## Answers and Teachers' Notes



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MINISTRYOFEDUCATION
Te Tāhuhu o te Mātauranga

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## Introduction

The Figure It Out series is designed to support Mathematics in the New Zealand Curriculum. The booklets have been developed and trialled by classroom teachers and mathematics educators. The series builds on the strengths of a previous series of mathematics booklets published by the Ministry of Education, the School Mathematics supplementary booklets.
Figure It Out is intended to supplement existing school mathematics programmes and can be used in various ways. It provides activities and investigations that students can work on independently or co-operatively in pairs or groups. Teachers can select particular activities that provide extension to work done in the regular classroom programme. Alternatively, teachers may wish to use all or most of the activities in combination with other activities to create a classroom programme. The booklets can be used for homework activities, and the relevant section in the teachers' notes could be copied for parents. These notes may also provide useful information that could be given as hints to students. The teachers' notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community
There are nine booklets for levels 3-4; one booklet for each content strand, one on problem solving, one on basic facts, and two theme booklets. Each booklet has its own Answers and Teachers' Notes. The notes include relevant achievement objectives, suggested teaching approaches, and suggested ways to extend the activities and link them with other curriculum statements. The booklets in this set (levels 3-4) are suitable for most students in year 6. However, teachers can decide whether to use the booklets with older or younger students who are also working at levels 3-4.
The booklets have been written in such a way that students should be able to work on the material independently, either alone or in groups. Where applicable, each page starts with a list of equipment that the students will need to do the activities. Students should be encouraged to be responsible for collecting the equipment they need and returning it at the end of the session.
Many of the activities suggest different ways of recording the solution to the problem. Teachers could encourage students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Where possible, suggestions have been made to encourage the discussion and oral presentation of answers, and teachers may wish to ask the students to do this even where the suggested instruction is to write down the answer.

The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.

Mathematics in the New Zealand Curriculum, page 7
Students will have various ways of solving problems or presenting the process they have used and the solution. Successful ways of solving problems should be acknowledged, and where more effective or efficient processes can be used, students can be encouraged to consider other ways of solving the problem.
The Figure It Out theme books are designed to explore mathematics around a unifying theme that draws on students' own life experiences. This means that the activities from the books can readily be used as part of broader units linked to objectives from other curricula.
Moving House features three children from families that are in transition and highlights some of the stresses and financial issues they encounter. Key Social Studies concepts touched on or explored include: access, management, needs, wants, rawa, resource, opportunity cost, change, distance, location, place, human rights, cultural interaction, and culture (Social Studies in the New Zealand Curriculum, page 14). A number of the activities will directly assist teachers in developing the financial literacy of their students.
Suggested links to other curricula are included at the end of the teachers' notes for most of the activities.

## Page 1: What's for Sale?

## Activity

1. Answers may vary.

First choice for Robert's family: "Luxury in Kurapo"; second choice: "Rural Kurapo". (Although advertised at $\$ 203 \mathrm{~K}$, an offer of less than $\$ 200,000$ may succeed.)

First choice for Mika's family: "Downtown Taurapa"; second choice: "Great for Nana!"

First choice for Melissa's family: "Family Paradise, Taurapa"; second choice: "Fun in Taurapa". (Note that these two choices both meet all criteria, and either could make an acceptable first choice.)
2. a.-b. Practical activities

## Pages 2-3: Rent or Buy?

## Activity One

1. a .

|  | Mean | Median |
| :--- | :--- | :---: |
| Central Taurapa | $\$ 288,400$ | $\$ 293,000$ |
| Normandy | $\$ 194,400$ | $\$ 207,000$ |
| Fitzherbert | $\$ 352,200$ | $\$ 89,000$ |

b. Fitzherbert
c. Central Taurapa
d. The median price gives the more accurate impression, because one very high or very low price, such as the $\$ 1,400,000$ house in Fitzherbert, can push up the mean price in what is evidently a cheaper area.
2. a.

|  | Mean | Median |
| :--- | :--- | :--- |
| Central Taurapa | $\$ 300,000$ | $\$ 294,000$ |
| Normandy | $\$ 138,500$ | $\$ 122,500$ |
| Fitzherbert | $\$ 301,000$ | $\$ 107,500$ |

b. (i) Normandy
(ii) Normandy
3. Normandy, because prices there are rising the most. If current trends continue, it will show the biggest increase in the foreseeable future.

## Activity Two

1. a. $\$ 260$
b. $\$ 270$
2. a.

| Owning a house in <br> Normandy | Cost |
| :--- | :--- |
| Repayments on mortgage | $\$ 9,600(800 \times 12)$ |
| Rates | $\$ 1,170(195 \times 6)$ |
| Maintenance | $\$ 1,000$ |
| Insurance | $\$ 450$ |
| Annual cost of owning a <br> house (excluding deposit) | $\$ 12,220$ |

b. Owning a house is cheaper unless the rent is less than $\$ 235$ per week. Both the mean annual rent $(52 \times \$ 260=\$ 13,520)$ and the mode annual rent ( $52 \times \$ 270=\$ 14,040$ ) are higher than the annual cost of owning a house. Note that this excludes the deposit needed to buy a house.
3.

| Owning a house in <br> Normandy (after increases) | Cost |
| :--- | :--- |
| Repayments on mortgage | $\$ 10,440(870 \times 12)$ |
| Rates | $\$ 1,560$ |
| Maintenance | $\$ 1,000$ |
| Insurance | $\$ 450$ |
| Annual cost of owning a <br> house (excluding deposit) | $\$ 13,450$ |

Now the annual cost of owning a house is $\$ 13,450$, but this is still cheaper than the mean annual rent. Weekly rent would need to be less than $\$ 258.65$ to be cheaper than the cost of owning.

## Investigation

Answers will vary.

## Page 4: Garage Sale

## Activity

1. a. About $\$ 100$. Estimates will vary.
b. $\$ 100$
c. Yes, if he sells most things.
2. a. Suggested estimates of the prices discounted by $10 \%$ are:
Water gun: between $\$ 2.20$ and $\$ 2.50$
Flying saucer: $\$ 0.90$
Softball glove: \$9
CDs: $\$ 1.80$ each
Bike: \$17
Train set: \$7
Soccer ball: between $\$ 4$ and $\$ 4.20$
Books \$0.90 each
Soccer boots: between $\$ 9$ and $\$ 9.50$
Skates: $\$ 13.50$

Remember, some estimates are better than others, but there is no one "right" estimate. However, unless the estimate gives a fair idea of the correct amount, it is not good enough. Where the calculation is very simple, the estimate and the actual answer may be the same.
b. Take $10 \%$ off the original total of $\$ 100$ to get $\$ 90$.
c. $\quad \$ 90$ is greater than $\$ 89$, so he will have enough.
3. a. $45 / 89$
b. Yes, $\$ 45$ is over half of $\$ 89$.
c. Answers will vary and need not be written down.

## Page 5: Getting a Loan

## Activity

1. $\$ 660$
2. a. $\$ 1,500$
b. $\$ 300$
c. No, the bank manager will not give them a loan, because $\$ 660$ is more than a third of their income.
3. Yes, the bank will now give them a loan. The payments would drop to $\$ 528$ a fortnight, which is less than a third of Robert's parents' combined income.

## Investigation

Answers will vary.

## Pages 6-7: Hangin' Out

## Activity

1. a. Possible answers are:

| Mika to $\ldots$ | How to get there | Time | Cost |
| :--- | :--- | :--- | :--- |
| Oscar | 8 km bus (3 sections) and walk 3.5 km | 54 min. | $\$ 1.10$ |
|  | 11.5 km bike | 58 min. | Nil |
| Viliamu | 8 km bike | 40 min. | Nil |
|  | 6 km train and 1 km walk | 19 min. | $\$ 1.50$ |
| Simi | 6.5 km bike | 33 min. | Nil |
|  | 5 km bus (2 sections) and 1.5 km walk | 26 min. | $\$ 1.00$ |
| Mareko | 11 km train and 1 km walk | 25 min. | $\$ 2.50$ |
|  | 12 km bus (5 sections) and 0.5 km walk | 24 min. | $\$ 1.30$ |

b. Answers will vary depending on reasoning. Possible answers are:

Oscar: bus and walk ( 11.5 km is a long way to bike)
Viliamu: train and walk (quick and convenient)
Simi: bike (not too far, cheaper, no waiting for transport)
Mareko: bus and walk (cheaper, less walking)
c. Answers will vary depending on reasoning:

Simi and Viliamu are probably easiest to get to. The hardest person to visit is Oscar: when visiting him there is no way of avoiding a fair bit of biking or walking.
d. Individual discussion. Teacher to check
2. a. Possible answers are:

| To club | How to get there | Time | Cost |
| :--- | :--- | :--- | :--- |
| Mika | 4 km train | 5 min. | $\$ 1.50$ |
|  | 7 km bike | 35 min. | Nil |
| Oscar | 7 km bike | 35 min. | Nil |
|  | 3.5 km walk, 2 km bus, and 1 km walk | 57 min. | $\$ 0.90$ |
|  | 3.5 km bike | 18 min. | Nil |
|  | 1 km walk and 2 km train | 14 min. | $\$ 1.50$ |
| Simi | 3.5 km bike | 18 min. | Nil |
|  | 3.5 km walk | 42 min. | Nil |
|  | 1 km walk and 7 km train | 20 min. | $\$ 1.50$ |
|  | 1.5 km walk and 6 km bus | 27 min. | $\$ 1.10$ |

b. Answers will vary depending on reasoning. Possible answers are:

Mika: train (very short, convenient stations)
Oscar: bike (faster and cheaper)
Viliamu: bike (cheaper and only takes slightly longer)
Simi: bike (faster)
Mareko: walk and train (faster, less walking)
c. Individual discussion. Teacher to check

## Page 8: Moving On

## Activity

1. 

## Moving Company Quotes


2. a. Tautoko Removals (\$300)
b. For each of the three companies, take the base charge and add to this the additional charge per km (if any) for 25 km . The lowest result will indicate the cheapest company.
3. Move It (\$350)
4. a. $\$ 250+(\$ 2 \times \mathrm{km}$ travelled $)$
b. km travelled $\leq 80: \$ 350$
km travelled >80: \$350 + (3×(km travelled -80))

## Page 9: Your Half or My Half?

## Activity

1. a. $58 \mathrm{~m}^{2}$
b. $29 \mathrm{~m}^{2}$
2. a. There are numerous possible solutions. One is:

b.-c. Individual activity
d. The boundary with the fewest changes of direction and fewest "dead" corners is likely to be the best solution.
3. Answers will vary.

## Pages 10-11: Who Goes Where?

## Activity

1. a.-b. Solutions include:

Room Solution 1 Solution 2 Solution 3 Solution 4

| TV | Mika | Nana | Nana | Nana |
| :--- | :--- | :--- | :--- | :--- |
| Bed 1 | Mum | Mum | Mika | Eseta |
|  | \& Dad | \& Dad | \& Siaosi | \& Reita |


| Bed 2 | Nana | Reita | Eseta | Mika |
| :--- | :--- | :--- | :--- | :--- |
| Bed 3 | Eseta | Mika \& | Mum | Mum |
|  | \& Reita | Siaosi | \& Dad | \& Dad |
| Bed 4 | Siaosi | Eseta | Reita | Siaosi |
| Bed 5 | TV | TV | TV | TV |

The two biggest rooms are shaded.
Other possible solutions need to be checked against all the requirements.
c. Answers will vary.
2. Practical activity

## Pages 12-13: Melissa's Room

## Activity

1. Bed: $0.9025 \mathrm{~m}^{3}$

Desk: $0.375 \mathrm{~m}^{3}$
Bedside table: $0.1 \mathrm{~m}^{3}$
Wardrobe: $1.25 \mathrm{~m}^{3}$
Bookshelf: $0.1875 \mathrm{~m}^{3}$
The largest item is the wardrobe.
2. Practical activity
3. 3 m
4. Practical activity. The final arrangement should have the wardrobe fitting into the taller part of the room, but not against the wall with the door in it.

The diagram shows one possible layout.
5. Answers will vary depending on the way the furniture (particularly the bed) is arranged. The space left in the centre could measure approximately $2 \mathrm{~m} \times 2 \mathrm{~m}$, as in the diagram:


## Pages 14-15: Making the Team

## Activity One

| Job | Rate | Calculation | Total |
| :--- | :--- | :--- | :--- |
| Uncle's stall | $\$ 12.00$ flat rate |  | $\$ 12.00$ |
| Circulars | $\$ 4.75$ per day | $\$ 4.75 \times 9$ | $\$ 42.75$ |
| Mr Murray | $\$ 3.50$ per hour | $\$ 3.50 \times 6.5$ | $\$ 22.75$ |
| Neighbour | $\$ 3.25$ per hour | $(\$ 3.25 \times 8)+5$ | $\$ 31.00$ |
| Nana | $\$ 1.50$ per week | $\$ 1.50 \times 6$ | $\$ 9.00$ |
| TOTAL |  |  | $\$ 117.50$ |

The amount Mika earned was $\$ 117.50$.

## Activity Two

12.5 min . (Mika's practice takes $60+25+25=60+50$ $=110 \mathrm{~min}$., so he has to do 137.5 minutes of homework this week. He has already done $30+25+$ $15+35+20=125 \mathrm{~min} .137 .5-125=12.5 \mathrm{~min}$.

## Pages 16-17: Cosmo's Kennel

## Activity

1. Practical activity
2. Check that:

- all six pieces are there
- the edges that have to fit together are the same length (the roof, however, can overlap)
- the width of the two roof pieces put together is greater than the width of the kennel
- there isn't too much plywood being wasted
- the doorway is likely to be a suitable size for a dog.

3. a.-b. Answers will vary.
4. a.-b. Answers will vary, depending on the amount of plywood wasted. As 1 L of paint covers about $12 \mathrm{~m}^{2}, 500 \mathrm{~mL}$ will cover $6 \mathrm{~m}^{2}$. Robert wants to put three coats of paint on the kennel, so he will need the bigger can if the total surface to be painted is greater than $2 \mathrm{~m}^{2}$.

## Investigation

Answers will vary.

## Pages 18-19: To'ona'i

## Activity One

1. $\quad \$ 63.81$ (individual amounts rounded up)
2. $\$ 191.43$
3. The cost for 22 people is $\$ 175.48$. The total cost for 46 people would be $\$ 366.91$. Answers will vary depending on the rounding method used.
4. Find the cost per person and multiply it by the number of people at the party. Number $\times \$ 7.98$ $=$ total cost. (In practice, catering requirements are not worked out quite like this!)

## Activity Two

1. Ingredients for 15 people would be 375 mL of orange juice, 1.5 L of water, 750 mL of mango juice, 180 mL of lemon juice, and sugar to taste.
2. Eseta would need 9 times the amount of ingredients needed for 5 people or 3 times the amount calculated for 15 people. This would be 1125 mL or 1.125 L of orange juice, 4.5 L of water, 2250 mL or 2.25 L of mango juice, 540 mL of lemon juice, and sugar to taste.
3. Find the amount of each ingredient required per person and multiply that amount by the number of people at the party to get the total needed: number of people $\times$ ingredients per person $=$ total ingredients required.

## Page 20: Solar-powered Shower

## Activity

1. $\$ 480$
2. $a$.

## Cost of Shower Heating


b. In just over 6 years
c. 6.25 years ( 75 months or 6 years and 3 months)
d. No, she won't install a solar-powered shower.
3. a. After 4.17 years $\left(\frac{3000}{60}=50\right.$ months or 4 years and 2 months)
b. Yes, she will, because 50 months is less than 5 years.

## Page 21: Water Everywhere

## Activity

1. a. One possible answer, which uses 170 m of piping, is:

b. Answers will vary. A useful strategy is to take out the longest pipe you can without missing a crop. Repeat if possible.
c. Practical activity
2. Practical activity

## Pages 22-23: Moving Survey

## Activity

1. a .

| Number of students <br> leaving in a term | Tally |  |
| :---: | :--- | :--- |
| 0 | H/ H/H |  |
| 1 | H/ H H H/H |  |
| 2 | H/ // |  |
| 3 | $/ / /$ |  |
| 4 | $/ / /$ |  |
| 5 | $/ /$ |  |

b. The median is 1 student leaving per term.
c. A tally chart is a quick and easy way of sorting and counting the data, and the median can easily be found from it.
2.

Students Leaving Class in the Term

3. a. 1
b. i. The mode is the tallest bar.
ii. The mode has the most tallies.
4. 3 leaving a class in a term is not particularly common, but neither is it unusual. Data was collected from 10 classes; this represented a total of 40 school terms. In 8 out of those 40 terms, 3 or more students left.
5. Statistically, there is no problem with talking about 1.5 people even though common sense tells us that you cannot have half a person. The "average" is a mathematical idea designed to help make sense of lots of bits of data. It is rare for it to be a whole number.

## Investigation

1.-3. Answers will vary. Teacher to check
4. Whatever your result, there will be schools that would produce quite a different result from a similar survey. Reasons for a high rate of transfer could be:

- a lot of rental housing in the community
- recent closure of a major industry
- lack of employment opportunities in the area
- students shifting to a more popular school


## Page 24: Farewell Party

## Activity

1. Each slice of cake should be $1 / 3$ of the total size, as in the diagram:

2. a. $1 / 3$
b. $2 / 3$
3. a. $120^{\circ}$
b. $240^{\circ}$
4. a. $15^{\circ}$
b. Practical activity
5. a. 12
b. The cake can be divided using 7 cuts. First slice it horizontally, then use 6 vertical cuts to divide it into twelve $30^{\circ}$ sectors. This will give 24 equal pieces.


## Page 1: What's for Sale?

## Achievement Objectives

- $\quad$ explain the meaning of the digits in any whole number (Number, level 3)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, levels 3 and 4)


## Activity

Buying a house is a complex business in which a match has to be found between what the family wants, what they can afford, and what is available. The family starts with criteria in mind and will usually have to change or compromise some of these in order to make a purchase. In this activity, the students are given information about a number of houses and then make decisions based on different sets of criteria. In a second task, they define their own set of criteria and search for a "best fit" match in the newspaper.

One approach is for the students to set out the details for each property in a table with headings such as Price, Location, Number of bedrooms, and Other features.

Some students find larger numbers difficult to order because they tend to disregard the zeros. If this is a problem, ask them to place the house prices on a number line or place value chart.

The students could also use the inequality symbols to compare the different prices: < (less than), > (greater than), $\leq$ (less than or equal to ), $\geq$ (greater than or equal to). These symbols are valuable mathematical tools, and the students must become comfortable with their use. Emphasise that the bigger number is always placed at the open end ("mouth") of the symbol, and the smaller number at the pointed end. Emphasise also that a sentence including any of these symbols is always read from left to right (like a normal English sentence).

The price for the house in rural Kurapo is written "\$203K". Ask the students to explain what this means and to write the amount in words and digits (two hundred and three thousand dollars or $\$ 203,000$ ). Get them to collect other examples of the use of " K " to mean thousand (for example, car prices and car odometer readings).

Question 2 is very open; for this reason, discuss with the students the parameters they are to work within. For example, you may wish to suggest a maximum price or size of section. Without parameters, the exercise is likely to have little practical value for students. Bear in mind that those who come from lower socio-economic backgrounds should not be embarrassed by the way this activity is conducted.

## Links

This activity could form part of a unit on Moving.
Have the students research a historic homestead in the local area (or nationally through the Internet), identify a family that once lived there, and give possible reasons for their choice of location and house (for example, resources, work, lifestyle, finances, and proximity of extended family). The students could then investigate the reasons behind the purchase or rent of their own homes.

Suggested achievement objectives are:

## Social Studies

- how and why people experience events in different ways (Time, Continuity, and Change, level 4)
- how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)
- make decisions about possible social action (Social Studies Processes, Social Decision Making, levels 3-4)


## Health and Physical Education

- identify factors that affect personal, physical, social, and emotional growth and develop skills to manage changes (Personal Health and Physical Development, level 3).


## Pages 2-3: Rent or Buy?

## Achievement Objectives

- use their own language to talk about the distinctive features, such as outliers and clusters, in their own and others' data displays (Statistics, level 3)
- report the distinctive features (outliers, clusters, and shape of data distribution) of data displays (Statistics, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3 and 4)


## Activity One

This activity requires the students to find the mean (the sum of the prices divided by the number of prices, often known as the average) and the median (the middle value in any ordered list). These measures are specifically identified in a level 5 objective and will require discussion before the students start work. The terms are defined in a panel on page 2 of the students' book.

The students should list the amounts in ascending or descending order so that they can easily find the median.

Students should realise that both mean and median are best attempts to find a single number to represent a group of numbers. The main disadvantage of the mean is that one unusually big or small number in the group can greatly distort it. The main disadvantage of the median is that it totally ignores all numbers in the group except the middle one. Both are used in real life, and students need to understand the difference so that they are not misled by statistics in advertising or propaganda.
Fitzherbert is clearly an area of modestly priced houses. This is evident from the median but not from the mean, which is severely distorted by one house priced at more than 10 times any other advertised property. For this reason, the median is a better indicator of house prices in the area. Let your students explore how to manipulate the numbers in one of these lists (for example, Normandy), without altering (i) the mean, (ii) the median, or (iii) both. They will discover that as long as the total of the asking prices remains the same, so will the mean, and as long as the middle value in the list is unchanged, the median will remain the same.

In question $\mathbf{2 b}$, the answer is Normandy in both cases. However, be careful when discussing question 3. The students should note that six house sales is a very small statistical sample on which to base judgments or make projections. They should also note that house prices cannot be guaranteed to increase in line with the current trend. A downturn in the national economy, or local factors such as the closure of a major industry, or the development of a new access road or motorway bypass, can impact heavily on property values and sales.

## Activity Two

This activity is an exercise in comparing the costs of renting and buying. This time, the students compare the mean and mode (rather than the mean and median, as in Activity One). They may find it easier in question $\mathbf{1}$ to list the weekly rents in order before working out the mode (most common value) and mean. This scenario is, of course, a simplified one: it ignores the fact that a person buying a house ends up with a valuable asset while the person who rents has nothing to show for the years of payments.

In question 2, the students need to note that the costs relate to different periods ( $\$ 800$ per month, $\$ 195$ per 2 months, and $\$ 1,000$ per year). In the third column, they calculate the yearly equivalent for each cost.

To compare renting and buying, the students first need to either convert weekly rent to yearly rent or divide the cost of owning a house by 52 to get the weekly cost.

When doing question 3 , the students may find it helpful to make another chart using the new data.

## Investigation

The students could explore the first part of question $\mathbf{1}$ by using the For Rent section of the classified advertisements in the newspaper or by contacting a local real estate agency that manages rental properties. Newspapers also regularly publish tables showing the number and value of house sales in their readership area; these could provide the data needed for the second part of this question. The students need to think about the statistics such tables present. Do they use the mean or median sale value? Why is this? These pages also often show the percentage increase or decrease in sale value from one period to another. More able students could do a study of one of these tables and present the data graphically, using a computer spreadsheet. In question 2 , the students think about the advantages and disadvantages of renting and buying. There are opportunities to go into the issues as deeply as you or the students wish. They may like, for example, to investigate what difference it makes if they start saving for a house early on in their earning career.

## Links

This activity could form part of a unit on Safeguarding Consumers.
The students imagine that their family is moving to a town in Queensland, Australia and needs to rent a property. Using the Internet, they investigate how their family could go about finding and choosing a suitable property. What factors should they take into account? The students should consider the possible pitfalls and the safeguards that are available to protect them during and after the process.

Suggested achievement objectives are:

## Social Studies

- how and why individuals and groups seek to safeguard the rights of consumers (Resources and Economic Activities, level 4)
- why and how people find out about places and environments (Place and Environment, level 4).


## Page 4: Garage Sale

## Achievement Objectives

- $\quad$ explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 3 and 4)
- find a given fraction or percentage of a quantity (Number, level 4)


## Activity

Estimation is an extremely important mathematical skill, but students often have trouble with the concept behind it. An estimate is an approximation that is close enough to the actual value to give a good idea of its size, but which is easier to work with. Estimates are often used in quick mental calculations that would be too difficult with the original numbers.

What is "close enough" depends on the size of the number you are approximating. Saying that there were 35000 people at a rugby match is likely to be an acceptable estimate even if it is wrong by two or three thousand. In other situations, however, an estimate that is out by two or three thousand is completely unacceptable (for example, saying that there were 4000 people at an exhibition when there were only 1 000).

Students need to understand these ideas:

- When asked for an estimate, there is no one right answer, though there is often a best answer ( $\$ 20$ is the best estimate for $\$ 19.95$ ).
- The estimate must point towards the actual amount, not away from it ( $\$ 10$ is not an acceptable estimate for \$17.95).
- The range of acceptable estimates is proportional to the number being approximated. An estimate is normally within about 10 percent of the actual value, although there is no rule that stipulates this.

Before the students estimate the totals in question 1, discuss with them how they might go about it. Should they just consider the dollar amounts? Should they round up or down? What method might give the closest estimate? Give them plenty of examples for practice. Supermarket dockets are a ready source of real-life numbers for which students could estimate totals. (Blank out the actual totals first.)

Question 2 is about percentages and estimates. Ask the students what 10 percent means (one-tenth or 10 parts out of 100) before asking them to estimate what 10 percent of each item is. Discuss with them the idea of rounding the amounts before estimating, for example, rounding $\$ 9.95$ to $\$ 10$. Then 10 percent off $\$ 10$ gives $\$ 9$, which is a much easier calculation.

Question 3. Before the students reach for their calculators, encourage them to look at the numbers and see that $\$ 45$ is approximately half of $\$ 89$.

As a further investigation, the students could look at the sales advertisements of retail stores. What do tickets like these mean: "Up to $50 \%$ off", " 5 to $50 \%$ off everything", " $15 \%$ off", "At least $10 \%$ off"?

## Page 5: Getting a Loan

## Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 3 and 4)
- find a given fraction or percentage of a quantity (Number, level 4)


## Activity

In question $\mathbf{1}$, the students need only to read the table to see that the fortnightly mortgage repayments on a loan of $\$ 180,000$ are $\$ 660$. Question 2 involves straightforward division, multiplication, and addition to determine that the family's income per fortnight is $\$ 1,800$. A third of this is $\$ 600$. The mortgage repayment per fortnight would be $\$ 660$, exceeding the one-third limit, so the family would not get a loan.

For question 3, discuss with the students different ways of finding 20 percent of an amount (find 10 percent and double it, divide the amount by 5 , or find $1 / 5$ of the amount). Discourage them from using a calculator for easily calculated percentages such as these. For practice, they could go on and find 20 percent of all the fortnightly payments. It is important to emphasise that taking 20 percent off leaves 80 percent behind. A common mistake will be to suggest that the new mortgage repayment will be just $\$ 132$ (20 percent) instead of $\$ 660-\$ 132=\$ 528$.

## Investigation

When doing this open investigation, students will need to contact organisations such as the SPCA or a veterinarian or search for information on the Internet about different dogs and the related costs. Useful websites are: the New Zealand Veterinary Association, www.vets.org.nz and the Royal New Zealand Society for the Prevention of Cruelty to Animals, www.rspcanz.org.nz This could be an excellent project for a pet-loving student.

Further investigation about home loans could focus on the advantages and disadvantages of longterm and short-term loans. The students could work out the total amount to be repaid on a $\$ 180,000$ loan, using the figures from the table. Most will be amazed to learn how much larger this sum is than the original amount. They may also not realise that rates quoted by banks are not normally fixed for long periods, so the final total could be even greater.

## Pages 6-7: Hangin' Out

## Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- perform calculations with time, including 24-hour clock times (Measurement, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3 and 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)


## Activity

Before beginning this activity, check that the students understand the concept of scale: that is, using something small to represent something big. Maps, house plans, drawings of engines, stage sets, and model aeroplane kits - all use a scale. To be of any use, the scale must be clearly defined. In this case, it is 2 centimetres to 1 kilometre (commonly expressed as $1: 50000$ ).

Ask the students how they are going to work out the distances between the various homes, bus stops, and stations. They need to do this to complete the chart and make decisions. They could use:

- a piece of string, which will follow the curves and can then be measured using a ruler
- a narrow strip of flexible card with the scale marked on the edge; this can be bent to conform to the curves of the roads or the railway line
- a pair of dividers, or a small compass, set to 1 centimetre ( 0.5 kilometres).

To complete the table in question $\mathbf{1}$, the students need to be able to work out time, given a certain speed and distance. Discuss with the class how to do this. A good strategy is to first work out the time it takes to travel 1 kilometre by each of the four modes. For example, the students should be able to see that if it takes 60 minutes ( 1 hour) to walk 5 kilometres, it will take 12 minutes $(60 \div 5=12$ ) to walk 1 kilometre, so walking times become multiples of 12 minutes. Ask them to work out the time it takes to travel 1 kilometre by the other modes, as in the following table:

| Mode | Time to cover 1 kilometre |
| :--- | :--- |
| Bus | $1 \mathrm{~min} .30 \mathrm{~s}(1.5 \mathrm{~min})$. |
| Train | $1 \mathrm{~min} .12 \mathrm{~s}(1.2 \mathrm{~min})$. |
| Bike | 5 min. |
| Walk | 12 min. |

There is no right answer to questions $\mathbf{l b}$ or $\mathbf{2 b}$; different people will make different decisions about the best way to travel. Even in a building, one person will take the stairs, while another uses the lift. Exercise, convenience, and speed are all factors that may influence such choices. Discuss with the students the advantages and disadvantages of using each mode of travel: fitness, convenience, cost, time taken, bus or train schedules, and time of day. For example, is it OK to bike home from rugby when it is dark in winter? What is important is that the students consider all modes of travel and can justify their decisions.

As an extension activity, the students could work out the cost of visiting a friend or relative in a different part of New Zealand or overseas. They would need to find out the costs and schedules of the different modes of transport and then make decisions as to the "best" way to travel.

## Links

This activity could form part of a unit on Our Local Environment or A Great Place to Live!
Ask your students to investigate why the families of people they know have moved to New Zealand from overseas. They should collate the responses, noting places of origin, reasons for the change, three positive differences about living in New Zealand, and three challenges that the families have faced in their new country. If members of the class can gather sufficient data, the students could try and make generalised statements about immigrant experiences.

Suggested achievement objectives are:

## Social Studies

- how different groups view and use places and the environment (Place and Environment, level 3)
- how and why people express a sense of belonging to particular places and environments (Place and Environment, level 3).


## Page 8: Moving On

## Achievement Objectives

- $\quad$ sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- $\quad$ solve simple linear equations such as $2 \square+4=16$ (Algebra, level 4)


## Activity

This activity requires the students to graph the costs for three removal firms. Each firm has a base charge and a rate per additional kilometre. If the students haven't had experience with linear graphs, they may need to be taught the basics of plotting points and using them to determine a line. There is no reason why they can't work directly onto square grid paper. However, an alternative strategy is to ask them to make a table listing each of the three firms and their base charges, and the costs for the various distances, and to plot these results on the grid.

|  | Base charge | Rate per km | Cost 10 km | 20 km | 25 km | 100 km |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Move It | $\$ 350$ | $>80 \mathrm{~km}, \$ 3$ |  |  |  |  |
|  |  |  |  |  |  |  |
| Tautoko |  |  | $\$ 270$ |  |  |  |

The graph can be drawn on paper, but if the students have access to a computer, they could enter the data into a spreadsheet program and use the program's chart-making facility. The spreadsheet would look like this:

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | c | D |  |
| 1 | km | Birs Movers | Tauteko | Move it |  |
| 2 | 0 | 200 | 250 | 350 |  |
| 3 | 20 | 300 | 290 | 350 |  |
| 4 | 40 | 490 | 330 | 350 |  |
| 5 | 60 | 500 | 370 | 350 |  |
| 6 | 80 | 600 | 410 | 350 |  |
| 7 | 100 | 700 | 450 | 410 |  |
| 8 | 120 | 800 | 490 | 470 |  |
| 9 | 140 | 900 | 530 | 530 |  |
| 10 - 11 [ |  |  |  |  |  |
| 11 |  |  |  |  | $\cdots$ |
|  |  |  | TI |  | 1.11 |

Choose the XY (Scatter) style of graph (straight line option) from the chart menu, not the line graph, to avoid problems with the horizontal axis and its scale.
It is important that the students learn to "read" graphs for their story instead of just looking at them. What does it mean if a line is horizontal? Uphill? Downhill? What does it mean if there is a bend in the line? What does it mean if two lines run parallel to each other? If one is steeper than another? If two lines intersect? Do lines continue beyond the boundary of the graph? (Note that the students should not put arrowheads on lines to indicate that they continue beyond the edge of their graph.)

If a table has been created as above, writing formulae for question 4 should be straightforward: cost $=$ base charge + (number of kilometres travelled $\times$ cost per kilometre). Be aware, however, that because of the way Move It charges, two formulae are required: one for up to 80 kilometres and one for $>80$ kilometres. Discuss with the students the advantages of graphs compared to those of formulae. (See the notes on Solar-powered Shower, page 27, for further discussion of this issue.)

## Page 9: Your Half or My Half?

## Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- carry out measuring tasks involving reading scales to the nearest gradation (Measurement, level 4)
- calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (Measurement, level 4)


## Activity

The basement is a composite shape, so it can't be found using a simple formula. Discuss the possible ways of determining this area:

- counting all the squares (what do you do about part squares?)
- dividing the basement into rectangles, squares, and triangles and then working out the various areas using formulae (for example, area of rectangle $=$ base $\times$ height).

Having found its area, the students need to divide the floor into two equal parts. Ask them to think about factors they might consider before drawing up their boundary, for example, the location of the doors. Note that the question does not imply that a physical wall is to be constructed along this line, only that the space is to be shared equally. The students may have interesting ideas about how
the two areas could be delineated. Once they have completed the basic task, you may like to ask them to produce a boundary line that has at least one diagonal section.

In question 3, it is probably best to restrict the students to straight-line boundaries. Suggest that to calculate the area of each "half", they should split the large shape into suitable squares, rectangles, and triangles (as they did for question 2) and add the areas of these rather than just count the squares of the grid. Although this approach may demand more effort, it will reinforce higher order skills.

When they have completed the set parts of this activity, the students could draw their own scale diagram of the basement using a larger scale, with the room divided into two parts according to their preferred plan. They could then cut out scale rectangles to represent beds, desks, sets of drawers, and other space-consuming items and see how these could best be fitted into the available shape. Is the area large enough for three equal-sized bedroom spaces? How could the area be divided into thirds?

An interesting activity to help the students find half of a shape (which also introduces the idea of symmetry) is to find all the possible ways of dividing a 5 by 5 geoboard into two symmetrical halves. If geoboards are not available, dot paper in a 5 by 5 array can be used instead:


## Pages 10-11: Who Goes Where?

## Achievement Objectives

- draw and interpret simple scale maps (Geometry, level 3)
- read and construct a variety of scales, timetables, and charts (Measurement, level 4)


## Activity

Trying to get the best possible living arrangements in a home is an activity that many students will relate to. The solution will always require certain criteria to be met. One strategy is to write the names of all the people in the household on small pieces of paper. These pieces can be moved around on the plan of the house until an arrangement that satisfies all the requirements is found. Start with whatever is non-negotiable, then work with the flexible elements. In this case, the fixed elements are that:

- Nana is downstairs
- Mum and Dad have one of the two largest bedrooms
- either the boys or the girls share and are in the other large bedroom

It soon becomes clear that no solution can have the TV room downstairs, and it would be good if the students could articulate the reason for this. It is as follows: bedroom 1 is one of the two largest bedrooms and will therefore be occupied by either Mum and Dad or two children sharing; this means that if the TV were in either of the other downstairs rooms, it would be next to Nana, which is unacceptable.

If Mum and Dad are upstairs and the girls are sharing bedroom 1, the TV room could be either bedroom 4 or bedroom 5. If Mum and Dad are upstairs and the girls are not sharing, Reita needs to be in bedroom 4 next to them, so the TV room must be bedroom 5. Some students may like to try listing all the possible solutions.
Question 2 asks students to draw a plan of their own house. They could start this at home in order to get the layout reasonably accurate. You will need to specify whether you expect a scale plan. If this is so, make sure that your students know how to go about this, that they have the necessary equipment, and that they know how much detail is expected. The task does not require a scale plan; a tidy sketch showing the rooms and how they relate to each other will do. Once the plan is drawn, the next step is to rearrange things to accommodate an extra person. Remember that some students live in homes with rooms to spare, so no rearrangement may be needed, while others may live in overcrowded situations. Some sensitivity could be required to ensure that those from overcrowded situations don't feel unnecessarily exposed.

Introduce the students to other logic-type activities that require rearrangements according to set criteria. There are a number of good books containing "co-operative bit problems", where each member of the group has a clue card and the group has to pool the clues to solve the problem. These include:

- Erickson, Tim et al. (1989). Get It Together: Math Problems for Groups, Grades 4-12. Berkley: Lawrence Hall of Science, University of California.
- Goodwin, Jan M. (1992). Group Solutions and Group Solutions, Too! Berkley: Lawrence Hall of Science, University of California.


## Links

This activity could form part of a unit on Living Here.
Create a plan of an imaginary 5-bedroom home for your students. Get them to make a list of the family members in their own or an imagined household. They then write down the ages, physical and emotional needs, interests, and responsibilities of each. Finally, they decide who goes where in the imaginary house, explaining their reasons and the compromises they have had to make.

A suggested achievement objective is:

## Social Studies

- how and why people exercise their rights and meet their responsibilities (Social Organisation, level 4).


## Pages 12-13: Melissa's Room

## Achievement Objectives

- demonstrate knowledge of the basic units of length, mass, area, volume (capacity), and temperature by making reasonable estimates (Measurement, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)
- calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (Measurement, level 4)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)


## Activity

Before the students begin this activity, you may need to help them revise the concept of volume and how to calculate it. One way is to use connecting blocks (multilink cubes or centicubes) to make various shapes and let the students find the quickest way to work out the volumes. After finding
the volumes of the furniture, ask the students to arrange them in ascending order as a check that they have not lost a decimal place along the way. Some may find it helpful to put the volumes on a place value chart like this so that the greatest volume can be clearly identified:

| Item | Ones | Tenths | Hundredths | Thousandths | Ten-thousandths |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Bed | 0 | 9 | 0 | 2 | 5 |
| Wardrobe | 1 | 2 | 5 |  |  |
| Table | 0 | 1 |  |  |  |

Students often have trouble visualising volume and, as a result, don't know if their answer is sensible or not. One way of giving them a benchmark is to ask a group to construct the frame of a "cubic metre" and to put it on display in the classroom. There is an activity on page 15 of Measurement, Figure It Out, level 3, which suggests how this can be done.
Some students also have trouble moving between volume expressed in cubic centimetres and volume expressed in cubic metres. Recognising that 1 cubic metre $=1000000$ cubic centimetres is a key to their understanding. They can imagine stacking 100 layers of wooden or plastic cubes, all with 1-centimetre sides, into the "cubic metre" they have made.

Before they begin question 2 , check that your students understand what the scale 1 centimetre : 25 centimetres means. Most of the measurements are multiples of 25 centimetres and are unlikely to pose a problem. The students do, however, need a strategy to help them find the scale lengths of those that are not (for example, 1.9 metres). Once they understand that 1 centimetre : 25 centimetres is the same as 10 millimetres : 25 centimetres, the next step is to see that 2 millimetres is the scale length of 5 centimetres.

Because the students are asked to construct a scale model of the room and its furniture, accuracy is important. It is worth emphasising the processes that will help them achieve this:

- how to draw up a net for the item, or individual pieces, with lines that are parallel and the right length and with corners that are right angles
- the mechanics of tidy construction: glue tabs, cutting neatly, scoring fold lines, using glue or tape.

The scale size of some of the pieces of furniture means that they will be fiddly to make, particularly for those who aren't naturally careful or who lack good hand-eye co-ordination.
Question 3 asks the students to draw a floor plan of the attic room. Question 4 says there is a door in the middle of the tallest wall. This should be added to the plan. The students can then place their models in the best configuration. Remind them that the roof slopes, so they won't be able to fit the wardrobe in parts of the room where the ceiling is low. Question 5 requires them to determine the dimensions of the rectangular free space remaining in the middle of the room. This can be done from the scale model and the dimensions converted to full size, using the scale 1 centimetre : 25
centimetres. Answers will vary depending on how the students have arranged the furniture.

## Pages 14-15: Making the Team

## Achievement Objectives

- $\quad$ write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- perform calculations with time, including 24-hour clock times (Measurement, level 4)


## Activity One

This activity gives students an excellent opportunity to use their mental strategies for addition and multiplication. Mika suggests two such strategies on page 14 . You could ask the students to suggest others. For example, to work out how much Mika earns from helping his neighbour with
the fence, the students might round the $\$ 3.25$ down to $\$ 3$, do the calculation, and then add on the $\$ 0.25 \times 8$. The students could record the amounts Mika earns in a table like the one below. If preferred, the calculation column could be replaced with a strategy column:

| Job | Rate | Calculation | Total |
| :--- | :--- | :--- | :---: |
| Uncle's stall | $\$ 12.00$ flat rate |  | $\$ 12.00$ |
| Circulars | $\$ 4.75$ per day | $\$(4.75 \times 6)+(4.75 \times 3)$ | $\$ 42.75$ |
| Mr Murray | $\$ 3.50$ per hour | $\$ 3.50 \times 6.5$ | $\$ 22.75$ |
| Neighbour | $\$ 3.25$ per hour | $(\$ 3.25 \times 8)+5$ | $\$ 31.00$ |
| Nana | $\$ 1.50$ per week | $\$ 1.50 \times 6$ | $\$ 9.00$ |
| TOTAL |  |  | $\$ 117.50$ |

Ask the students to estimate how much Mika has earned altogether before they calculate the exact answer. (See the discussion of estimates in the notes for Garage Sale, pages 15-16.)

## Activity Two

This problem has a number of elements in it: time spent at rugby practice, time spent on homework, the ratio of homework to practice, and the amount of homework still to be completed.

Students are often confused by problems of this kind; they find it hard to know where to begin, how to pull the details together, and how to judge whether their answer is sensible. Finding the total amount of time already spent on homework (125 minutes) and at rugby practices (110 minutes) is a good first move. The students are likely to find it easiest to work entirely in minutes rather than to try and deal with hours and minutes. " 25 minutes on homework for every 20 minutes at practice" can be equally well expressed as " 12.5 minutes of homework for every 10 minutes of practice". Mika spends 11 multiples of 10 minutes on his rugby (110), so he must spend 11 multiples of 12.5 (137.5) on his homework. He has already done 125 minutes, so this will mean he has another 12.5 minutes to do.

For the students who have difficulty understanding ratios, a double number line can be used. For more on these, see Book 7: Teaching Fractions, Decimals, and Percentages, Numeracy Professional Development Projects 2003, (Wellington: Ministry of Education, 2003). This material is available online at Www.nzmaths.co.nz/numeracy/


This activity may well provoke some discussion on the amount and regularity of homework. Is Mika's dad expecting too much when he says that Mika must do 25 minutes' homework for every 20 minutes of sports practice, or is this amount too small? A survey of the class could be done to see how much time the students spend on sports and cultural practices, compared with the time they spend on homework.

## Links

This activity could form part of a unit on Rights, Rules, and Responsibilities.
Have the students imagine that, in co-operation with a group of friends, they are going to paint a community building in the local area over one weekend. They consider a plan to complete the task effectively, identifying possible conflict situations and taking account of these in their planning.

Suggested achievement objectives are:

## Social Studies

- how and why people make and implement rules and laws (Social Organisation, level 3)
- how and why people manage resources (Resources and Economic Activities, level 3)
- how and why people exercise their rights and meet their responsibilities (Social Organisation, level 4).


## Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)
- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (Measurement, level 4)
- effectively plan mathematical exploration (Mathematical Processes, problem solving, levels 3 and 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3 and 4)


## Activity

This activity asks students to draw to scale, on square grid paper, a sheet of plywood measuring 2400 millimetres $\times 1200$ millimetres. Discuss choosing the scale. It should be convenient to use while at the same time allowing the student to draw a good-sized rectangle.

The next task is to draw within this rectangle the six pieces that will make up the kennel. Discuss the shapes of the pieces needed: two sides, two ends, and two sloping planes for the roof. Ensure that the students are aware that the sum of the widths of the two roof pieces must be greater than the width of the kennel. They will need to check that this is the case when they review their classmate's plan as part of question 2. (See page 21 for further discussion on this.)

Determining the exact size, shape, and placement of each piece on the scale diagram is a trial-anderror exercise. The students may find that they won't have enough plywood to make a kennel with the dimensions they originally planned. For this reason, they may find it best to work with pencil and eraser and may need a number of sheets of grid paper. Remind them that they are to use as much of the sheet of plywood as possible.

Some students may realise that a more systematic approach to the problem is to imagine the scale sheet of plywood divided into six equal pieces with a single line down its length and two lines across its width. If they do this, they can see the average size of the six pieces that will make up the kennel. They will probably go on from here to conclude that the most economical length for the kennel is 800 millimetres. They will need to experiment with the dimensions of the front and back pieces so that the proportions seem right and so that the two roof planes are wide enough to overlap the walls when assembled. This diagram shows an economical way of making the cuts:


Before the students begin question 3, revise the concept of area (the size, or measure, of a surface, expressed in square units), and the formulae for a square, rectangle, and triangle. Also discuss the units of measurement. The linear dimensions of sheet timber are normally given in millimetres, but the area is expressed in square metres. Millimetres are convenient for measuring length because the builder only has to remember a simple number (for example, 1152 millimetres, instead of 1 metre, 15 centimetres, and 2 millimetres) and avoids using a decimal point. Square millimetres are, however, unsuitable for measuring area, because they are so tiny. Square centimetres are unsuitable for the same reason. In this activity, therefore, the students should convert millimetres to metres before calculating areas. They will find this much easier than the alternative of working in square millimetres then trying to convert the result into square metres. They need to learn the skill of selecting suitable units for a particular measurement task and moving confidently between different units.

For question 4, make sure that the students know that Robert wants to put 3 coats of paint on the kennel. They also need to know that they must be able to justify their decision as to which is the best size of can to buy. The answers have a full discussion of this decision, but as long as Robert uses at least 69 percent of the sheet of plywood, he will need to buy 1 litre of paint.
If you wish, the students could do a useful investigation into the reasons why the sum of the widths of the sides of the roof has to be greater than the width of the kennel itself. Give them various lengths, such as $2,2,2,3,3,4,5,6,6$, and 6 . Ask them to choose three lengths and see if they can make a triangle from them. When all the possibilities have been explored, the students should see that the sum of the lengths of any two sides of a triangle has to be greater than the length of the third.


Interested students could go on to make a scale model of their kennel, using card.

## Pages 18-19: To'ona'i

## Achievement Objectives

- $\quad$ write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- $\quad$ solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, levels 3 and 4)


## Activity One

This page builds on Making the Team (pages 14-15) in that students can use their mental strategies to find the cost of each item. As in the earlier task, some strategies are suggested on the page, and the students can use these or their own to work out the costs. You could ask the students to estimate the total cost before they calculate it exactly. Encourage them to make a table with the items, rates, amounts, and costs so that they can find what the total cost is.

Mika suggests one way of working out the cost for 46 people. You could ask the students to suggest additional ways. For example, they could find the cost for 2 people by dividing the cost for 8 people by 4 . Finding the cost for 1 is the key to writing the rule in question 4 : total cost $=$ number of people $\times \$ 7.98$.
Some students will realise that the above method of calculating requirements does not correspond closely to the realities of catering. If 1 chicken is enough for 8 people, that does not mean that $1 / \frac{1}{8}$ chickens can be purchased for 9 people. In practice, catering requires a lot of judgment calls. For example, can a chicken that was enough for 8 be stretched for 9 , or does another need to be bought? If another is bought, several more people could then join the party without chicken being bought specifically for them. Eseta's mother will not necessarily buy another tin of coconut cream just because another person is coming. She may decide to make do with what she has.

## Activity Two

This activity requires a similar approach to Activity One. Questions $\mathbf{1}$ and $\mathbf{3}$ can be answered by multiplying by 3 and 9 . In question 3, first find the amounts for 1 glass and multiply this by the number of glasses required. Discuss with the students whether 1.250 litres is a better unit than 1250 millilitres. (This is debatable for quantities of this size.)
As a group extension activity, the students could plan the catering for a real or imagined situation. This would include costing the catering and working within a budget.

## Links

This activity could form part of a unit on Celebrating Diversity.
Ask the students to compare the ways that people meet, greet, and host each other in a range of settings (for example, at home, at a local marae, at civic functions, in the community, at church, at a sports club, at school, or in a shopping mall).

Suggested achievement objectives are:

## Social Studies

- how practices of cultural groups vary but reflect similar purposes (Culture and Heritage, level 3)
- why and how individuals and groups pass on and sustain their culture and heritage (Culture and Heritage, level 4)


## Health and Physical Education

- participate in communal events and describe how such events enhance the well-being of the community (Healthy Communities and Environments, level 3).


## Page 20: Solar-powered Shower

## Achievement Objectives

- read and construct a variety of scales, timetables, and charts (Measurement, level 4)
- $\quad$ sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)


## Activity

This activity is a simple cost-benefit analysis: is it worth spending money now to save money in the future? Question 1 requires the students to find the cost per year ( $\$ 40 \times 12$ ) of using electricity. This information is needed to complete the graph comparing solar power with electricity. Most students will find it easiest to plot the relative costs at the end of 2 years, 3 years, 4 years, and up to 7 years and join the dots. Check that they understand why the line representing the cost of solar power is horizontal and why the cost of electricity is shown by a sloping line. (Solar power involves a one-off payment, but electricity has to be paid for each month.) The students could use a computer spreadsheet to produce a finished graph.
Question $\mathbf{2 b}$ can be answered from the graph by reading the number of years off the horizontal scale immediately below the point of intersection of the two lines. Ask the students what it means when two lines intersect on a graph. (Two lines always meet at a point unless they are parallel. At that point, the two lines share common values on both scales.) Here the intersection means that, at one moment in time, the cost of solar power and electricity will be the same. Will the two lines continue to be straight if the graph is extended beyond 7 years? Not necessarily. The solarpowered system may need costly maintenance, and the cost of electricity may increase. But it is unlikely that the two lines will cross again, meaning that the solar-powered system will continue to save Melissa's family money.
In question $\mathbf{2 c}$, the students need to calculate how many monthly payments of $\$ 40$ it will take before Melissa's mum has spent the $\$ 3,000$ needed for the solar shower. Discuss the relative merits of graphs and calculations. (Graphs are great for spotting trends, patterns, or special features of data, and for estimating values, but they don't usually allow you to read off precise values. For exact answers, we need to do calculations using the raw data.)
Also discuss the benefits of having a formula for working out costs. (As a variable changes, it is easy to recalculate the total cost.) The answer to question 2 c can be found using the formula: number of months $=3,000 \div$ cost per month of electricity.

For question 3, the students could add a third line to their graph, showing the new cost of electricity at $\$ 60$ per month.
With the permission of their parents, the students could bring along an electricity account from home, and you could photocopy these in such a way that they don't show the name or address of the family. These could then be used to investigate how consumers are charged for electricity. More able students could prepare a presentation on the actual cost of running appliances such as a light bulb, heater, and computer. One way of doing this would be to set up a spreadsheet and graph the time in hours that each appliance would take to use $\$ 1$ worth of electricity.

## Links

This and the following activity could form part of a unit on Our Precious Resources.
Have the students consider possible options available to New Zealand power utilities or politicians for maintaining a reliable power supply through the winter months. After feedback and class discussion, students reach a consensus on preferred options, giving their reasons.
Suggested achievement objectives are:

## Social Studies

- how and why people manage resources (Resources and Economic Activities, level 3)
- how and why people view and use resources differently and the consequences of this (Resources and Economic Activities, level 4)
- make decisions about possible social action (Social Studies Processes, Social Decision Making, levels 3-4)


## Page 21: Water Everywhere

## Achievement Objectives

- calculate perimeters of circles, rectangles, and triangles, areas of rectangles, and volumes of cuboids from measurements of length (Measurement, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3 and 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)


## Activity

Discuss Dad's plan. Would it work? (Yes.) What is wrong with it? (There are unnecessary sections of pipe, which will waste money and installation time.) The students can then set about deciding how they will redesign the irrigation system, using as few pipes as possible. The most likely problemsolving strategy will be trial and error. The students may find it helpful to make a list of their various attempts, adding the lengths so that comparisons can be made. If they want to explore whether the best solution is to lay pipes in a completely different configuration (for example, a single trunk line with feeder pipes going off at right angles), they will need to construct a scale diagram. This will require careful use of a compass to draw the triangles accurately and would be a valuable exercise in its own right. (In reality, a trunk line does not minimise pipe length.)

Question 2 is a very open question. It may help if the students use square grid paper or if you give them a scale plan of the garden of a private home (real or imagined) and ask them to see how they can irrigate it using the shortest length of pipe. Gardening stores have brochures with quite detailed information on home irrigation systems. These will tell the students what coverage to expect from different fittings.

For able students, this activity leads naturally into investigations of networks and the discovery of Euler's Law. Mathematical Investigations, A Series of Situational Lessons, Book Two, by Souviney et al. (California: Dale Seymour Publications, 1992) has three investigations looking at networks. Connected Networks is an investigation involving irrigation networks.

Alternatively, you could give the students various patterns and ask if they can draw the pattern without lifting their pencil off the paper and/or going over the same line twice. Can they work out why some patterns can be traced in this manner but others cannot? (The clue is in the nature of the "nodes" - the points where lines meet or cross.) This could lead into an investigation of the famous Königsberg Bridge problem. See also page 18 of At Camp, Figure It Out, level 3.


OK


Can't do this


OK

## Pages 22-23: Moving Survey

## Achievement Objectives

- plan a statistical investigation of an assertion about a situation (Statistics, level 3)
- make sensible statements about an assertion on the basis of the evidence of a statistical investigation (Statistics, level 3)
- predict the likelihood of outcomes on the basis of a set of observations (Statistics, level 3)
- plan a statistical investigation arising from the consideration of an issue or an experiment of interest (Statistics, level 4)
- $\quad$ collect appropriate data (Statistics, level 4)
- choose and construct quality data displays (frequency tables, bar charts, and histograms) to communicate significant features in measurement data (Statistics, level 4)


## Activity

This activity begins with the assertion " 3 students leaving the class in 1 term - that's unusual!" and some data that might or might not support the assertion. The students then analyse the data using different tools (a tally chart, a bar graph, and the median, mode and mean) and conclude by saying whether they agree with the original assertion. The activity is an opportunity to emphasise that unsorted, unprocessed data (such as in the first table) is little help in reaching a conclusion. Analysis is the process by which data is sorted, measured, and presented in a useful way.
A useful starting point is the statement itself, which needs clarification. By " 3 ", does Mr Rohe mean " 3 or more"? (Yes.) Then ask the students if they can conclude anything from looking at the raw data on page 22. Ask them to suggest what they could do with this data to help work out whether Mr Rohe's statement is right. The questions then guide them through a series of steps that should enable them to reach an informed conclusion.

Question $\mathbf{l}$ asks the students to make a tally chart. If they have not been introduced to tally marks, this very useful technique will need to be explained, especially the grouping in fives for easy counting, as illustrated in the second diagram. Question 2 asks the students to use a spreadsheet to make a bar graph. It should look like this:

| $\square$ | Ematoma |  | E1 |
| :---: | :---: | :---: | :---: |
|  | A | B | $C$ |
| 1 | Number of leavers | Number of classes |  |
| 2 | 0 | 10 |  |
| 3 | 1 | 15 |  |
| 4 | 2 | 7 |  |
| 5 | 3 | 3 |  |
| 6 | 4 | 3 |  |
| 7 | 5 | 2 |  |
| 8 |  |  |  |
| 9 |  |  |  |
| 10 |  |  | $\bigcirc$ |
| 4.7 |  | IT | . 1 |

In question 3, the mode can easily be found from either the bar graph or the tally chart - just look for the tallest bar or the line with the most tallies.

Once they have crunched the numbers, question 4 asks the students to decide whether they support the original assertion that it is unusual for 3 (or more) students to leave from a class in a term. In fact, this happened 8 times out of 40 . The students can discuss whether something that happens 20 percent of the time should be called unusual. Statisticians would say that it should not. Three students leaving in a term is not particularly common, but neither is it unusual. If students are still uncertain about this, ask if they think it is "unusual" to be left-handed. It is estimated that about one in seven people are left-handed, but this does not make left-handedness unusual.

Question 5 asks what it means when the average of whole-number (discrete) data is not itself a whole number. The students need to see that there is meaning to this concept even if there is no such thing in reality. The average of the two whole numbers 8 and 9 cannot itself be expressed by a whole number, but this does not stop us usefully defining the middle ground (average) with the fraction 8.5. You could ask the students to check Robert's calculation of the mean: 60 students leave over 40 term equivalents, which is exactly 1.5 students per term.

## Investigation

Assigning one student to collect the data from each class is the most efficient way to start this investigation. This will provide quite a pool of data that can then be graphed and analysed as suggested. It could be interesting to break the data down by, say, gender or year level. Different groups within your class could analyse the data, checking for the influence of different factors. The answer to question 4 needs to be "yes"; some schools have considerably higher rates of transfer than others. It will be interesting to hear the reasons students come up with for this fact.

## Links

This activity and the following one could form part of a unit on Changes.
Suggested achievement objectives are:

## Health and Physical Education

- identify factors that affect personal, physical, social, and emotional growth and develop skills to manage changes (Personal Health and Physical Development, level 3)
- identify and compare ways of establishing relationships and managing changing relationships (Relationships with Other People, level 3)
- identify the effects of changing situations, roles and responsibilities on relationships and describe appropriate responses (Relationships with Other People, level 4).

Because this topic is a sensitive one, teachers should make themselves familiar with the Ministry of Education's safety guidelines. These are found in Change, Loss, and Grief, in the series The Curriculum in Action.

## Page 24: Farewell Party

## Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- carry out measuring tasks involving reading scales to the nearest gradation (Measurement, level 4)


## Activity

In this activity, students divide a circular cake into 3 equal parts and then 24 equal parts. To do this, they need to understand:

- that angle is the measure of turn
- that there are 360 degrees in one complete turn (that is, a circle)
- how to relate a fraction of a circle to a measurement in degrees
- how to use a protractor to measure angle in degrees.

One way of introducing the concept of angle is to ask how you might measure how far a door has been opened. Why not just use centimetres? Centimetres are a measure of length, and when you open a door, different parts of the door actually travel different lengths. The leading edge of the
door travels much further than the edge by the hinge, which hardly moves at all. So angle is very different to length and needs a different system of measurement.

Why are there 360 degrees in a circle? The number is arbitrary (an alternative system is also occasionally used, gradians, in which a complete turn is divided into 400). Nevertheless, 360 is a good choice, and this could be discussed with the class. Because it is a multiple of a remarkably large set of numbers: $2,3,4,5,6,8,9,10,12,15,18,20,24,30,36,40,45,60,72,90,120$, and $180-22$ in all - it is possible to conveniently measure many angles without having to use fractions.

Students need practice at relating fractions of a circle to 360 degrees. It will help if they can see that the numbers on the above list of factors work in pairs: just as a circle can be divided into nine parts, each of 40 degrees, it can also be divided into 40 parts, each of 9 degrees.

Check that all the students know how to use a protractor before they begin this activity. A protractor is to angle what a ruler is to length. Given that protractors are normally made of clear plastic, it is easy to demonstrate their use to a class by laying them on an overhead projector and projecting the image onto a screen or wall. An alternative is to use a large-scale wooden or plastic protractor, sold as a teaching aid.

Question 4 requires the students to divide 360 degrees by 24. Ask them to demonstrate with their arms how big they think 15 degrees is. Learning to visualise the size of an angle is the key to avoiding foolish answers. The most common error when using a protractor (apart from inaccuracy resulting from carelessness) is to read off the supplement of the desired angle (for example, 100 degrees instead of 80 degrees). If they know what they are looking for, your students are unlikely to fall into this trap.

Question 5 calls for some lateral thinking. A horizontal cut followed by 6 vertical cuts through the centre will divide the cake into 24 equal pieces. (Some people don't like icing anyway!)


## Acknowledgments

Learning Media and the Ministry of Education would like to thank Len Cooper for developing these teachers' notes and reviewing the answers. Thanks also to Barbara Batchelor and Vikki Pink of Christchurch College of Education for linking the activities to the social studies and health and physical education curricula.

The illustration on the front cover and page 1 is by Lisa Paton and the photo by Adrian Heke and Mark Coote. The background illustrations on the contents page and on page 2 are by Lisa Paton and Ali Teo.

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Series Editor: Susan Roche
Editor: Ian Reid
Designer: Rose Miller
Published 2003 for the Ministry of Education by
Learning Media Limited, Box 3293, Wellington, New Zealand.
www.learningmedia.co.nz
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All rights reserved. Enquiries should be made to the publisher.
Dewey number 510.76
ISBN 0478273762
Item number 27376
Students' book: item number 27373

