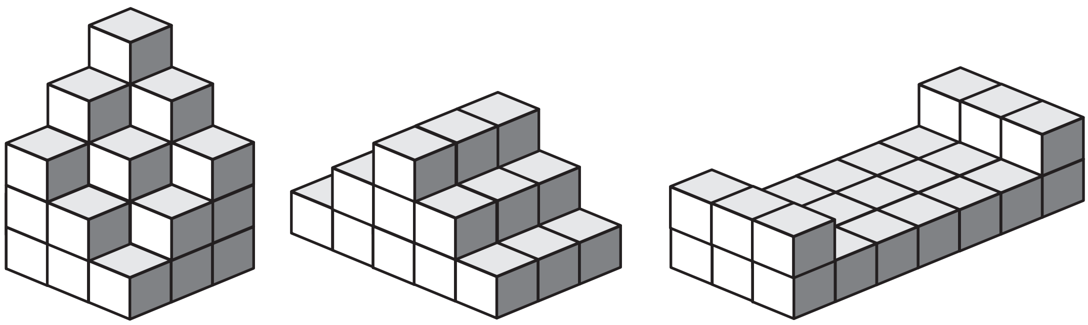


1 2 3 4



5 6 7

As an extension, the students could use the seven soma pieces to make target shapes such as those shown below or to create and draw their own shapes for classmates to model. (Students may have difficulty constructing the first two shapes below.)



crystal stairs bed

Page 14: Building Boldly

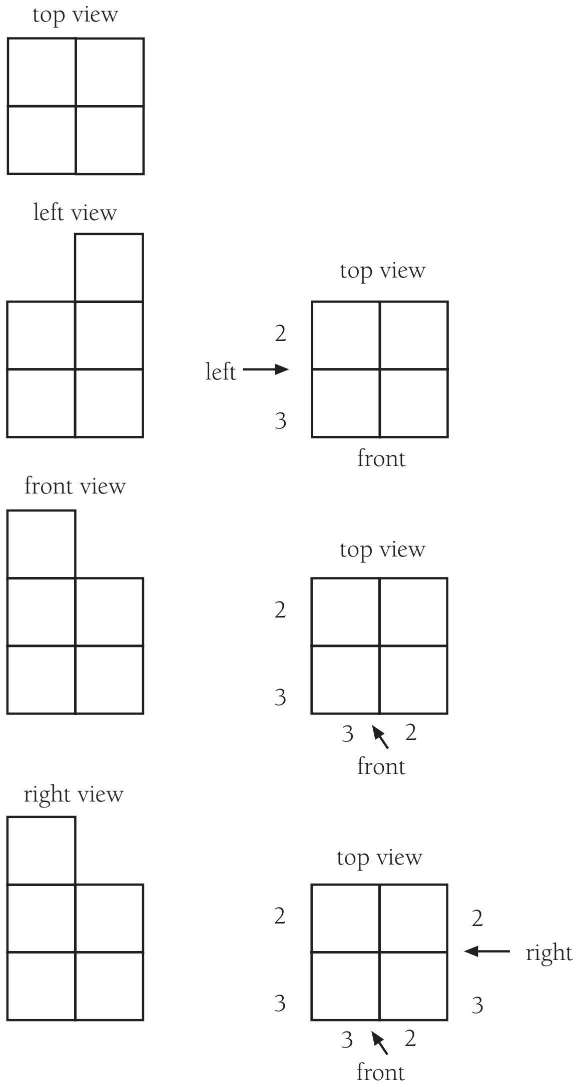
Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)

Activity

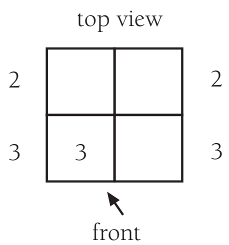
This activity uses the skills that students have developed in the activities on pages 8–9 of *Geometry*, Figure It Out, Level 3; Problem Two on page 20 of *Problem Solving*, Figure It Out, Level 3; and the activities on page 11 of *Geometry*, Figure It Out, Levels 2–3. You may find the teachers' notes for these activities useful and you could use the activities as a warm-up.

The students could make models of the buildings and use trial and improvement to find the smallest and largest number of cubes, but there is a more efficient method. They can use the view from the top and write in the number of cubes in each column. For example, for question **1**:

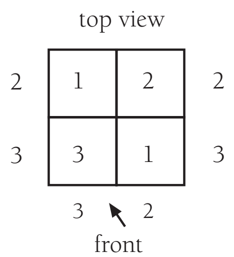


From this information, the students can answer **1a**, again using the top view. For example, the bottom row has to contain one 3. A 3 in the left square works from both the side and the front.

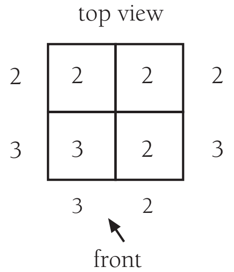
A 3 in the second square works from the side, but it does not work from the front because the front can only have a maximum of 2.



The students can use the same sort of reasoning to fill in the other squares:



For **1b**, the students can use the same method, but this time they are looking for the largest number of cubes:



The students could make models of the buildings with multilink cubes to check their answers.

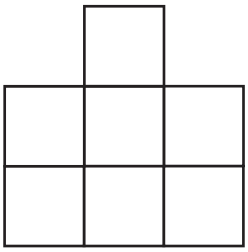
Page 15: Blocked Plans

Achievement Objective

- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)

Activity One

This activity uses students' skills in visualisation. They have to match various plan views of buildings with the perspective view. Encourage them to work systematically to match the perspectives and plans. For example, to find the perspective drawing for Plan A, the students could start with the plan view on the left.



This could be the side view of Perspective 1, and it could be the top view of Perspective 2, but it doesn't match at all with Perspective 3, so it is definitely not Perspective 3. Next, the students could work with the plan view on the right of Plan A. This doesn't match Perspective 1, but it does match with the front view of Perspective 2, so Plan A must go with Perspective 2. The students can continue to match the other plans and perspectives in this way.

If the students are finding this very difficult, you could tell them which plan views are top views, which are side views, and which are front views.

Activity Two

Although this activity requires the students to make the models from cubes, some students may have already made models to help solve the problems given in **Activity One**. The objective is to have the students involved in the process of viewing, drawing, and creating models. A real challenge for students at this level is to use the isometric paper to draw their models.

Page 16: Growing Up and Down!

Achievement Objective

- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)

Activity

This activity further develops students' understanding of enlargement. It builds on earlier activities on pages 23 and 24 of *Geometry,* Figure It Out, Level 3, where students use a grid method to enlarge pictures by a scale factor of 2.

In this activity, students use a variety of scale factors, namely 2, $^1/_2$, $^1/_4$, 3, $^1/_3$, and 1. A scale factor of 1 means that the image stays the same as the original. The key understanding reinforced by this activity is that the image (the enlarged shape) has the same proportions as the original.

Question **1** gives the students an example of an enlargement and a reduction. In the first example, the knob is on 2, so the side length of one unit is multiplied by 2. In the second example, the knob is on $^1/_2$. The original is two units, and $2 \times {}^1/_2 = 1$. This is a good opportunity to reinforce that dividing a unit by 2 is the same as multiplying it by $^1/_2$.

In question **2**, the students are given original images and their scale factors and have to work out the relevant enlargement or reduction. In each case, encourage them to focus on multiplying by the figure on the knob before they draw their models.

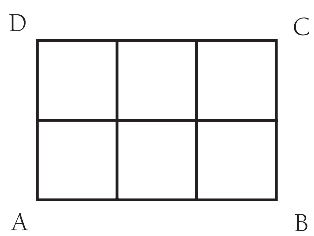
The students can check their answers by considering whether reversing the process would give the original image. Students who are having difficulty with this activity may also find grid paper useful. Pattern blocks are also useful for modelling the enlargements and reductions. Have the students begin by modelling the enlargement shown in question **1** so that they understand how to use the pattern blocks to enlarge or reduce.

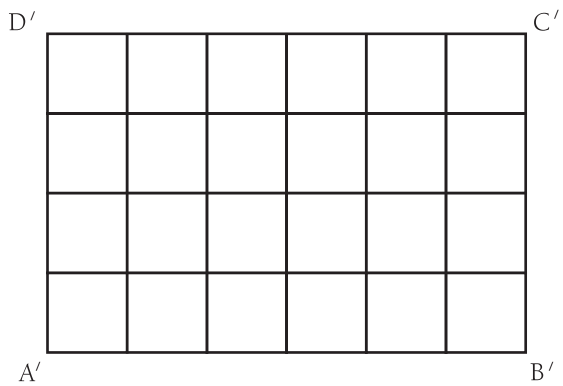
An alternative way of considering enlargement problems is: given the image and scale factor, what does the original look like? For example, if a model has been reduced by $^1/_3$, what does it need to be multiplied by to get it back to its original size?

As an extension, you could give the students some original images and enlargements or reductions of these images and then ask the students to find the scale factor. The key aspects of enlargement that the students should understand are:

- The scale factor of an enlargement is the factor by which the length of an object is increased.
- The scale factor can be found by taking the ratio of the length of a side of an enlarged or reduced image to the length of the corresponding side on the original image.
- When a 2-dimensional shape is enlarged by a scale factor of 2, the area is increased by a factor of 2^2 (or 4).
- When a 3-dimensional shape is enlarged by a scale factor of 2, the volume is increased by a factor of 2^3 (or 8).

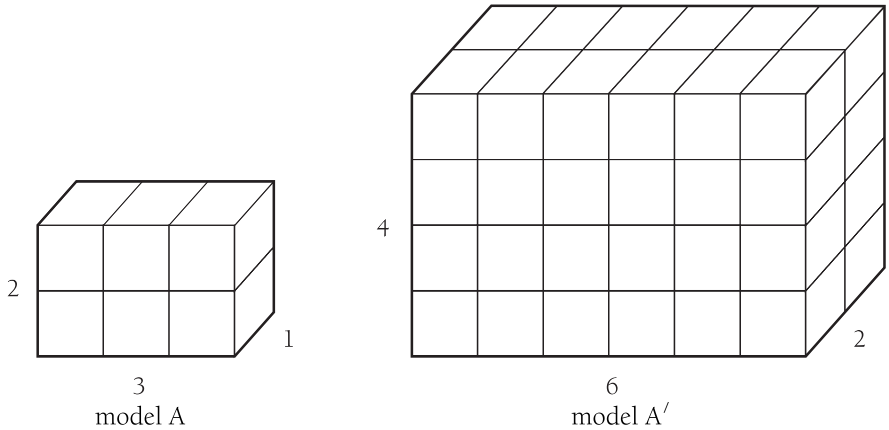
Rectangle ABCD is enlarged by a scale factor of 2. The length of each side is multiplied by 2.





The area of the rectangle has increased from 6 square units to 24 square units.

Model A is enlarged by a scale factor of 2. The volume of the model has increased from 6 cubic units to 48 cubic units.



Page 17: Cut It Out!

Achievement Objectives

- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)
- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)

Activity One

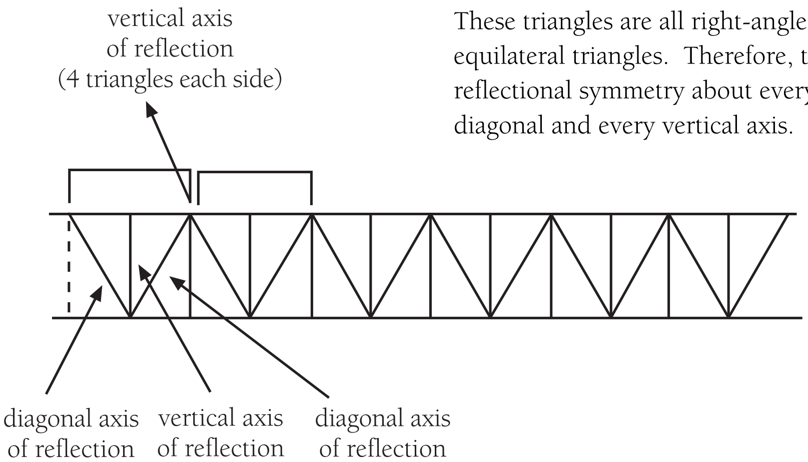
Using the folding method shown in this activity, the students create a design that has reflective symmetry. They could use a mirror to help them to recognise the lines of symmetry in both their motif and the pattern.

As a follow-up activity, the students could examine friezes (also called strip patterns) for symmetries. Kōwhaiwhai are good examples of friezes. They often have reflective and rotational symmetry and translations. The simplest way for the students to either describe or create a pattern is to identify a basic shape and then look for the different ways by which the shape has been transformed to generate the frieze. There are some photographs of kōwhaiwhai on pages 20 and 21 in *School Journal*, Part 1 Number 2, 2001.

Activity Two

In question **1**, the folding technique means that the motif has both diagonal and vertical lines of reflection. The frieze pattern has reflective symmetry only after the motif is repeated four times.

These triangles are all right-angled equilateral triangles. Therefore, there is reflectional symmetry about every diagonal and every vertical axis.



The strip in question **2** uses rotation to create a pattern that is repeated after four rotations of the motif. There is no reflective symmetry in this pattern. So the only symmetry here is rotational symmetry of order 4 and translation symmetry after four moves.

The students may like to use the method described on page 16 of *Geometry*, Figure It Out, Levels 2–3 to make a block for their own design. Encourage the students to talk about their patterns and the transformations that they have used to create them.

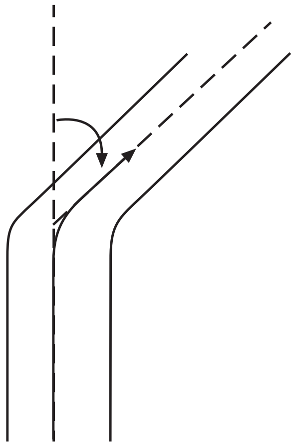
Pages 18–19: Sharp Corners

Achievement Objectives

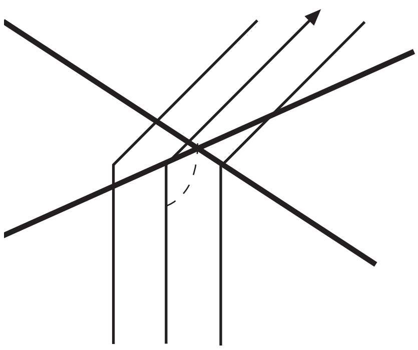
- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- use equipment appropriately when exploring mathematical ideas (Problem Solving, levels 3–4)

Activity One

Before the students begin this activity, ensure that they understand how the angles are measured. The angle to measure is the one between an extension of the original line of direction and the new line of direction:



Make sure that the students are *not* measuring the angle in the path:



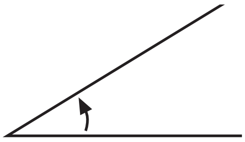
The students can either estimate or measure the size of the angles.

Activity Two

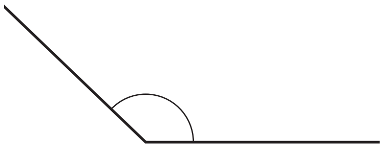
In the process of designing their own course, the students get an opportunity to apply their understanding of angles. Some students may do this without using a protractor. However, when the students are checking their classmate's course, you could ask them to measure and record the angle size rather than simply record the grade of each corner.

The *Answers and Teachers' Notes* for *Geometry,* Figure It Out, Level 3, pages 14–15 has very useful notes about acute and obtuse angles. You could discuss these angles with the class.

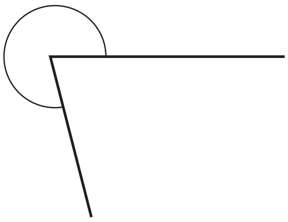
The acute angle is less than 90°:



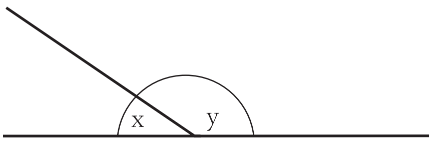
The obtuse angle is greater than 90° but less than 180°:



The reflex angle is greater than 180° but less than 360°:



You could also reinforce the students' understanding that the adjacent angles (x and y) on a straight line add up to 180°:



Pages 20–21: Who Lives Where?

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- specify location, using bearings or grid references (Geometry, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

This activity gives students an opportunity to use logical thinking and direction clues to locate houses on a street map. Some students may need to revise the compass points for NE, SE, SW, and NW. Before the students begin question **3**, you could ask them to estimate who travels the shortest distance to school. Paora, in house D, looks the closest, but does he travel the shortest distance when he goes to school along the roads? (Note that the students need to calculate the distance travelled along the roads, not the distance of a straight line from each house to the school.) The students can then measure the route each friend takes to school and use the scale to work out who travels the shortest distance.

In question **4**, the students need to use a local map to investigate different pathways, including the shortest path between two locations. They could use grid references to identify the location of places of interest such as the school, shopping centre, museum, sports fields, and so on. If a rural map is used, the students could identify farms, golf courses, and any other key features of the local area. They can then investigate the shortest paths between two locations, applying understanding of scale.

Pages 22–23: Ringing the Road

Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- specify location, using bearings and grid references (Geometry, level 4)

Activity

This activity is similar to the preceding one on pages **20–21**. The students need to apply an understanding of the compass points and scale to follow the route of Paulo's brother and sister on their cycling tour. You may need to use the answers to help the students work out the clues. An A3 copymaster of the Waikato district map is provided in the centre section of these notes.

As an extension, the students could use this map or a map of their own area to plan a cycling holiday. You could set parameters. For example, the students need to:

- travel at least 25 kilometres per day
- reach home within 5 days
- allow for bad weather on the third day.

It is important to point out the scale of the map. In this case, the scale is 1 centimetre : 2.5 kilometres or 1 millimetre : 0.25 kilometres. To succeed with this exercise, the students must be familiar with measuring to scale. It would be a good idea to get them to measure the distances between towns and convert their ruler measurements to kilometres before doing the exercise. For example, for clue 1, a measurement of 154 millimetres will give a distance of 38.5 kilometres ($154 \div 4$).

As noted on the student page, the distances measured are not the actual distances travelled (this sort of measuring would be very difficult on this sort of map). Instead, the distances are marked in a straight line from the starting point to the finishing point.

As an extension, use maps of the same area but different scales in order to show the difference in graphical representation. You could discuss what the different maps could be needed for.

Page 24: Around School

Achievement Objective

- draw and interpret simple scale maps (Geometry, level 3)

Activity

In this activity, the students follow directions (north, south, east, and west) on a dice and plot their path on a scale map of the school. The dice adds an element of chance as to which direction the students end up going in. The numbers and directions on the dice cancel each other out, so they will probably stay in the court area. If a school map is not available, the students could draw a scale map to plot their movements.

As an extension, the students could make another set of dice with the bearings of NE, SE, NW, and SW. They will need to make sure that the distances and directions cancel each other out.

Copymaster: Isometric Dot Paper

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