Answers and Teachers' Notes

MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The Figure It Out series is designed to support *Mathematics in the New Zealand Curriculum*. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.

Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students.

There are eight booklets for levels 2–3: one booklet for each content strand, one on problem solving, one on basic facts, and a theme booklet. Each booklet has its own *Answers and Teachers' Notes*. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities. The booklets in this set (levels 2–3) are suitable for most students in year 4. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 2–3.

The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.

Figure It Out

Basic Facts

Answers

Page 1: One Liner

Game

Game of addition

Page 2: Dicing Times

Game

Game of multiplication

Page 3: Quick Add

Game

Game of addition

Page 4: 31 or None

Game

Game of addition, subtraction, multiplication, and division

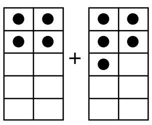
Activity

64 plums

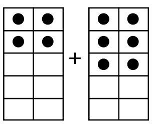
Page 5: Fizzing It Up

Activity

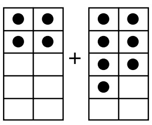
1. 4 + 4
2. 4 + 5 = 9



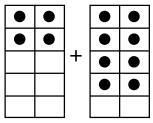
4 + 6 = 10



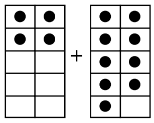
4 + 7 = 11



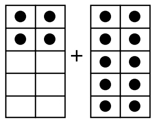
4 + 8 = 12



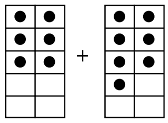
4 + 9 = 13



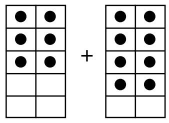
4 + 10 = 14



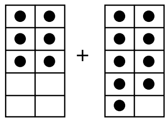
3. 6 + 6, as shown on the page
4. 6 + 7 = 13



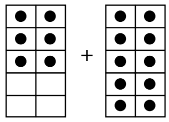
6 + 8 = 14



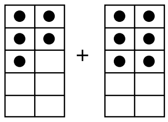
6 + 9 = 15



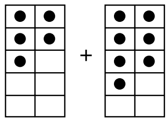
6 + 10 = 16



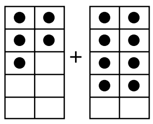
5. 5 + 6 = 11



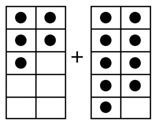
5 + 7 = 12



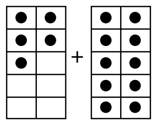
5 + 8 = 13



5 + 9 = 14



5 + 10 = 15



Page 6: Add It On

Activity

Answers are provided on the page as the bottom line of the table, shown in orange.

Page 7: Adding Adders

Activity One

1.	9	7 + 2 = 9 9 – 7 = 2	2 + 7 = 9 9 – 2 = 7
2.	5	8 + 5 = 13 13 – 8 = 5	5 + 8 = 13 13 – 5 = 8
3.	13	9 + 4 = 13 13 – 9 = 4	4 + 9 = 13 13 – 4 = 9
4.	5	5 + 5 = 10	10 – 5 = 5
5.	7	7 + 1 = 8 8 – 7 = 1	1 + 7 = 8 8 – 1 = 7
6.	4	4 + 3 = 7 7 – 4 = 3	3 + 4 = 7 7 – 3 = 4
7.	1 + 1	1 + 1 = 2	2 – 1 = 1
8.	9 + 9	9 + 9 = 18	18 – 9 = 9

Note: Each family of facts contains either two or four equations.

Activity Two

1.
   - **a.** 5
   - **b.** 6
   - **c.** 7
   - **d.** 3
   - **e.** 13
   - **f.** 12

2. Answers will vary. For **a–d**, students need to find two of the numbers on the same line. The third number to complete the equation will be on a horizontal or vertical line with the larger of the other two numbers.

   For **e** and **f**, students need to find the two numbers in the shaded areas. The largest number will be in the square where they meet.

   Each set of three numbers can be used to make and answer a variety of ☐ problems.

Page 8: Time's Up

Activity One

1.	12	2 x 6 = 12 12 ÷ 6 = 2	6 x 2 = 12 12 ÷ 2 = 6
2.	21	7 x 3 = 21 21 ÷ 7 = 3	3 x 7 = 21 21 ÷ 3 = 7
3.	8	8 x 7 = 56 56 ÷ 8 = 7	7 x 8 = 56 56 ÷ 7 = 8
4.	4	9 x 4 = 36 36 ÷ 9 = 4	4 x 9 = 36 36 ÷ 4 = 9
5.	6	3 x 6 = 18 18 ÷ 6 = 3	6 x 3 = 18 18 ÷ 3 = 6
6.	4	4 x 6 = 24 24 ÷ 6 = 4	6 x 4 = 24 24 ÷ 4 = 6
7.	3 x 3	3 x 3 = 9	9 ÷ 3 = 3
8.	7 x 7	7 x 7 = 49	49 ÷ 7 = 7

Note: Each family of facts contains either two or four equations.

Activity Two

1. 8 equations (2 x 6 = 12, 6 x 2 = 12, 12 ÷ 6 = 2, 12 ÷ 2 = 6, 4 x 3 = 12, 3 x 4 = 12, 12 ÷ 4 = 3, 12 ÷ 3 = 4)
2. 4 equations (6 x 7 = 42, 7 x 6 = 42, 42 ÷ 6 = 7, 42 ÷ 7 = 6)
3. 8 equations (6 x 4 = 24, 4 x 6 = 24, 24 ÷ 6 = 4, 24 ÷ 4 = 6, 8 x 3 = 24, 3 x 8 = 24, 24 ÷ 8 = 3, 24 ÷ 3 = 8)
   Other equations for 24 not shown on this array: 2 x 12 = 24, 12 x 2 = 24, 24 ÷ 2 = 12, 24 ÷ 12 = 2
4. 8 equations (6 x 3 = 18, 3 x 6 = 18, 18 ÷ 6 = 3, 18 ÷ 3 = 6, 2 x 9 = 18, 9 x 2 = 18, 18 ÷ 2 = 9, 18 ÷ 9 = 2)
5. 2 equations (9 x 9 = 81, 81 ÷ 9 = 9)
6. 6 equations (4 x 4 = 16, 16 ÷ 4 = 4, 8 x 2 = 16, 2 x 8 = 16, 16 ÷ 2 = 8, 16 ÷ 8 = 2)
7. 2 equations (5 x 5 = 25, 25 ÷ 5 = 5)
8. 4 equations (9 x 3 = 27, 3 x 9 = 27, 27 ÷ 9 = 3, 27 ÷ 3 = 9)

Note: The full family of facts is not required for this activity but is provided here for reference.

Page 9: An Apple a Day

Activity One

1. 24
2. Answers will vary but will be based on addition and multiplication.
3. Answers will vary, for example, rows of 6 x 6, 4 x 9, or 3 x 12.

Activity Two

1. Various addition methods, such as 3 + 3 + 3 + 3 + 3 + 3 + 3 = 21
2. a. 18
   b. 14
   c. 15
   d. 12
   e. 12
   f. 9

Page 10: On Track

Activity One

1. Pat could also use white (1 cm), red (2 cm), light green (3 cm), and pink (4 cm) Cuisenaire rods.
2. 1 x 24 = 24, 2 x 12 = 24, 3 x 8 = 24, 4 x 6 = 24

Activity Two

1. Rachel could make five different 20 centimetre tracks:
   white (20 x 1)
   red (10 x 2)
   pink (5 x 4)
   yellow (4 x 5)
   orange (2 x 10)
2. She could make six different 36 centimetre tracks:
   white (36 x 1)
   red (18 x 2)
   light green (12 x 3)
   pink (9 x 4)
   dark green (6 x 6)
   blue (4 x 9)

Page 11: Heading for Home

Game

A game of division

Page 12: Takeaway Numbers

Activity

Answers are provided on the page as the bottom line of the table, shown in blue.

Page 13: Domino Facts

Game

A game of multiplication and division

Page 14: Flying-fish Soup

Activity

1. 2 onions
   2 tsps curry powder
   2 tbsps oil
   2 tsps salt
   2 tsps pepper
   3 cups coconut cream
   3 kg fish
2. $^1/_2$ onion
   $^1/_2$ tsp curry powder
   $^1/_2$ tbsp oil
   $^1/_2$ tsp salt
   $^1/_2$ tsp pepper
   $^3/_4$ cup coconut cream
   $^3/_4$ kg (750 grams) fish
3. The students could either multiply the two-person recipe by three or add the two-person and the four-person recipes together.

Page 15: Twenty-seven

Game

A game of strategy with addition

Page 16: Pata Tika

Game

A game to demonstrate systematic thinking

Page 17: Secret Codes

Activity One

If you multiply a number by zero, the answer will be zero.

Activity Two

A blue whale may weigh 134 tonnes.

Page 18: Magnificent Multiples

Activity

Answers are provided on the page as the bottom line of the table, shown in orange.

Page 19: Stay on Line

Game

A game of addition and subtraction

Page 20: A Sticky Problem

Activity

1. 4

2. **a.** 6

b. Yes, 4

3. 5

4. $4

5. $2.50

6. She will have 37 pieces of fudge to sell. At 10 cents a piece, she will make $3.70. Once Rachel subtracts her costs, she will be left with $2.20 profit.

Page 21: Testing Triangles

Activity One

An activity with no set answers. Outcomes will vary depending on the tiles that are given out to the students.

Activity Two

Students will create their own triangle tiles.

Page 22: Division Revision

Activity

Answers are provided on the page as the bottom line of the table, shown in blue.

Page 23: Loopy

Game

A game of multiplication

Page 24: Six Shooters

Game

A game of multiplication

Figure It Out

Basic Facts

Teachers' Notes

Overview: Basic Facts

About Basic Facts

Mathematics in the New Zealand Curriculum defines the addition and multiplication facts in the following way:

- Addition facts 1 + 1, … 9 + 9
- Multiplication facts 1 x 1, … 10 x 10

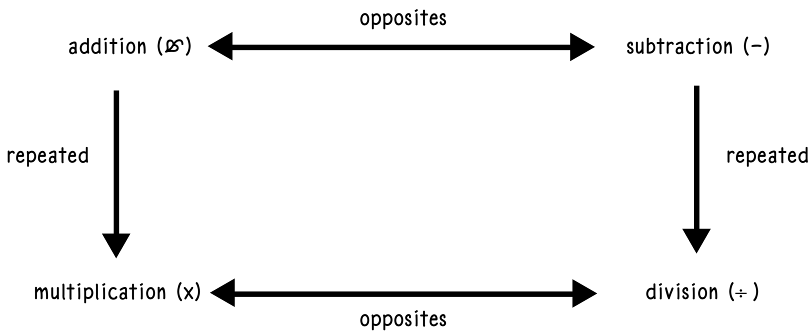
The relevant achievement objectives are:

- recall the basic addition and subtraction facts (Number, level 2)
- recall the basic multiplication facts (Number, level 3)

Clearly, the requirement for multiplication facts at level 3 should not prohibit students at level 2 from exploring and learning these facts. Nor should the basic addition and subtraction facts objective at level 2 prohibit maintaining recall of these facts at later levels.

Three key principles should guide the learning of basic facts:

1. Understanding the meaning of each operation (+, –, x, ÷) and the relationships between the operations must underpin the learning. The following diagram shows those relationships:



   This means that basic division facts must be learned alongside multiplication facts at level 3.

2. Learning activities should use pattern wherever possible so that students develop strategies to work out unknown facts from those they know. For example, in learning multiplication facts, arrays are very useful. Given the array below, you could ask students:

x	3	4	5	6	7
3	9	12	15	18	21
4	12	16	20	24	28
5	15	20	25	30	35

"What multiplication and division facts can you see in this picture?"

The students can then use these facts to work out 4 x 6, 5 x 5, 6 x 5, 4 x 4, and so on.

3. Students need to encounter basic facts in all their symbolic forms. For example:

4 + 6 = ❑	4 + ❑ = 10	❑ + 6 = 10
9 – 3 = ❑	9 – ❑ = 6	❑ – 3 = 6
5 x 8 = ❑	❑ x 8 = 40	5 x ❑ = 40
36 ÷ 4 = ❑	36 ÷ ❑ = 9	$^{36}/_{4} = \begin{array}{r}9\\4\overline{)\square}\end{array}$

The activities in this booklet represent both learning and practice activities. Students cannot practise what they do not know how to do. Opportunities must be provided in lesson time for learning basic facts on an individual basis rather than overemphasising testing.

Page 1: One Liner

Game

This is a game to practise addition facts. It requires students to either know these facts before playing or have some visual means of working them out (for example, counters, tens frames). The board is weighted in favour of the sums 13 and 14.

Discussion after the game might focus on why there are two 13s and 14s on the board. The table below shows that there are more ways for 13 to be made. There are five ways to make either 12 or 14, so either could have been selected to give students more opportunities to place their counters.

+	4	5	6	7	8	9
4	8	9	10	11	12	13
5	9	10	11	12	13	14
6	10	11	12	13	14	15
7	11	12	13	14	15	16
8	12	13	14	15	16	17
9	13	14	15	16	17	18

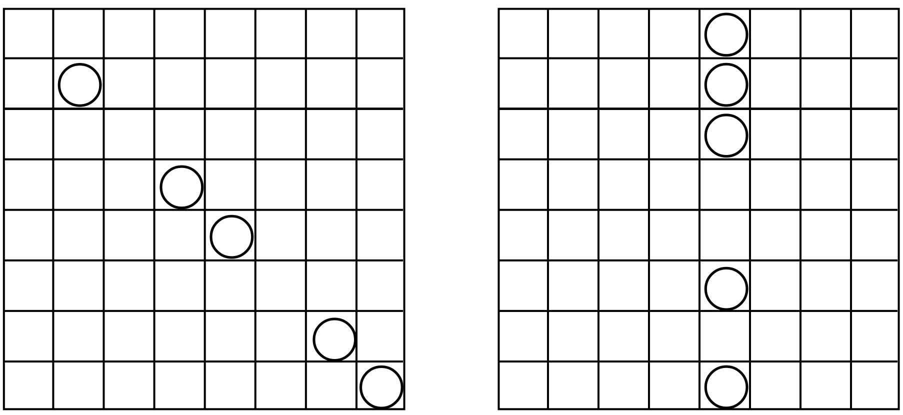
Note that you could ask students what sums they can make that are not on the board or which board numbers are hardest to get.

Page 2: Dicing Times

Game

Students can practise multiplication facts through this activity. This means that they must either know most of the facts involved or have some way of visualising them (for example, 10 x 10 array of dots).

To introduce the game to the class, model it with two students. This will help clarify the winning possibilities. Note that the five counters in line can be placed vertically, horizontally, or diagonally and do not have to be adjacent. For example:



Game

This is an excellent strategy game for practising basic addition facts. To introduce the game, begin with its simplest form, two columns, and model the progression of a game with two players. Most two-column games will end with the numbers 5, 6, or 7 placed. Here is another example:

A	B
1	

⇨

A	B
1	
2	

⇨

A	B
1	3
2	

⇨

A	B
1	3
2	4

At this point in the game, the even-numbered player could be attempting to trap the odd player at 7 by thinking ahead.

A	B
1	3
2	4
	5

⇨

A	B
1	3
2	4
6	5

The odd-numbered player is beaten because he/she is unable to write 7 in any column.

This "think ahead" strategy becomes more pronounced in the three-column game. With three columns, the games usually end with numbers about 14, 15, or 16 unable to be placed. If games progress into the twenties, students have usually missed pairs of numbers that block a number being written. This provides another opportunity for basic addition practice. For example:

A	B	C
1	2	4
3	5	7
6	8	12
10	9	13
	11	

This game has gone on for too long.
Which player has missed a chance to win?

Page 4: 31 or None

Game

This activity is based on an ancient Chinese game in which players had to make given totals by using four single-digit numbers and the four operations. Thirty-one is a good target as there are many ways in which students can reach this score. Students could use a tally sheet to record who has the highest score in each round.

After students have played the game several times, the target number can be changed. Students may wish to describe why it is easier to get a target of 31 than a target of 13. Similarly, they can be given a target total such as 27 and asked to write as many four-digit combinations for it as they can.
For example:

$(4 \times 5) + 3 + 4 = 27$ $\quad$ $9 + 6 + 4 + 8 = 27$

$(7 \times 6) - 9 - 6 = 27$ $\quad$ $(8 \times 5) \div 2 + 7 = 27$

Where students use different types of calculators, particularly scientific and four-function, the issue of order for operations may arise. For example, given $4 + 6 \times 3 - 5 = \square$, a scientific calculator will get 17 whereas a four-function calculator will get 25. Four-function calculators perform calculations in the order of keying, that is, $4 + 6 = 10$, then $10 \times 3 = 30$, and then $30 - 5 = 25$.

A scientific calculator uses the convention for operations, that is, multiplication and division are calculated before addition and subtraction. With $4 + 6 \times 3 - 5 = \square$, the $6 \times 3 = 18$ is performed first, and then 4 is added and 5 subtracted: $18 + 4 - 5 = 17$.

Activity

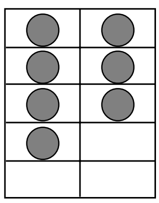
Students will need to approach this systematically. A table would be useful.

Mon	4
Tues	8
Wed	16
…	…

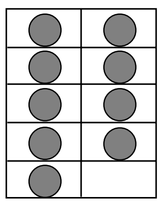
Page 5: Fizzing It Up

Activity

Tens frames are particularly good aids to help students see patterns that connect the basic addition and subtraction facts. Students should have enough prior experience with tens frames to be able to instantly recognise patterns of numbers up to 10 when they are quickly shown to them and then hidden. (This is known as "subitising".) An interesting result of this is that students develop a "subtract from ten" view of the numbers six to nine, that is:

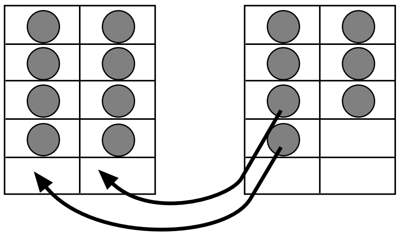


7 = 10 – 3; seven full spaces, three empty.



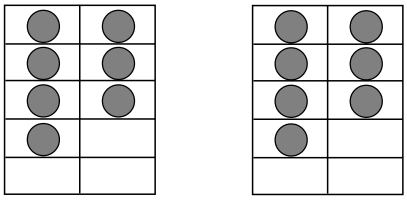
9 = 10 – 1; nine full spaces, one empty.

This facilitates the development of an "up over ten" strategy for the more difficult addition facts. For example, consider 8 + 7:



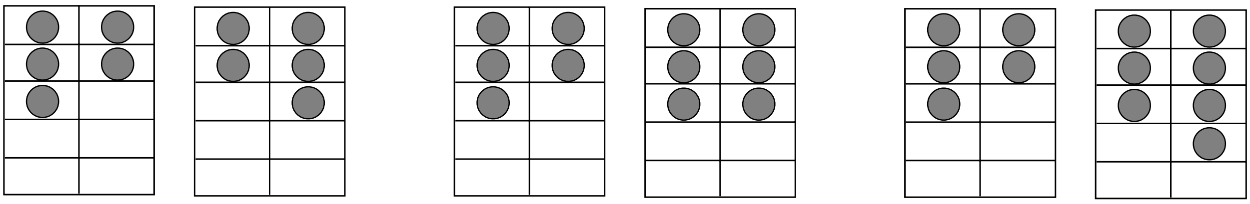
The student may visualise two counters from the tens frame with seven in it being placed into the tens frame with eight in it, making a full 10 plus five.

You could ask students to draw a picture in the air of how they worked out solutions. Another possible strategy for 8 + 7 is a comparison with doubles patterns:



Seven and seven are 14, so seven and eight is one more than that … 15.

The activity on page 5 suggests a growing series of tens frames. It is also a useful method for learning basic facts through pattern. For example:



5 + 5 = 10 5 + 6 = 11 5 + 7 = 12

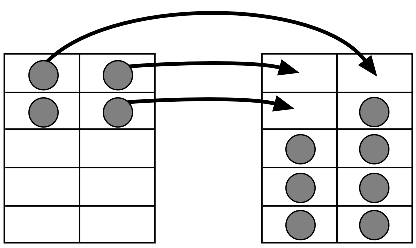
Page 6: Add It On

Activity

Self-testers are an excellent way for students to review which facts they can recall and which they cannot. It is important that students record the facts they do not know for future learning.

The following strategies can assist learning:

- Do you know another fact close to this one (the unknown)? For example, you don't know 4 + 7, but you do know 4 + 4.
  How could you get from (known fact) to find the answer to (unknown fact)?
- Picture what the numbers look like in tens frames.
  Can you make a full 10, with some over?
  Draw a picture of this in your book.



4 + 7 = 10 + 1 = 11

- What other new facts will you be able to work out from (unknown fact)? For example, 4 + 7 = 11, so 5 + 7 = 12, 4 + 8 = 12, 3 + 7 = 10, 4 + 6 = 10, and 11 – 4 = 7, 11 – 7 = 4, etc.

Page 7: Adding Adders

Activity One

Students may enjoy exploring patterns on a copy of the addition array. For example, the odd and even answers can be shaded with different colours. This produces a diagonal pattern that can be explained by the fact that the answers in each row and column grow consecutively by one. Since odd and even numbers alternate, this produces the odd-even-odd-even shading pattern.

Similarly, students can shade all the spaces with answers of 10, for example, and write the equations:

1 + 9 = 10

2 + 8 = 10

3 + 7 = 10.

They may notice that as one addend increases by one, the other decreases by one.

Subtraction patterns can be investigated by taking an addend (for example, six) and looking across the row to create subtraction equations with that answer.

5				
6	7	8	9	10
7				

This creates patterns like this:

7 – 1 = 6

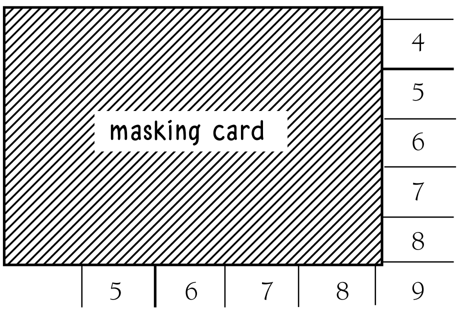
8 – 2 = 6

9 – 3 = 6.

As the number taken from increases by one, the number that is taken must also increase by one to get the same answer. This can be extended with problems such as:

	7 – 1 = 6		9 – 1 = 8
so	12 – ❑ = 6	so	12 – ❑ = 8
so	14 – ❑ = 6	so	17 – ❑ = 8

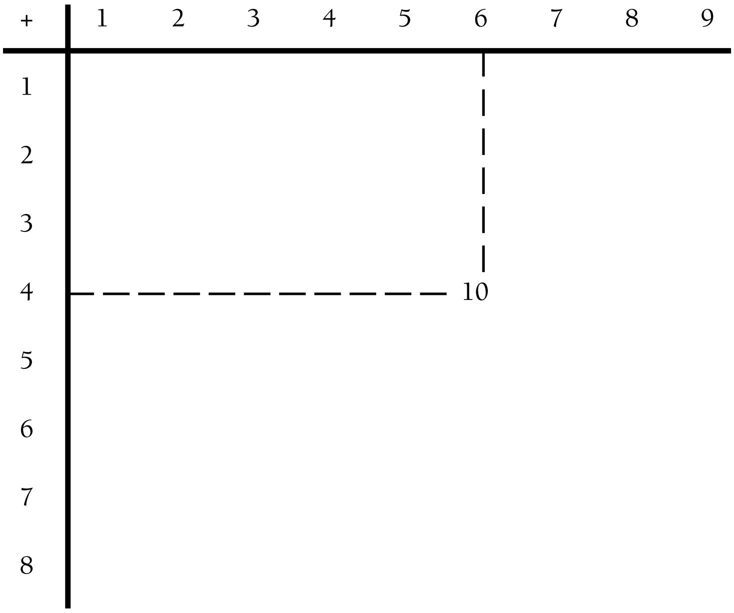
For students who have difficulty reading the array, a rectangular masking card can be useful.



Visually, this helps students to read off 4 + 5 = 9 etc.

Activity Two

Using an addition array to solve subtraction problems involves the idea of an inverse operation. In this case, subtraction and addition are inverses, that is, one undoes the other. Adding three, for example, can be undone by subtracting three. Any particular triad of numbers in the addition array can be used to solve a variety of problems. For example, consider the triad 4, 6, and 10:



6 + 4 = ❑	10 – 6 = ❑
4 + 6 = ❑	10 – 4 = ❑
4 + ❑ = 10	10 – ❑ = 4
6 + ❑ = 10	10 – ❑ = 6
❑ + 4 = 10	❑ – 4 = 6
❑ + 6 = 10	❑ – 6 = 4

Page 8: Time's Up

Activities similar to those suggested for page 7 can be introduced. Shading odd and even answers on a copy of the multiplication array produces a pattern of a few odd numbers surrounded by even numbers. Students should be encouraged to explain why this occurs. This may lead to general rules about the product of numbers:

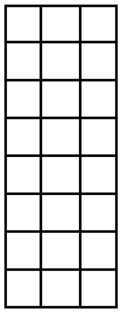
O x O = O   O x E = E   E x O = E   E x E = E

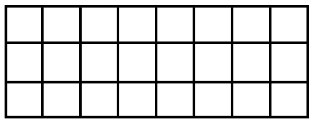
where O is any odd number and E is any even number.

Students might look for factors of a given product, say 24 or 18, write the equations, and look for patterns:

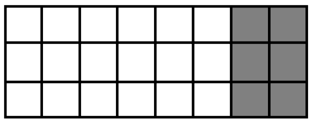
3 x 8 = 24	2 x 9 = 18
4 x 6 = 24	3 x 6 = 18
6 x 4 = 24	6 x 3 = 18
8 x 3 = 24	9 x 2 = 18

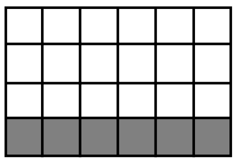
These equations might be modelled with square tiles to investigate why the patterns occur. For example, 3 x 8 and 8 x 3 can be seen as rotations of each other:





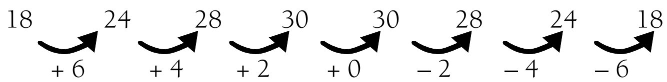
3 x 8 can be mapped onto 4 x 6.





Students might also investigate the numbers along the diagonals of the table, for example:

4, 9, 16, 25, 36, … (square numbers)



2 x 2 squares of numbers can also be looked at for patterns. For example:

4	6
6	9

4 x 9 = 36
6 x 6 = 36

4 + 9 = 13
6 + 6 = 12

25	30
30	36

25 x 36 = 900
30 x 30 = 900

25 + 36 = 61
30 + 30 = 60

Students may need calculators to find the products. Note that the products of the numbers on each diagonal are equal, and the sums of these numbers have a difference of one.

Page 9: An Apple a Day

Activities One and Two

The array view is probably the most powerful visual model for multiplication. It has important connections to measuring area and volume and to finding possible outcomes in probability.

Given an array, as in the apple tray example, students will demonstrate various strategies for counting the objects. These progress in sophistication:

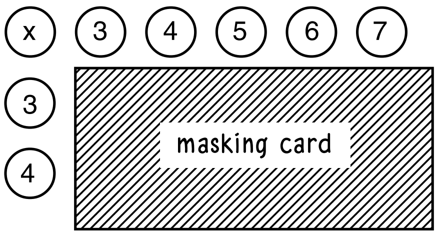
i. One-by-one counting

ii. Skip counting of rows or columns, for example, 4, 8, 12, …

iii. Equal additions, for example, $4 + 4 + 4 + 4 + 4 + 4$

iv. Use of multiplication facts, for example, 6×4.

The above stages constitute a developmental sequence for students that can help teachers design appropriate interventions. Students at the first stage need experience with stressed and then skip counting, that is, 1, 2, **3**, 4, 5, **6**, 7, 8, **9**, … to 3, 6, 9, … Students at the third stage need experience with arrays being masked. For example:



This encourages students to visualise the pattern and consider both factors, 6 and 3, simultaneously.

Where students are given instructions to create arrays for a given number of objects, say 24, you can develop the use of pattern by recording their results on a table:

wide	1	2	3	4	6	8	12	24
long	24	12	8	6	4	3	2	1

Students may notice that a square array can be made only for numbers like 9, 16, and 25.

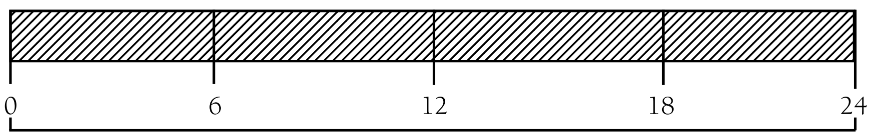
Page 10: On Track

Activity One

This activity uses a length model for multiplication by getting students to make Cuisenaire rod tracks. Students who are not familiar with the length of the coloured rods can easily check these on the ruler.

Building tracks reinforces the links between the operations (see the relationships diagram for page 7).

Consider the 24 centimetre track below:

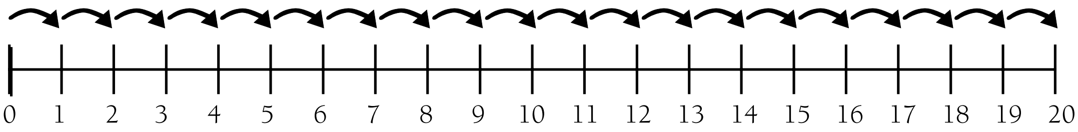


Equations that can be written about this track include:

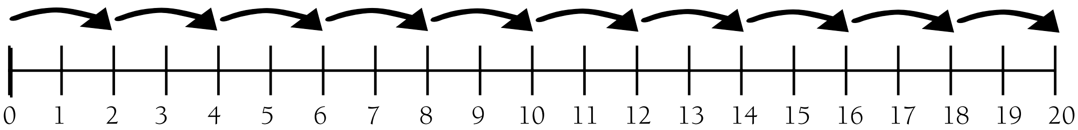
$6 + 6 + 6 + 6 = 24$	$4 \times 6 = 24$
$24 - 6 - 6 - 6 - 6 = 0$	$24 \div 6 = 4$

Activity Two

In this activity, students must systematically find all the possible tracks of a given length. This requires them to identify all the multiplication facts for that length. For example, with a track that is 20 centimetres long, the solutions are:



20 x 1 = 20

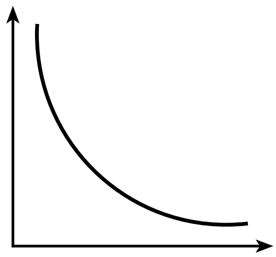


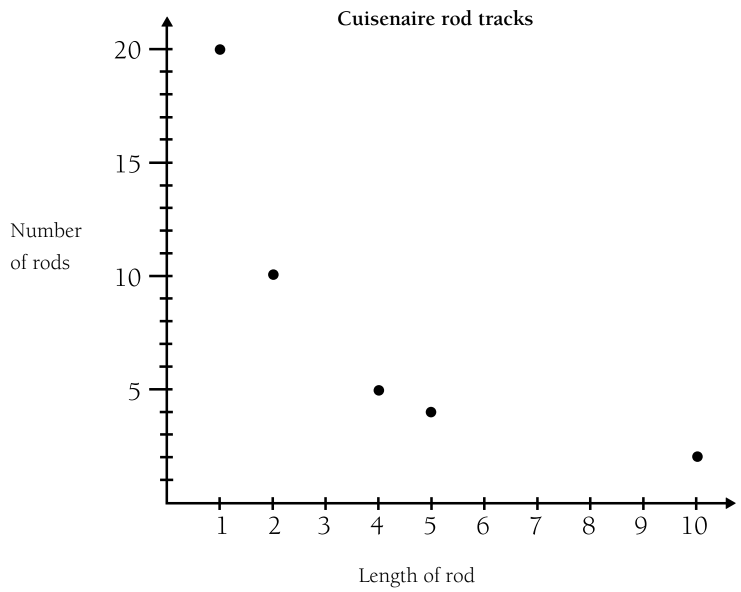
10 x 2 = 20

Results can be organised in a table:

length of rod	1	2	4	5	10
number of rods	20	10	5	4	2

This relationship produces an interesting pattern when graphed. The points lie along a hyperbola.





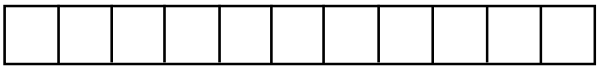
Page 11: Heading for Home

This game provides students with experience in simple division. It is interesting to see what the board numbers are exactly divisible by, that is, what they can be divided by to leave no remainder.

Students will realise from playing the game that some numbers are easier to move off than others. For example, 14 is troublesome as only dice throws of one and two will result in a move. Twelve, by contrast, has many factors, so throws of one, two, three, four, and six will result in a move.

Students enjoy creating their own board for this game. Remind students to choose the numbers carefully so that players do not become frustrated by a lack of moves.

To investigate prime numbers, put 11 and 17 on a board. Students will find that only a throw of one will result in a move. This is because prime numbers have only two factors, themselves and one. Note the connection between this and the number of rectangular arrays that can be made from a prime number of square tiles. With 11 tiles, only a 1 x 11 array is possible:



Page 12: Takeaway Numbers

Activity

Refer to the notes for page 6.

Encourage students to look for easy ways to subtract numbers. For example, subtracting nine is like subtracting 10 and adding one. For example:

$$\begin{array}{r} 14 \\ -\ 9 \\ \hline \end{array} \quad \text{is the same as } (14 - 10) + 1 = 4 + 1$$
$$= 5$$

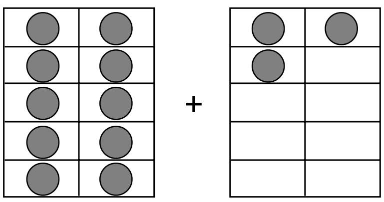
Similarly, subtracting eight is like subtracting 10 and adding two.

As with addition facts, encourage the students to link unknown facts with those they already know. Consider 13 – 7. Ask questions like:

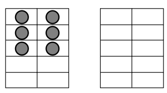
"Do you know 7 + ☐ = 13?"

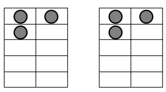
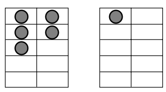
"Do you know 14 – 7 = ☐?" (using doubles)

"Look at these tens frames. Can you imagine 13 – 7?"



Students will provide a variety of answers.

Given these tens frames, most will probably imagine 13 – 7 as  .

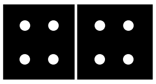
Some will imagine the answer as  or  .

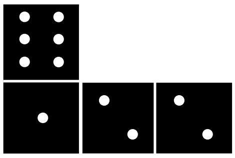
Page 13: Domino Facts

Game

Due to the constraints with the numbers on a standard set of dominoes, the facts practised here are limited to elementary multiplication and division up to 3 x 2 = 6.

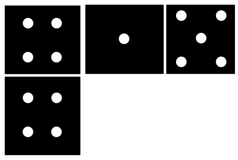
After some games, the students will realise the rules that apply to placing some dominoes.
For example:

- A "double" domino, such as , can only be joined to a single dot on a domino. For example:



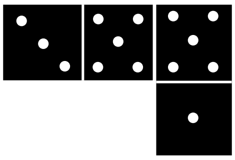
1 x 2 = 2

and



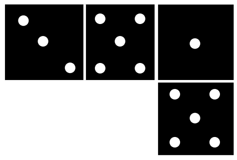
1 x 4 = 4

- Five-dot dominoes are difficult to match.



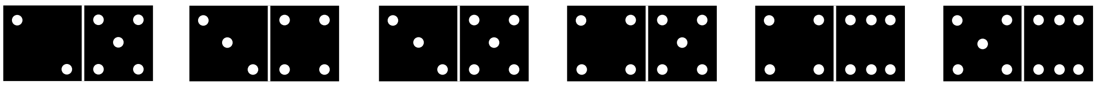
5 ÷ 5 = 1

or

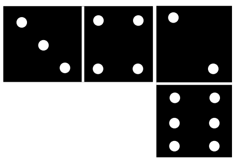


5 x 1 = 5

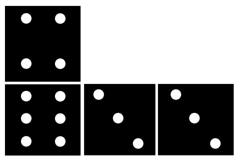
- Some combinations cannot be matched to anything. These are:



As an alternative game, the rules for joining can be adjusted to include all the operations. This would make it possible to join all dominoes to others. For example:



4 + 2 = 6



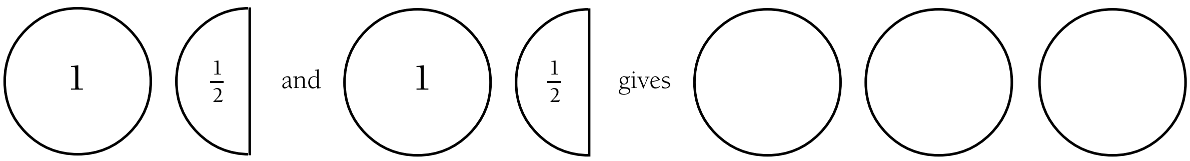
6 – 3 = 3

Page 14: Flying-fish Soup

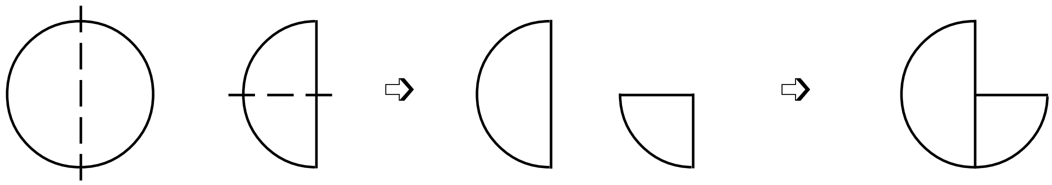
Activity

This problem page uses ratio, which is a difficult concept for many students. Ratio involves the idea that the relative proportions of two or more things stay constant as the quantities change. For example, someone mixing concrete might use five shovels of builders' mix to one shovel of cement. This ratio can be expressed as 5:1. Using this "recipe", the builder might put 20 shovels of builders' mix and four shovels of cement into the mixer. Although the quantities are 20 shovels to four shovels, the ratio remains the same.

In the case of making Pretend Flying-fish Soup for eight people, the recipe for four people is doubled. The quantity of each ingredient is therefore doubled. Finding two times one and a half can be modelled with regions:



Conversely, for question **2**, the recipe quantities are halved. Half of one and a half can be demonstrated by folding one and a half paper circles in half.

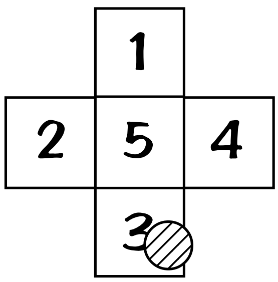


So half of one and a half is three-quarters.

Page 15: Twenty-seven

Game

This is an excellent strategy game as well as providing a good opportunity for basic facts practice. Students can investigate winning positions, for example:



The score is 19. It is Joel's turn. Can he win? How?

Students will need to look at the implications of Joel's potential moves: to 4, with the score on 23; to 5, with the score on 24; to 2, with the score on 21. If he moves to 4, his opponent will push the counter to 3. This will trap Joel since any next move will push the score over 27. If he moves to 5, his opponent will push the counter to 3 to make the score up to 27. If he moves to 2, his opponent will go to 3 and put the score to 24. Joel can then only go to 2, leaving his opponent to move to 1 and bringing the score total to 27. In short, Joel cannot win.

Other winning positions depend on the number the counter is on at a particular total.

Page 16: Pata Tika

Game

This traditional game involves students thinking logically about the possible digits in a two-digit whole number. It does not demonstrate an understanding of place value: it demonstrates systematic thinking. As a prop to learning the terms “pata” and “tika”, coloured counters can be used in an analogy to traffic lights.

R red (wrong digit, wrong place)

O orange (right digit, wrong place – “pata”)

G green (right digit, right place – “tika”)

A three-digit game might progress like this:

4 5 6	O	R	R	The digit 4 is correct but in the wrong place.
1 4 7	R	O	O	Both the 4 and 7 are correct but in the wrong places.
2 7 4	R	O	G	4 is correct and in the right place; 7 is correct but in the wrong place.
7 9 4	G	R	G	7 and 4 are correct and in the right places.
7 3 4	G	G	G	7, 3, and 4 are correct and in the right places.

Recording systems can help students by easing the load on their working memory. For example,

0 1 2 3 4 5 6 7 8 9

can be used to eliminate non-applicable digits as they are cancelled out by red counters (or crosses).

Page 17: Secret Codes

Activities One and Two

Students will enjoy making up their own codes for others to solve. It is important for them to realise that the answers that match given letters must have a number of possible facts in case that letter is used more than once. A process for making such codes is:

Write a message.

⇩

List the letters used.

⇩

Assign an individual answer to each letter.

⇩

Write a basic fact for each letter in the message.

⇩

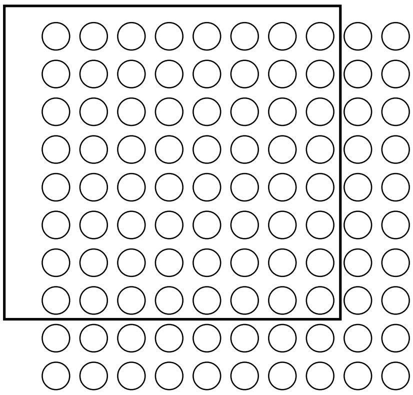
Check the code to see that it works.

Page 18: Magnificent Multiples

Activity

Refer to the notes for pages 6 and 12.

Encourage students to list facts they do not know and attempt to learn them. It is beneficial to focus on pattern (see pages 8 and 9 of *Basic Facts*). For example, if a student knows 8 x 8 = 64 but does not know 9 x 8 = 72, provide the student with a 10 x 10 array of objects and two masking cards:



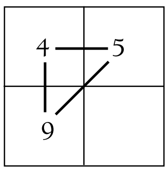
Ask the student to use the masking cards to show 8 x 8.
Then ask the student questions such as:
"How many is that altogether?"
"How many more will you need to uncover to show 9 x 8?" (another row/column of eight)
"How much will that be?" (64 + 8 is 72)

You could also link this unknown fact to 10 x 8 = 80 and ask "How much less is 9 x 8?" Students may wish to make flash cards of their unknown facts with the facts on one side and the answers on the other. Students may self-test or work with a partner.

Page 19: Stay on Line

Game

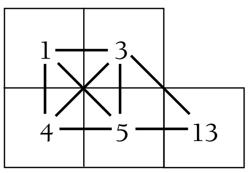
This activity is a number version of the word game Boggle. Encourage students to be more creative with their equations by introducing a points scoring system, one point for every number used in an equation. This will push them to strive for complex equations, such as 4 + 5 + 8 = 14 + 3. The game also develops the "family of facts" idea.
For example, from this triangle:



the following equations can be written:

4 + 5 = 9   5 + 4 = 9   9 – 4 = 5   9 – 5 = 4

With this arrangement:



many equations are possible, such as:

13 – 3 = 1 + 4 + 5   13 – 5 = 4 + 3 + 1   1 + 3 + 4 + 5 = 13

After a few attempts at playing the game, you could ask students to make up their own board. This is best done by starting with four single numbers in the centre squares and placing larger numbers around them to give the greatest possible number of equations. For example:

	4	3	
	5	2	

⇨

		7	8
	4	3	6
7	5	2	
	9		

⇨

20	13	7	8
11	4	3	6
7	5	2	10
16	9	12	16

Page 20: A Sticky Problem

Activity

This page involves a series of connected word problems that can be solved by using basic multiplication and division facts. Have multilink cubes, counters, or similar equipment available to act out the problems.

Encourage students to record their answers and the strategies they used to obtain them. For example, question **2** might be solved in many ways (see notes for page 9):

6 + 6 + 6 + 6 + 6 + 6 = 36 ⇦ so there will be four left over

6 + 6 + 6 + 6 + 6 + 6 + 6 = 42 ⇦ too many – four in last bag

6 x 6 = 36

6 x 7 = 42

Students at the one-by-one counting stage will count 40 objects and count away sets of six. It is important that students are shown the efficiency of multiplication strategies.

Similarly, questions **4–6** can be solved by counting methods with toy money or by calculation. Some students will count out 40 10-cent pieces and attempt to find the total by counting in tens. Encourage them to use more powerful strategies by asking questions such as:

"How many pieces of fudge must Rachel sell to get $1? How do you know?"
(10, because 10 10-cent coins are $1)

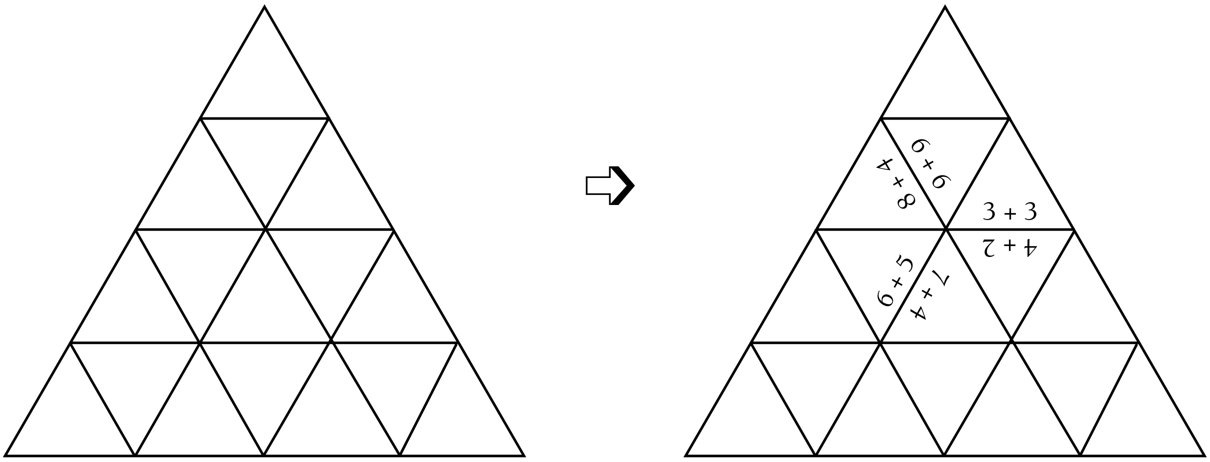
"How many pieces has Rachel got?" (40)

"How much money will that be?" ($4)

Page 21: Testing Triangles

Activities One and Two

These activities are matching games for basic addition facts. There is a copymaster of both a blank and completed set of triangle tiles at the back of this booklet. If the students are drawing up their own jigsaw puzzles, they should begin by marking out a composite triangle on isometric dot paper, like this:



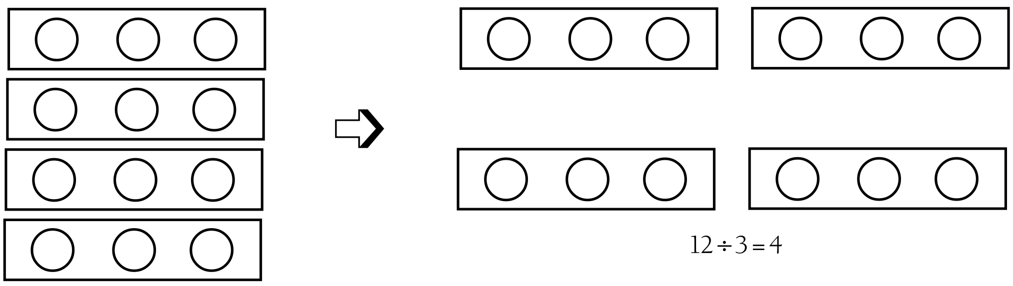
Mark out triangle. Fill in the facts.

Before they cut up the triangle, get students to draw a model solution plan to keep on the back of the triangle pieces' storage envelope.

Page 22: Division Revision

Activity

Refer to the notes for pages 6, 12, and 18. Arrays are a powerful model for investigating patterns. For example, it is useful to make a set of strip cards that can be removed in multiples.



12 ÷ 3 = ? 12 ÷ 3 = 4

Encourage students to link their knowledge of basic multiplication facts to division by varying the form of the problems presented to them.

4 x 6 = ❑	"Four sets of six are …?"
4 x ❑ = 24	"Four sets of what are 24?"
❑ x 6 = 24	"How many sets of six make 24?"
24 ÷ 4 = ❑	"24 divided into sets of four. How many sets?"
24 ÷ ❑ = 6	"24 divided into equal sets. Six sets are made. How many in each set?"
❑ ÷ 4 = 6	"Some things have been divided into sets of four. Six sets were made. How many were there to start with?"

Page 23: Loopy

Game

Loopy is a competitive game, so it is important that students of similar proficiency play together. One way to alleviate the competitive element is to deal out the cards so that each student has their own set of cards. For this alternative, a starter card needs to be created that will look something like this example:

Start
4 x 5

The player getting the Start card reads out the fact and turns the card over. Other players search for the answer in their cards, and the first player to call out the answer (provided they have it!) is the next caller, for example:

20
5 x 3

Then they will read out the fact at the bottom of the card and turn that card over. The first player to turn over all their cards is the winner or, alternatively, play can continue until all players have finished using their cards.

Sets of Loopy cards can easily be made for any particular collection of basic facts. Follow these steps:

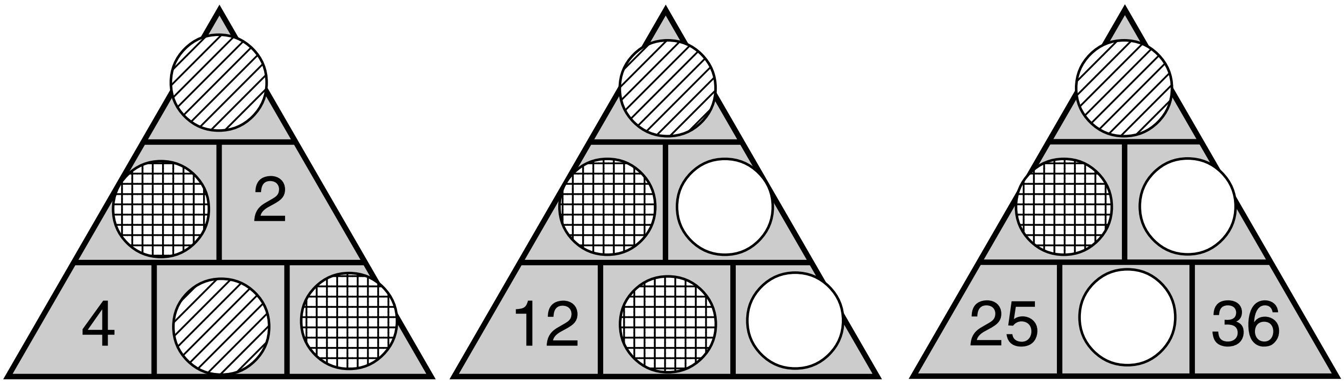
i. Cut up about 40 cards and write the facts on them. Include reversals of order.	ii. Write the answer to each fact on a different card so that it is not the correct solution to the fact below it. Use a marker to help keep track of which facts have had answers supplied.	iii. Make up the starter card with a fact that has its answer in each set of cards.
[ ] / 3 x 7 [ ] / 7 x 3 [ ] / 3 x 6 [ ] / 6 x 3 : [ ] / 9 x 3	18 / 3 x 7 18 / 7 x 3 21 / 3 x 6 27 / 6 x 3 : 21 / 9 x 3	Start / 9 x 2

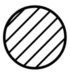
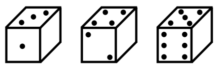
Page 24: Six Shooters

Game

This game involves basic multiplication facts. The use of three dice allows students some choice in manipulating which fact they select. A key point of the game is that as the game progresses and more counters are placed, the number of options decreases.

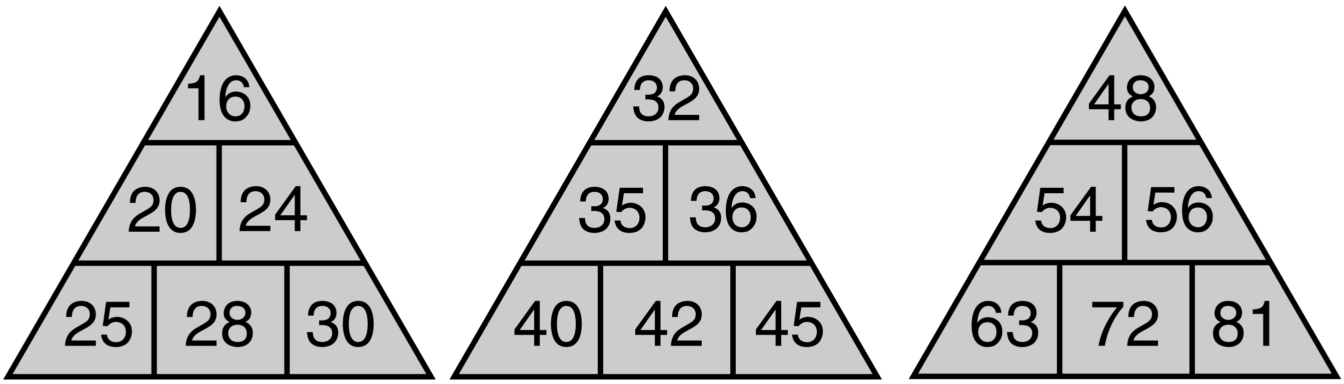
Once a counter is placed, it cannot be removed, nor can that answer be used by another player. Consider this situation:



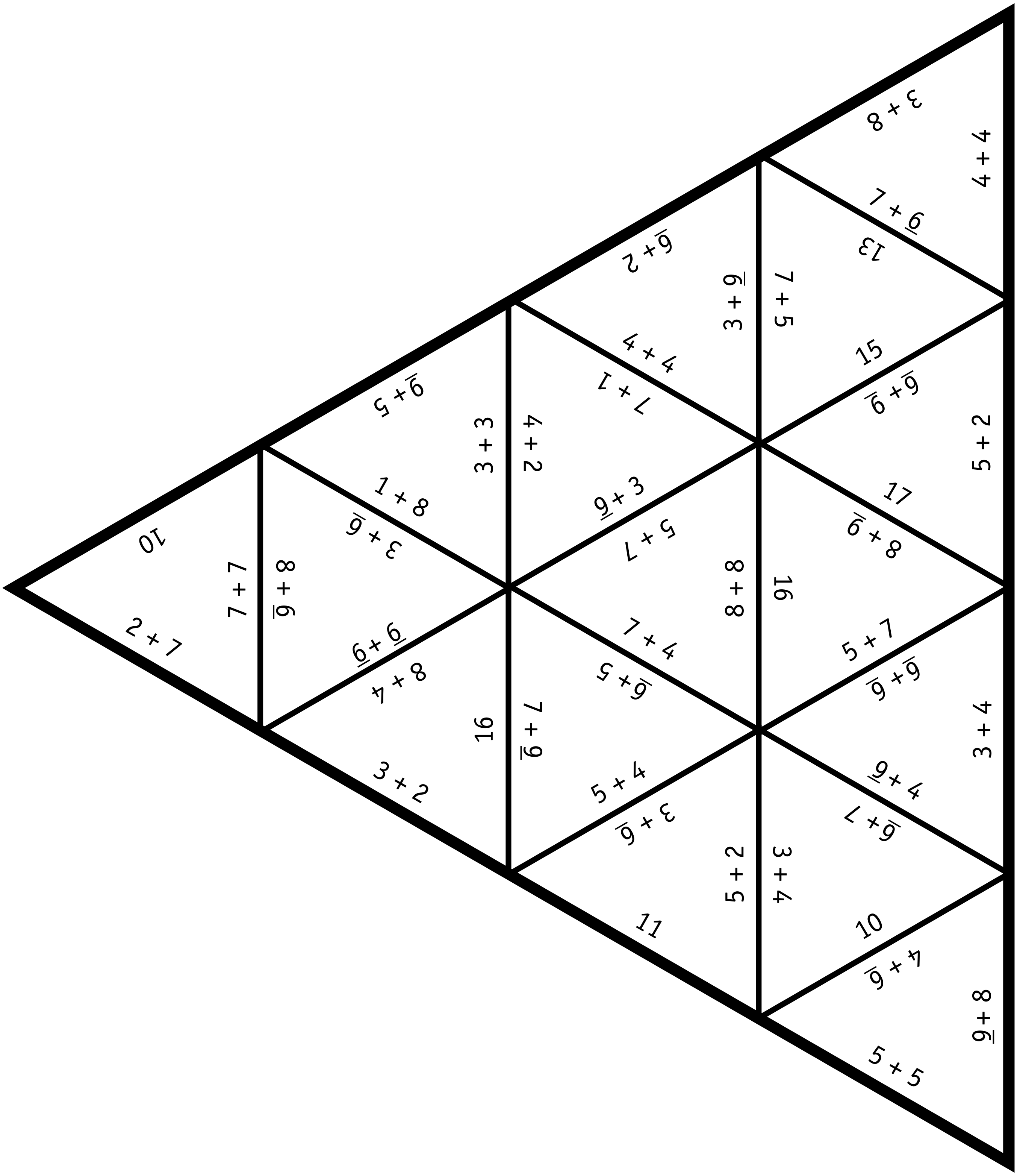
Moana is using  counters and throws: 

She rejects the five to form 4 x 3 = 12, which will give her two counters in the middle triangle.

To practise more difficult multiplication facts, the dice can be modified so that they have the numbers 4, 5, 6, 7, 8, and 9 on the faces. The triangles should also be modified to become:

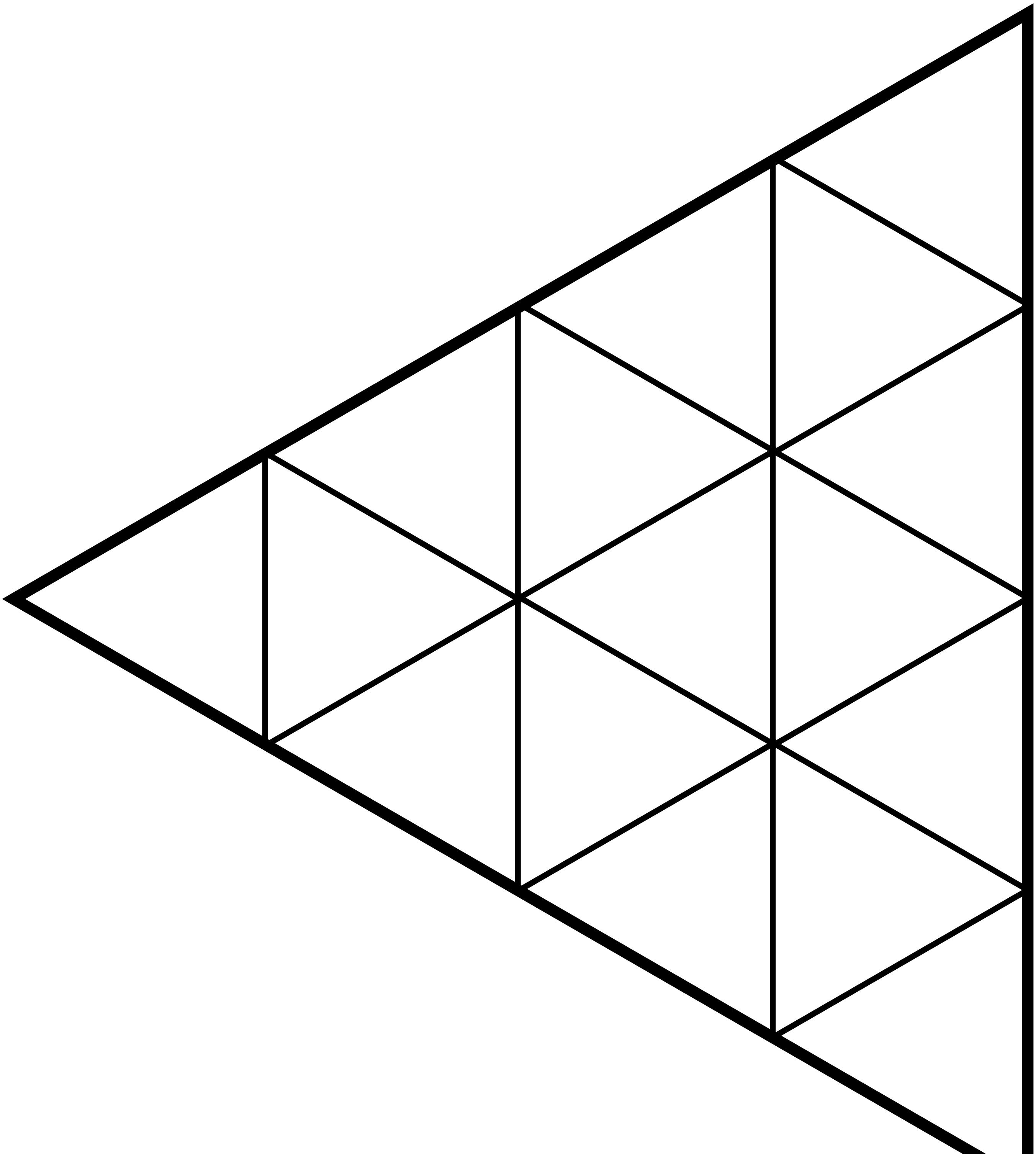


Copymaster: Testing Triangles



A blank triangle tiles sheet is on the following page. ⇨

Copymaster: **Testing Triangles**



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