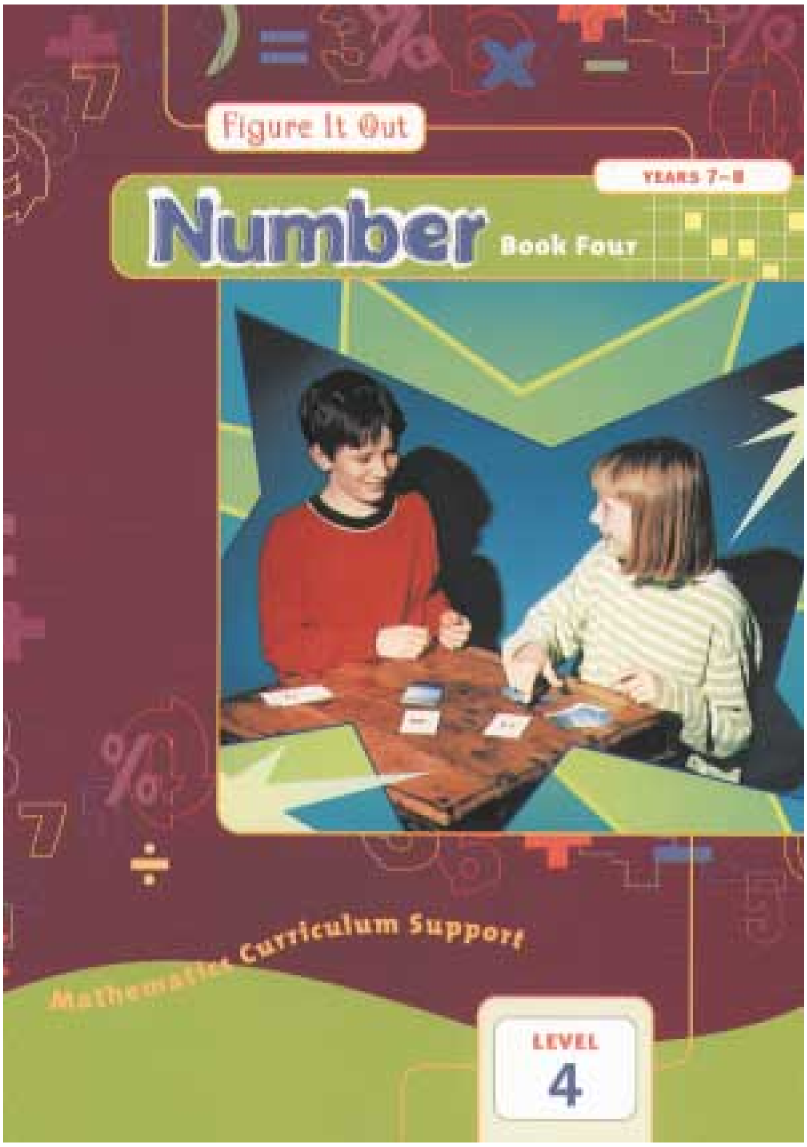
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+)
Number Sense (one linking, one level 4)
Algebra (one linking, two level 4, one level 4+)
Geometry (one level 4, one level 4+)
Measurement (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters, Getting Around*
These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent activities, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number: Book Four

Page 1 In Your Prime

ACTIVITY

1. **a.** 10, 14, 15, 35

   b. 4, 9, 25, 49

2. 2 x 2 x 2 = 8, 2 x 2 x 3 = 12, 2 x 2 x 5 = 20, 2 x 2 x 7 = 28, 2 x 3 x 3 = 18, 2 x 3 x 5 = 30, 2 x 3 x 7 = 42, 2 x 5 x 5 = 50, 2 x 5 x 7 = 70, 2 x 7 x 7 = 98, 3 x 3 x 3 = 27, 3 x 3 x 5 = 45, 3 x 3 x 7 = 63, 3 x 5 x 5 = 75, 3 x 5 x 7 = 105, 3 x 7 x 7 = 147, 5 x 5 x 5 = 125, 5 x 5 x 7 = 175, 5 x 7 x 7 = 245, 7 x 7 x 7 = 343

3. **a.** No.

   b. The factor order can be rearranged (for example, 5 x 3 x 2 x 2), but there is no other combination of prime numbers that can be multiplied to make 60. All other combinations (for example, 2 x 3 x 3 x 3 = 54 and 17 x 2 x 2 = 68) give products other than 60.

4. Mei Ling would end up with 1 x 60, 2 x 30, 3 x 20, 4 x 15, 5 x 12, and 6 x 10. After that, she would be repeating factors (for example, 10 x 6). So the factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

   Hine would end up with:
   2 x 2 x 3 x 5 = 60
   2 x (2 x 3 x 5) = 2 x 30
   3 x (2 x 2 x 5) = 3 x 20
   (2 x 2) x (3 x 5) = 4 x 15
   5 x (2 x 2 x 3) = 5 x 12
   (2 x 3) x (2 x 5) = 6 x 10
   From these, she would get the same factors of 60 as Mei Ling. (Although 1 is not a prime factor, Hine would need to include 1 as a factor to balance the 60 from 2 x 2 x 3 x 5, which is 1 x 60.)

Pages 2–3 Going Bananas

ACTIVITY

1. **a.** 15. (30 divides into 15 groups of 2. One from each group goes into the new pile.)

   b. No, 15 does not divide by 2 without a remainder.

   c. Any odd number

2. **a.** 10. (15 divides into 3 groups of 5. One from each group goes into the new pile. Mother gets the 10 left over.)

   b. A bunch that is not a multiple of 3 or a multiple of 3 after it has been halved by Father. (For example, he could pick prime numbers greater than 3 or doubles of primes.)

3. To get some bananas, Gus should get multiples of 5, for example, 5, 10, 15, 20. With powers of 5, such as 25, he gets first pick of the bunch.

4. Junior should choose prime numbers higher than 5 (for example, 7, 11, 13).

5. **a.** 14: Father 7, Mother 0, Gus 0, Junior 7

   b. 18: Father 9, Mother 8 (6 in first split and then 2 out of the 3 left), Gus 0, Junior 1

   c. 17: Father 0, Mother 0, Gus 0, Junior 17

   d. 6: Father 3, Mother 2, Gus 0, Junior 1

Page 4 Going to Extraordinary Lengths

ACTIVITY

1. The boss needs to keep these machines:

2	3	5	7	11
13	17	19	23	29
31	37	41	43	47
53	59	61	67	71
73	79	83	89	97

2. These machines have only two factors, themselves and 1. (These machine numbers are called prime numbers.) Therefore, shoelaces needing to be stretched by these scale factors cannot be made using any other machines.

Page 5 Boxing Balls

ACTIVITY

1. **a.** Three arrangements: 1 x 18, 2 x 9, 3 x 6

   b. Three arrangements: 1 x 20, 2 x 10, 4 x 5

   c. Four arrangements: 1 x 24, 2 x 12, 3 x 8, 4 x 6

   d. One arrangement: 1 x 17

2. **a.** Numbers with one arrangement are:
   1, 2, 3, 5, 7, 11, 13, 17, 19, 23, and all other prime numbers.

   b. They are prime numbers and have only two factors, themselves and 1. For example,
   13 = 13 x 1, 5 = 5 x 1.

Page 6 Prime Sites

INVESTIGATION ONE

No, not all whole numbers on either side of a multiple of 6 are prime numbers.
For example, with 23, 24, 25: only 23 is a prime number.

INVESTIGATION TWO

Chebyshev's theorem holds for any whole numbers greater than 1.

Page 7 Igloo Iceblocks

ACTIVITY

1. Using prime factors of 60 helps to find most of the possible arrangements.

   60 = 2 x 2 x 3 x 5, so the possible arrangements using prime numbers are:
   (2 x 2) x 3 x 5 = 4 x 3 x 5
   (2 x 3) x 2 x 5 = 6 x 2 x 5
   (2 x 5) x 2 x 3 = 10 x 2 x 3
   (3 x 5) x 2 x 2 = 15 x 2 x 2
   Other possible arrangements are 20 x 3 x 1 and 30 x 2 x 1.

2. The prime factors of 180 are 2 x 2 x 3 x 3 x 5.
   Grouping these gives the following possibilities:
   4 x 15 x 3, 4 x 9 x 5, 6 x 10 x 3, 6 x 6 x 5,
   6 x 15 x 2, 10 x 9 x 2, 12 x 3 x 5, 18 x 2 x 5,
   20 x 3 x 3, 30 x 3 x 2, 45 x 2 x 2.
   Other possible arrangements are 1 x 4 x 45,
   1 x 5 x 36, 1 x 6 x 30, 1 x 9 x 20, 1 x 10 x 18, and 1 x 12 x 15.

3. No. *97* is a prime number. The only factors are 97 and 1, and the sleigh is not long enough or wide enough to stack the cubes as a single layer.

Page 8 It's a Try!

ACTIVITY

1. **a.** ¯0:00:10

   b. ¯0:00:25

   c. 23 s

   d. ¯0:00:43

2. **a.** 34 s

   b. 0:00:14

3. ¯0:00:57

Page 9 Lifting Weights

ACTIVITY

1. Answers may vary, but Lefu is probably the most likely answer because he lifts the heaviest mass.

2. **a.**

Josh	¯3.4
Chris	¯5.5
Mike and Mark	¯6
Lefu	¯7
Harry	¯9
Dan	¯9.5
Nick	¯11
Wiremu	¯13.5
Pete	¯24

   b. Josh. He can nearly lift the equivalent of his own body mass.

Page 10 Integer Zap

GAME

A game using integers

Page 11 Integer Slam

GAME

Two games using integers

Page 12 Integer Links

GAME

A game using integers

Page 13 Family Trees

ACTIVITY

1. 14. $(2^1 + 2^2 + 2^3)$
2. a. 7

   b. i. 128

   ii. 2^7

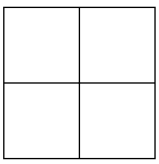
   c. 254
   $(2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7$
   $= 2 + 4 + 8 + 16 + 32 + 64 + 128$
   $= 254)$
3. By the ninth generation ($2^9 = 512$), making a total of 1 022 people
4. a. $2^{19} = 524\ 288$

   b. 1 048 574 $(2^1 + 2^2 + 2^3 \dots + 2^{19}$
   or $524\ 288 \times 2 - 2)$

Page 14 Building Squares

ACTIVITY

1. a. 2^2 is known as "two squared" because it gives the area of a square with sides of two units:



   b. 7. (To make a 4 x 4 square, 16 tiles will be needed altogether.)

2. a. Practical activity

   b. The differences are consecutive odd numbers (+ 3, + 5, + 7, + 9, …) or double the side length of each square plus 1.

3. a. i.

double 10 plus 1

$21.\ (10^2 + 21 = 11^2)$

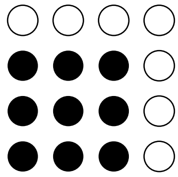
These add up to 21 (10 + 11).

   ii. $10^2 - 9^2 = 19$

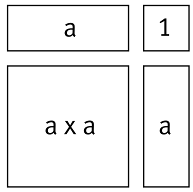
These add up to 19 (10 + 9).

(When you subtract a perfect square from the next largest perfect square, the answer is the sum of the two numbers.)

   b. i. Answers will vary. You can show $3^2 + 7 = 4^2$ as



You can also show this as a diagram. To build the next biggest square from a x a (a^2):



you need to add two lengths of the current side length plus 1. That explains why the differences are odd.

   ii. Answers will vary. The simplest explanation, following the pattern shown, is to look for two consecutive numbers that add up to the solution.

Page 15 Shifty Subtraction

ACTIVITY ONE

a. 20

b. Any two numbers, as long as the first number is 3 less than the second number, for example:
0 – 3, 1 – 4, 2 – 5, 3 – 6, $^-8 - {^-5}$, …

c. Any two numbers, as long as the first number is 5 less than the second number, for example:
0 – 5, 1 – 6, 2 – 7, 3 – 8, $^-10 - {^-5}$, …

ACTIVITY TWO

a. $5 - 6 = {^-1}$
$80 - 40 = 40$
$^-1 + 40 = 39$

b. $5 - 7 = {^-2}$
$100 - 60 = 40$
$^-2 + 40 = 38$

c. $2 - 8 = {^-6}$
$80 - 60 = 20$
$400 - 200 = 200$
$^-6 + 20 + 200 = 214$

d. $0 - 8 = {}^{-}8$
$90 - 60 = 30$
$500 - 400 = 100$
${}^{-}8 + 30 + 100 = 122$

e. $7 - 4 = 3$
$40 - 60 = {}^{-}20$
$200 - 100 = 100$
$3 + {}^{-}20 + 100 = 83$

f. $9 - 3 = 6$
$10 - 60 = {}^{-}50$
$800 - 400 = 400$
$6 + {}^{-}50 + 400 = 356$

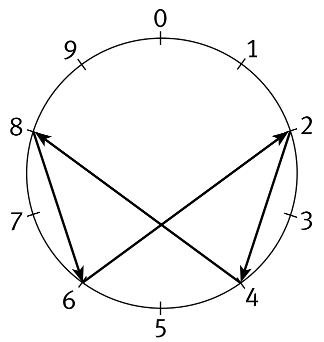
Page 16 Calculator Power

ACTIVITY

1. a. 2, 4, 8, 16, 32, 64

b. They make a recurring pattern:
2, 4, 8, 6, 2, 4, 8, 6, …

c.



2. a. Reasoning may vary. The pattern is 2, 4, 8, 6, 2, … So, in every cycle of four powers of 2, the ones digit reverts to 2 (that is, 2^1, 2^5, 2^9, 2^{13}, and so on). So:

i. 2. 2^9 will have 2 as the ones digit because the power is first in the cycle.

ii. 8. 2^{15} will have 8 as the ones digit because the power is third in the cycle.

iii. 6. 2^{20} will have 6 as the ones digit because the power is fourth in the cycle.

b. 2^{100} will have 6 as the ones digit because the power is fourth in the cycle (100 can be divided by 4 with no remainder).

3. 2^{20} is much larger than double 2^{10} because 2^{11} is double 2^{10}.

INVESTIGATION

a. 3, 9, 27, 81, 243 … ⟶ 3, 9, 7, 1, 3, 9, 7, …

b. 5, 25, 125, 625, … ⟶ 5, 5, 5, 5, …
6, 36, 216, 1 296, … ⟶ 6, 6, 6, 6, …

Both are boring because 5 x 5 = 2$\underline{5}$, so the 5 recurs, and 6 x 6 = 3$\underline{6}$, so the 6 recurs.

c. 7, 49, 343, 2 401, 16 807, … ⟶ 7, 9, 3, 1, 7, …
(These are the same ones digits as the powers of 3 but in a different order.)

d. 8, 64, 512, 4 096, 32 768, … ⟶ 8, 4, 2, 6, 8, …

Page 17 Cubic Capacity

ACTIVITY

1. a. i. 8 (because 2 x 2 x 2 = 2^3, which is 8)

ii. 27 (because 3 x 3 x 3 = 3^3, which is 27)

iii. 64 (because 4 x 4 x 4 = 4^3, which is 64)

iv. 125 (because 5 x 5 x 5 = 5^3, which is 125)

b. Answers will vary.

2. a. $5^3 = 125$

b. $3^3 = 27$

c. $10^3 = 1\ 000$

d. $4^3 = 64$

e. $8^3 = 512$

3. ii. 27 – 1 = 26 cubes

iii. 64 – 8 = 56 cubes

iv. 125 – 27 = 98 cubes

4. 2^3 gives the volume of a cube that has edges of two cubes long.
$2^3 = 2 \times 2 \times 2$
$= 8$ cubic units

Page 18 Growing Pains

ACTIVITY

1. A table for a two-split shrub will look like this:

Month	0	1	2	3	4	5	6	7	8
Number of shoots	1	2	4	8	16	32	64	128	256

So the shrub will need pruning after the eighth month.

For a three-split shrub, the table is:

Month	0	1	2	3	4	5
Number of shoots	1	3	9	27	81	243

So the shrub will need pruning after the fifth month.

2. Both shrubs would be pruned after 4 months.
(The sequences are:
Four-split: 1 4 16 64 256
Five-split: 1 5 25 125 625)

Page 19 Fold and Crease

INVESTIGATION ONE

After a certain number of folds, the sections can be worked out by: $\underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{number of folds}}$

A strip can be folded, at the most, seven times.

Number of times folded in half	1	2	3	4	5	6	7
Sections	2	4	8	16	32	64	128

The number of creases is always one fewer than the number of sections.

INVESTIGATION TWO

a. Answers may vary. To get more than 50 creases or sections, you need at least four folds.

b. Four times. You can show this in a table:

Number of times folded into thirds	1	2	3	4
Creases	2	8	26	80
Sections	3	9	27	81

INVESTIGATION THREE

The number of creases and sections are:

Sections	2	6 (2 x 3)	12 (2^2 x 3)	36 (2^2 x 3^2 = 36)
Creases	1	5	11	35

Note that the number of creases is always one fewer than the number of sections.

Page 20 Pip's Pay

ACTIVITY

1. **a.** Toline and Lefu are correct.

   b. You could argue that multiplying by 2 is faster than adding repeatedly. Both strategies are effective. Kirsty's method, if used as 100 x 2^5, would also be efficient.

2. Answers will vary. Kai-Yu has not realised that doubling the pay for season 3 only gives the pay for season 4. He needs to continue doubling.
Kirsty has an extra multiplication by 2
(100 x 2^5 = 32 000).

Page 21 Money Matters

ACTIVITY

1. **a.** \$23. (\$50 – \$27)

   b. Yes. (Chris has \$27, but he still owes \$50, so he is not debt-free.)

2. \$5. (Dion earns \$35 by working for the restaurant. Maria finishes with \$35, the same as Dion, so her guitar strings cost \$5.)
3. \$159. (Ana finishes with \$20 more than Dion and Maria, which takes her to \$55. Her radio advertisements cost \$104. \$104 + \$55 = \$159)
4. Answers will vary. (Note that contestants are disqualified if they are not debt-free at this point.)

Pages 22–23 Starting with Stamps

ACTIVITY ONE

1. For a 3 x 3 block, there are 9 + 4 + 1 = 14 different squares. (There are nine ways to sell a 1 x 1 stamp, four ways to sell a 2 x 2 block, and one way to sell a 3 x 3 block.)
2. For a 4 x 4 block, there are 16 + 9 + 4 + 1 = 30 different ways.

ACTIVITY TWO

50 m.
(One way to work this out is 2[1 + 3 + 5 + 7 + 9] m.)

ACTIVITY THREE

Each different sprinkler covers a square area. Dividing this area into 144 (that is, 12 x 12) gives the number of sprinklers needed:

Wet Wimp	144 ÷ 1 = 144 needed
So-soak	144 ÷ 4 = 36 needed
Deep Drench	144 ÷ 9 = 16 needed
Super Squirt	144 ÷ 16 = 9 needed.

Page 24 Superior Side Lengths

ACTIVITY

1. **a.** 49. (7 x 7)

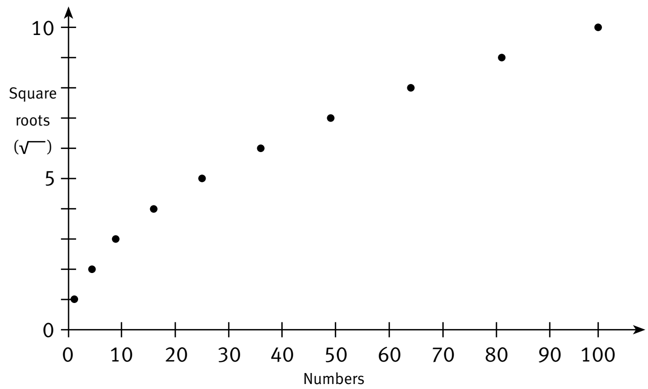
   b. $\sqrt{49} = 7$. This gives the side length of the square.

2. **a.** **i.** 4 **ii.** 9

   iii. 78 **iv.** 2.5

   v. 4.76

   b. Answers will vary. You always end up with the number you started with because the square root undoes the squaring. A general rule for this is $\sqrt{n \times n} = n$.

3. 11 tiles long
4. 

   The pattern is a curved line continuing upwards.

5. Estimates will vary. The answers, rounded to 1 d.p., are:

   a. 5.5

   b. 7.5

   c. 8.4

   d. 9.7

Teachers' Notes

Overview **Number: Book Four**

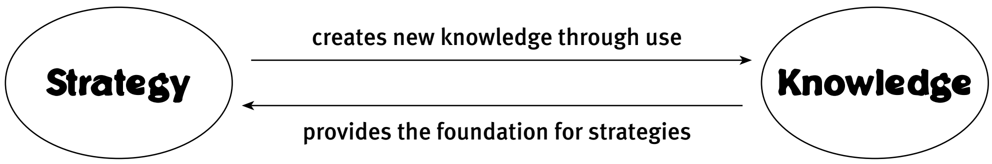
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence of machines in society that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation. The *Number Sense* books are aimed at developing students' ability and willingness to apply their number understanding to make mathematical judgments. Teaching number sense requires an emphasis on openness and flexibility in solving problems and the use of communication and interpretation skills.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged students. A key element of this drive has been the creation of the Number Framework, developed as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



Learning activities in the series are aimed both at developing efficient and effective mental strategies and at increasing the students' knowledge base. Broadly speaking, the levels given in the six year 7–8 *Number* books can be equated to the strategy stages of the Number Framework in the following way:

Link (Book One):	Advanced counting to early additive part-whole
Link (Book Two):	Advanced additive part-whole
Level 4 (Books Three to Five):	Advanced multiplicative to advanced proportional part-whole
Level 4+ (Book Six):	Advanced proportional part-whole.

Page 1 In Your Prime

Achievement Objective

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also:

- explore factors of numbers by investigating prime numbers
- maintain basic facts
- devise and use strategies to find prime numbers.

ACTIVITY

Exploring factors and prime factors enables students to use their knowledge of basic multiplication facts as they make observations about numbers. This will help them later when they work with fractions and simplify algebraic expressions. The students may work in small groups or individually. Small-group work will have the advantage of introducing the students to a range of possible solutions.

The students can attempt question **1** randomly with cards, but you could encourage a more systematic response by asking "How do you know you've got them all?"

2 x 3 = 6
2 x 5 = 10
2 x 7 = 14
3 x 5 = 15
3 x 7 = 21
5 x 7 = 35
2 x 2 = 4
3 x 3 = 9
5 x 5 = 25
7 x 7 = 49

You could limit the activity initially to just three cards or conversely extend it by adding the fifth prime number, 11.

In the initial discussion for question **2**, or if the students need help later, you could use a range of examples, such as 3 x 5 x 7, 2 x 5 x 5, and 3 x 3 x 3, to help clarify the types of equations that are possible. Again, as in question **1**, sharing ideas in a group can lead to a range of possible solutions being recorded, perhaps in a systematic table. Class discussion should revolve around recording all the possible numbers and making sure that none are missed. Again, you could extend the question by including the fifth prime number, 11.

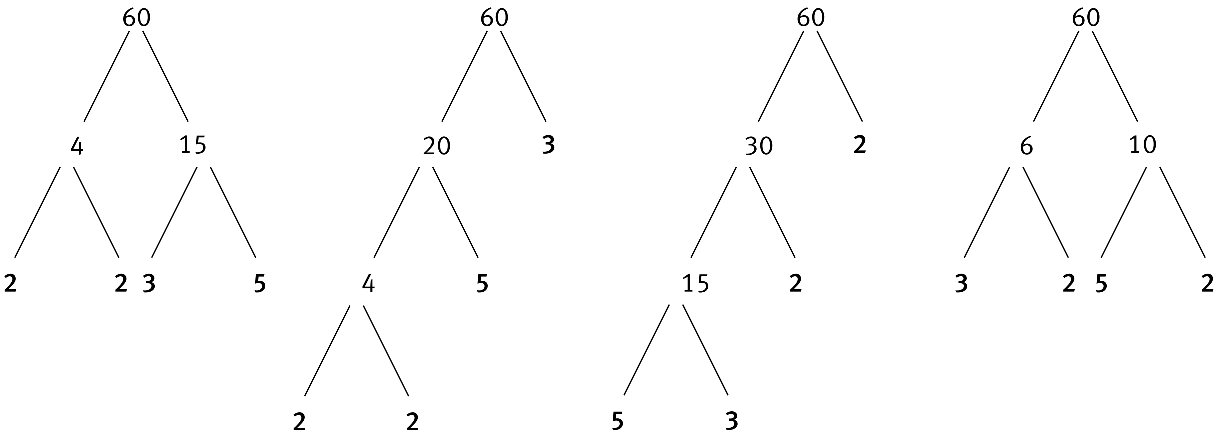
As a follow-up investigation, you could ask "Can all the numbers from 1 to 50 be products of these cards? What is unusual about the numbers that can't? How about the numbers from 51 to 100?"

For question **3**, you need to discuss two types of whole numbers: prime numbers and composite numbers. A prime number is a number greater than 1 that is divisible only by itself and 1. A composite number is any number that is not prime, that is, it has more than two factors.

Ask your students: "Is 57 prime or composite?" Discuss other examples such as 19, 21, 29, 27, and so on, and the special nature of 1. (1 is not considered a prime number because 1 = 1 x 1 is one times itself, with no other factor involved, and therefore it breaks the pattern. See also the sieve of Eratosthenes, which is described in the notes for page **4** of the students' book.) The students can also look up the word "prime" in their dictionaries and record definitions related to the mathematical meaning.

At this stage, they may not have a formal prime factorisation strategy, so they may link question **3** to the earlier questions and try other combinations of 2, 2, 3, and 5. To deter them from rearranging the factors given, a note has been added to the question.

The students can verify the answer by splitting composite numbers into their factors until there are no more composite numbers to split, using a method known as a factor tree. For 60, this is:



Regardless of how the composites used to initially split the 60 are ordered, you end up with the same set of prime factors (2, 2, 3, 5).

You could ask your students to complete factor trees for three other specific numbers after you have modelled 60 with them. They could then investigate their own choice of composite numbers.

Question **4** requires the students to list all the factors of a number (both composite and prime). Mei Ling's method is quite logical and systematic. Starting at 1, she moves systematically through paired factors of 60 until she reaches the halfway point, which is after 6 x 10. Beyond that, because of the commutative property of multiplication, the factors are just reversals of the previous ones (for example, 10 x 6).

Hine's method is really using the factor tree system in reverse, but this can get confusing, and the students will need to be systematic in keeping track of how they have used parentheses to make the different groups.

You can organise further practice in finding sets of factors and prime factors by having groups chart and share their solutions to different numbers, for example, 36, 24, 336, and so on.

Pages 2–3 Going Bananas

Achievement Objectives

- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also:

- explore factors of numbers by investigating prime numbers
- use division.

ACTIVITY

This activity leads students through a process that involves dividing composite (non-prime) numbers by a prime factor and follows a set procedure. You can help your students to understand this process by modelling the distribution of bananas in the flow chart on the students' page using counters or cubes. (The flow chart uses 18 bananas, but it is set out in a way that covers any number of bananas.) In pairs, the students could then use counters or cubes to model the distribution of the 30 bananas in questions **1–4**.

The students need to realise that the total pool of bananas is not re-formed before each division calculation and that the bananas that are not going into a new pile are kept by the gorilla who is doing the dividing.

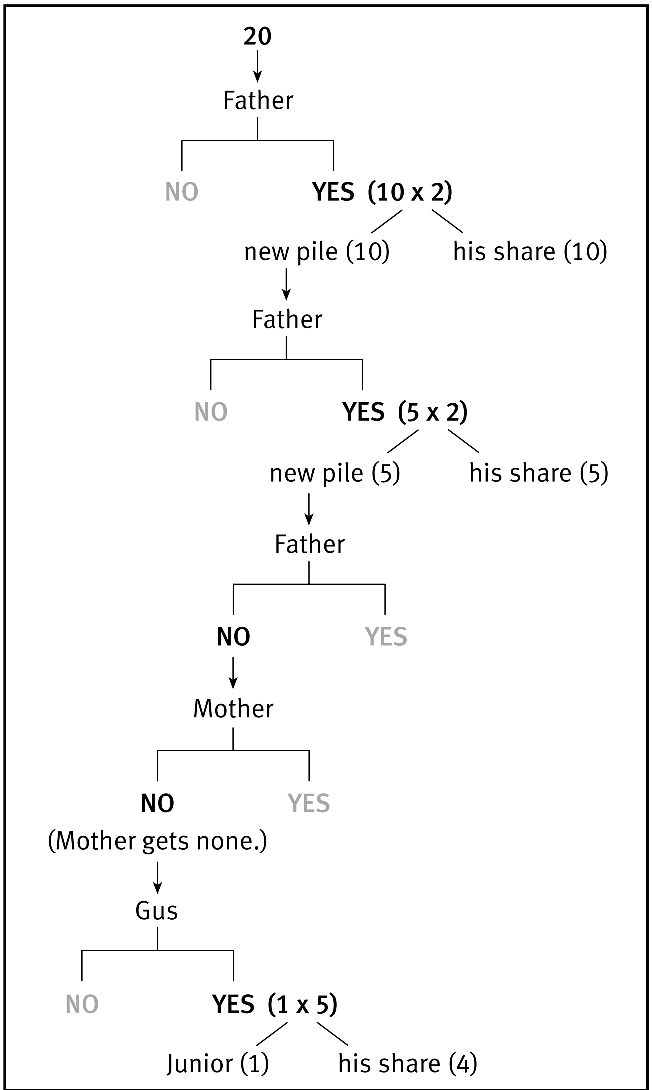
For example, in the first distribution of the 30 bananas shared out in questions **1–4**, the bananas are divided into groups of two bananas: 30 ÷ 2 = 15. One banana from each group goes into the new pile, which means there are 15 bananas in the second round. Father can't use his prime number on this new pile, so he has to be satisfied with the 15 bananas that are not in this pile.

Mother divides the 15 bananas in the new pile into five groups of three (her prime number): 15 ÷ 3 = 5. Again, students need to remember that one banana from each group goes into a new pile. Mother can't use her prime number on this new pile of five bananas, so she gets only the 10 bananas that don't go into this pile.

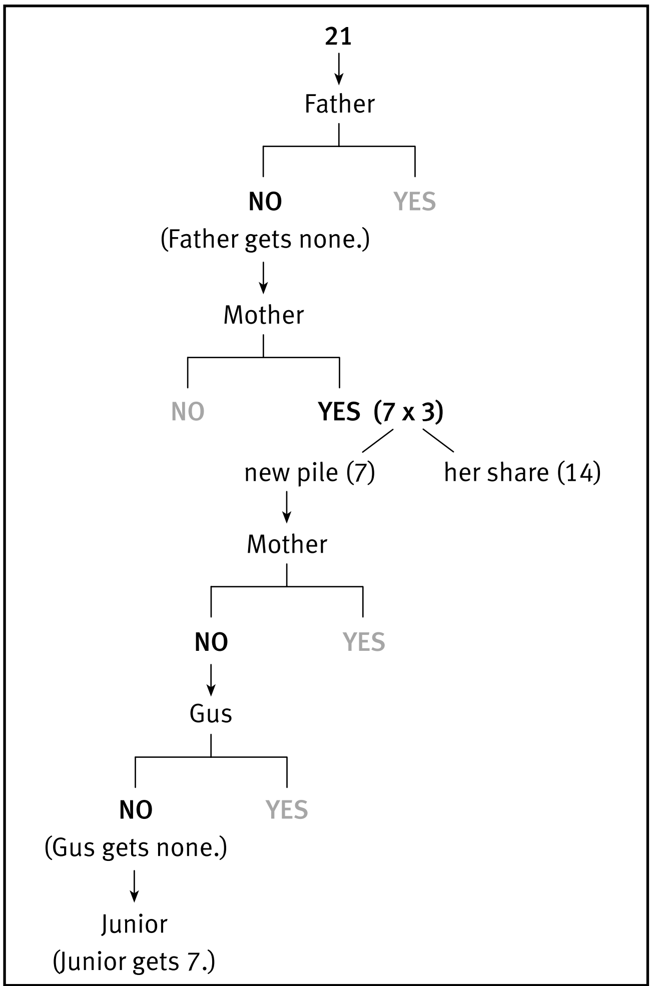
On Gus's turn, he has to divide the new pile created by Mother into groups of five, his prime number. There are only five bananas in the pile, so that is only one group: 5 ÷ 5 = 1. One banana from that pile goes into a new pile (that's what Junior gets), so Gus will get the four bananas that don't go into that pile.

In this process, the students need to think of division as grouping (that is, the number of groups of twos in 30, threes in 15, and fives in 5), not as partition (sharing) into equal parts.

Flow charts for 20 and 21 could look like this:



So:
Father gets 15.
Mother gets 0.
Gus gets 4.
Junior gets 1.



So:
Father gets 0.
Mother gets 14.
Gus gets 0.
Junior gets 7.

The more confident students could try keeping track of developments in a table or using a table as their overall strategy. For example, for question **5b**, with 18 in a bunch, a table could look like this:

Gorilla	Bananas available	First turn (number of groups)	New pile (one from each group)	Second turn (number of groups)	Total received
Father (2)	18	18 ÷ 2 = 9	18 – 9 = 9	Not possible	9
Mother (3)	9	9 ÷ 3 = 3	9 – 6 = 3	3 ÷ 3 = 1	6 + 2 = 8
Gus (5)	1	Not possible	0	0	0
Junior (remainder)	1				1

There are no third-turn splits for 17, so a column for this is not necessary at this stage. You could extend the activity by getting the students to try other numbers to see if they can obtain more equitable shares for the gorillas or by suggesting to the students that they use their knowledge of number to create an equally interesting but perhaps fairer system for sharing the bananas.

Page 4

Going to Extraordinary Lengths

Achievement Objectives

- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also:

- recognise prime numbers
- explore the power of prime factors.

ACTIVITY

This activity provides an unusual context for students to work with prime numbers and discover the power of prime factors.

Discuss what the machines do, perhaps using string or different-length laces to model the situation. You could simulate the process by stretching a rubber band. Ask: “What will the new lace look like when stretched through the 2 machine? … the 6 machine?” and so on. “How long would the lace be after going through a 10 machine? … a 15 machine?” and so on. (This is a good chance to practise mental multiplication strategies.)

When you are sure that the students understand the problem, it may be best to settle them into small groups to explore the problem and plan how they might work through their solution.

Some possible discussion points may include:

- discussing the statement that for any even number, at least one factor is 2.
- taking a general number like 24 and determining which factors can be used for a stretch of this length.
- discussing the statement that many of the numbers appear to be multiples of 2 and 3. (You could show this on a hundreds board.)
- looking at the different ways the prime numbers 2, 3, and 5 can be combined to get the same composites.

Encourage the students to keep a record of numbers that they can eliminate on their hundreds chart.

At the end of the session, discuss the students’ answers and compare the numbers that are left on everyone’s charts. Have the students share the different strategies they used to eliminate unnecessary machines.

A common way to sort through the prime numbers in 1 to 100 is to use a sieve approach (draining out the composite numbers and leaving the primes behind). Eratosthenes (275–194 BC) was a Greek mathematician who developed what has become known as the sieve of Eratosthenes.

The Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The students can use Eratosthenes' idea on a hundreds board, like this:

1. Cross off 1 because it's not prime.
2. Leave 2 because it's the smallest prime, but cross off every multiple of 2.
3. The number 3 is the next prime, so leave this too. Cross off all the multiples of 3. Some, like 6, may already be crossed off because they are also multiples of 2.
4. Leave 5, then cross off all its remaining multiples.
5. The only other number left in the first row will be 7, so cross off its multiples.
6. All the surviving numbers on the hundreds board are prime numbers. These are the machines that the boss needs to keep.

You can extend this problem by increasing the stretch up to 200 times. It should also be possible to look for other features of prime numbers on the hundreds board, such as finding twin primes (prime numbers that have a difference of 2). Ask your students:
"Is there a pattern of difference in the sequence of primes?"
"Are there more primes between 50 and 100 or between 100 and 200?"
"How far apart are prime numbers through the decades?"

Page 5 Boxing Balls

Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also explore factors of numbers by investigating rectangular numbers and prime numbers.

ACTIVITY

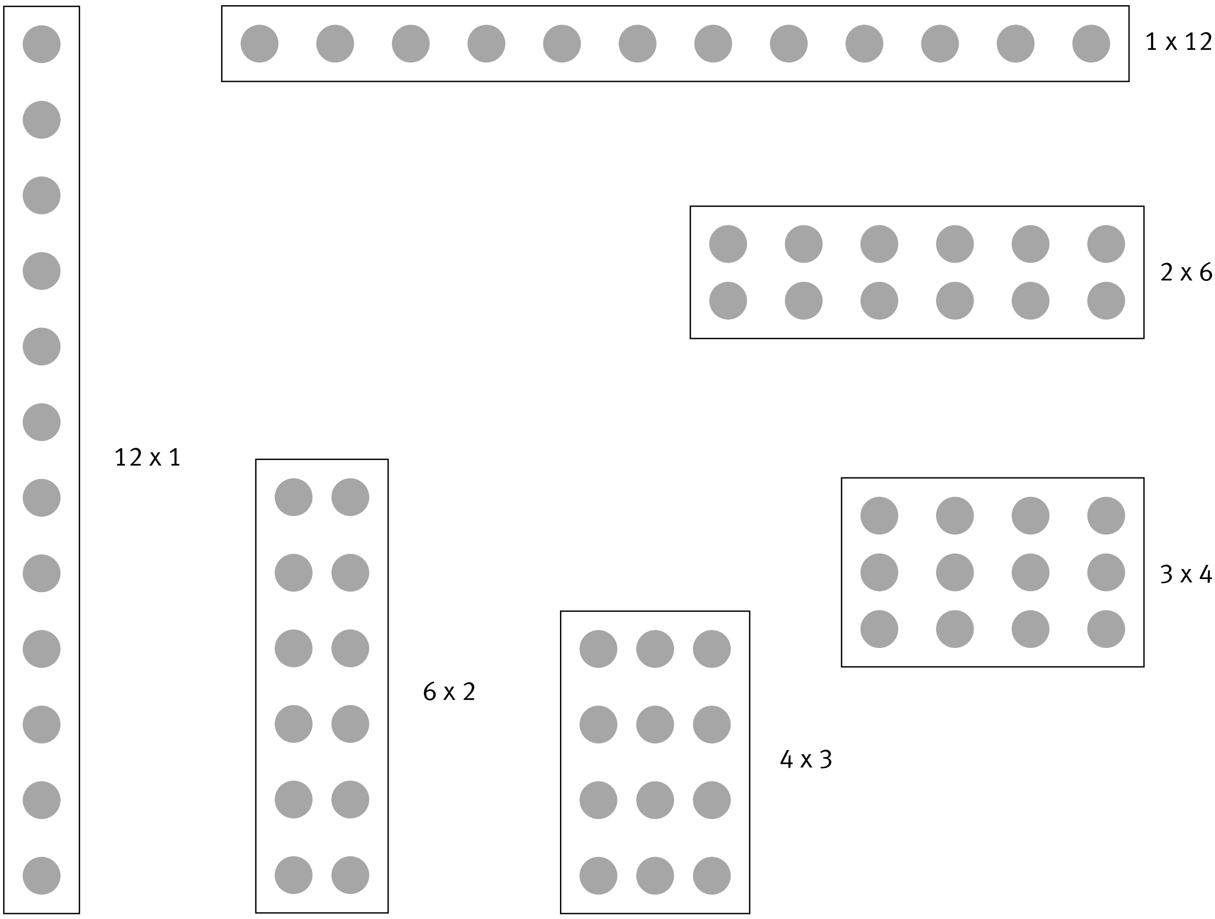
In this activity, the students can manipulate concrete objects or drawings to explore the physical relationship between prime and composite numbers. They will become aware of how prime numbers can only be represented by single-row rectangles, whereas composite numbers have two or more possibilities. The activity also provides an opportunity to introduce a further strategy for quickly finding the factors of a larger number and checking if a number is prime.

As an introduction, you could have the students pair up to find out how many rectangular patterns can be made for 12. Record the patterns on the board. Note that patterns of 1, 2, 3, 4, 6, and 12 rows can be made but that half of them are the same pattern turned 90 degrees. The students can compare this with the patterns for 8 and then, in pairs, complete the rectangular patterns for question **1**. They can record their answers by shading them on grid paper and labelling their shadings with matching equations.

The students need to know how they can be sure that they have found all the arrangements (factors) of a number. This challenge should be explored during teacher support time and when you have the follow-up discussion. Possible questions are:
“Do you need to try out all the numbers to find all the factors?”
“Where can you stop checking to be certain you have found all the factors?”
“Which type of rectangle shows that a number is prime?”
“Which number has the fewest factors? … the most factors?” and so on.

By exploring the problem with counters, cubes, or diagrams, the students may come to discover that checking up to the square root of a number is sufficient and that if they find no factors other than 1 and the number itself, they can be sure the number is prime. You can demonstrate this, if necessary, by structuring a progression of factors on an overhead projector using transparent counters.

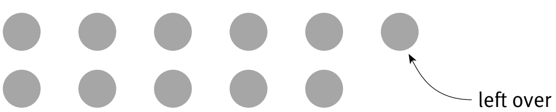
Checking 12:



Once past 3, the factors begin to repeat themselves, and that is how you know that you have found them all.

This idea of repetition will help with question **2**, where the students can check numbers quickly on a calculator up to the square root, which can also be determined on the calculator: $\boxed{149}\,\boxed{\sqrt{}}$.

Prime numbers have only two factors, themselves and 1, so rectangles beyond a single row cannot form prime numbers. There will always be one or more left over. For example, for the prime number 11:



Page 6 Prime Sites

Achievement Objectives

- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also investigate patterns with prime numbers.

INVESTIGATION ONE

One of the most interesting fields of mathematics is number theory, which is the investigation of numbers and their properties and relationships. Mathematicians have studied prime numbers for thousands of years.

The students can show the details of this investigation by circling prime numbers and multiples of 6 in different colours on the hundreds board supplied. A modified grid on squared paper will enable them to see the results of the conjecture in **Investigation One** more clearly:

2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31
32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49
50	51	52	53	54	55

This shows that while the original conjecture does not hold true, each prime number greater than 3 is either one before or one after a multiple of 6.

INVESTIGATION TWO

This investigation is similar to **Investigation One**. It should give the students an opportunity to use a calculator and test some larger numbers up to 200 and beyond, both for the theorem and for their relationship to prime numbers.

As an extension, you could ask the students to:

- explore a prime number theorem known as Goldbach's conjecture. In 1742, Christian Goldbach conjectured that every positive, even integer greater than 2 can be written as the sum of two prime numbers and that every odd number greater than 7 can be expressed as the sum of three odd prime numbers. Find out if the conjecture is right.
- compare the gaps between successive prime numbers in 1 to 100 with those for the prime numbers in 101 to 200. (In general, they are more spread out, and this trend continues as the numbers get larger.)
- explore grids of sizes other than that in **Investigation One** to see if other number relationships exist.
- explore the "perfect number". This is another special number of interest to mathematicians. It is a number that equals the sum of its factors apart from itself. The first one is 6 (from 3 + 2 + 1 = 6). The students can be challenged to find the next two or three (28, 496, and 8 128). After 8 128, the perfect numbers become enormous.

Page 7 Igloo Iceblocks

Achievement Objectives

- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)
- classify objects, numbers, and ideas (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

Students will also use prime factors in a problem-solving context.

ACTIVITY

The students can use the associative property (multiplicative strategies) to solve these problems by regrouping prime factors. Through the activities on previous pages, the students should have developed strategies for breaking down composite numbers into prime factor units (for example, factor trees, dividing up to square roots, and so on). An important feature of these problems is that the students need to realise that they should arrange the cubes in three dimensions. Therefore, three factors (other than 1) are needed to solve the problems.

It could be useful to introduce this idea with a simpler problem. For example, how many different arrangements can be made up from 12 cubes (other than a single-row arrangement)?

H L W
2 x 2 x 3
3 x 2 x 2
2 x 3 x 2

You can arrange the same dimensions in different ways and ask the students what they notice about these numbers. (They are all prime.)

Extending this to an exploration of 24 will establish that dimensions can be a mix of composite and prime numbers, but since all composite numbers can be expressed as a product of prime factors, prime factors are a good starting point for a systematic response using multiplicative strategies.

For example:

24 = 3 x 2 x 2 x 2
= 6 x 2 x 2 from (3 x 2) x 2 x 2
or 4 x 3 x 2 from (2 x 2) x 3 x 2

You can consolidate this activity by encouraging the students to write and share their own Igloo Iceblock problems.

Page 8 It's a Try!

Achievement Objective

- explain the meaning of negative numbers (Number, level 4)

Other mathematical ideas and processes

Students will also add and subtract positive and negative integers.

ACTIVITY

This very practical application and context introduces students to working with integers. The students will have varying degrees of skill in terms of addition and subtraction. Most should have some experience with using number lines with games and studying such things as temperatures, sea levels, bank balances, and so on. The development of mental fluency is made more difficult because it is often difficult to set up realistic situations that produce negative results, for example, 3 – 7 cannot be easily represented directly by physical objects.

Some important basic principles of number lines include:

- The set of integers includes zero, with positive and negative integers spaced evenly on the number line to the left and right of (or above and below) the zero.
- Adding a positive integer is represented by movement to the right on the number line. Adding a negative integer is represented by movement to the left.
- Subtracting a positive integer is represented by a movement to the left and subtracting a negative integer by movement to the right.

Begin the activity with class discussion to make sure that the mathematical information is well understood. The students could work through questions **1a** and **1b** in pairs before again sharing their understanding and answers with the class.

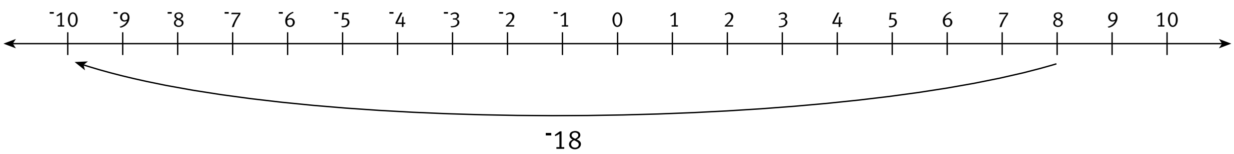
You need to prompt the students' understanding with questions such as:
"What happens when a positive number is subtracted from a positive number?"
"What happens when a positive number is subtracted from a negative number?"

A table of progress will help the students to keep track of each stage. For example:

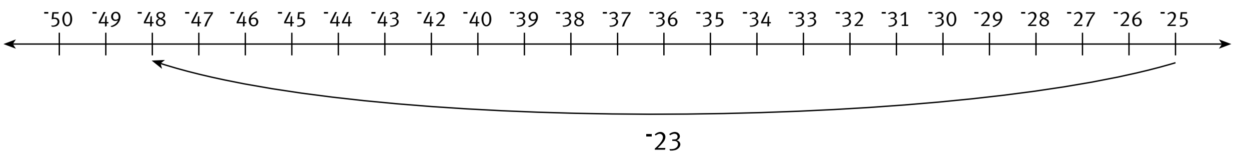
Question number	Start (s)	Wind (s)	Stop (s)	Equation from number line
1a	0:00:08	← 18	$^{-}$0:00:10	$8 - 18 = {}^{-}10$
1b	$^{-}$0:00:10	← 15	$^{-}$0:00:25	${}^{-}10 - 15 = {}^{-}25$
1c	$^{-}$0:00:25	← 23	$^{-}$0:00:48	${}^{-}25 - \square = {}^{-}48$
1d	$^{-}$0:00:48	5 →	$^{-}$0:00:43	${}^{-}48 + 5 = {}^{-}43$
2a	$^{-}$0:00:43	34 →	$^{-}$0:00:09	${}^{-}43 + \square = {}^{-}9$
2b	$^{-}$0:00:09	23 →	0:00:14	${}^{-}9 + 23 = 14$
3	0:00:00	← 23	$^{-}$0:00:23	$0 - 23 = {}^{-}23$
	$^{-}$0:00:23	← 34	$^{-}$0:00:57	${}^{-}23 - 34 = {}^{-}57$

You could encourage your students to support their findings with a number line.

For example, for $8 - 18 = {}^{-}10$:



Using the additive (reverse) method of subtraction would help the students to solve question **1c**. Ask the students to find the two numbers $^{-}25$ and $^{-}48$ on the number line and count or add the difference between them. In this case, the students need to find out the distance from $^{-}25$ to $^{-}48$ in the negative direction.



${}^{-}25 - \square = {}^{-}48$ or ${}^{-}48 + \square = {}^{-}25$

The students could extend this practice context for integers by creating their own bank balance stories using black and red balances, deposits, and withdrawals.

Page 9 Lifting Weights

Achievement Objective

- explain the meaning of negative numbers (Number, level 4)

Other mathematical ideas and processes

Students will also order negative integers.

ACTIVITY

This activity gives the students valuable practice in ordering negative numbers. They will need to subtract two-digit whole and one-place decimal numbers, resulting in an answer that is a negative number.

Question **1** lends itself to discussion. Some students may see the mass lifted as the best gauge of strength, whereas others may realise that assessing strength in terms of body mass and mass lifted has its own merit. This question links up with question **2b**.

In question **2a**, the students need to use the formula given. They could discuss the likely outcome of subtracting 38 from 32 as being six places to the left of zero on a number line before checking their ideas on a calculator, that is, $32 - 38 = {}^{-}6$. (The negative sign may be located on the left-hand side of the calculator display.)

Some initial discussion about ordering negative numbers may also be valuable. Part of this can involve:

- placing different-coloured counters or a peg on a blank number line for given negative numbers and ordering the different colours;
- having a variety of negative numbers printed in large letters on individual pieces of A4 card or paper and asking students to select one of the numbers and line up in the correct sequence at the front of the classroom (the rest of the class can assist with the process);
- small groups doing the two activities listed above but with smaller sets of numbered cards.

Have the students copy and complete the table in question **1**, but get them to add a fourth column for the "mass lifted minus body mass" figures. Check that the students realise that if they subtract the mass lifted from the body mass (for example, 38 – 32), they need to show the lift as a negative number (in this case, ${}^{-}6$). They could add a further column to rank the boys numerically according to strength.

Competitor	Mass lifted (kg)	Body mass (kg)	Strength (Lift – body mass)	Rank of strength
Mike	32	38	-6	3=
Lefu	52	59	-7	5
Pete	26	50	-24	10
Chris	39	44.5	-5.5	2
Nick	40	51	-11	8
Harry	49	58	-9	6
Wiremu	35	48.5	-13.5	9
Dan	41	50.5	-9.5	7
Josh	30	33.4	-3.4	1
Mark	50	56	-6	3=

Note that there are two boys with an equal third ranking, so the rank order for the next boy after that jumps to fifth.

The students could work in groups and record the results for the strength column on a large sheet of paper to share with the class later. They could also show the results visually by making a bar graph of the final scores of the boys' strength.

An extension for Question **2** is to plot points for the mass lifted against body mass on a scatter plot, labelling each point with the rank order for upper body strength and noting the relationship to the diagonal line (which indicates the median) drawn through the middle. Some students may also be keen to investigate this formula, using data derived from an international sports event, such as the Olympic or Commonwealth Games.

For further interest, some students may also enjoy the challenge of expressing strength as a percentage, writing the mass data as a fraction: $\frac{\text{mass lifted}}{\text{body mass}}$, and using the calculator to determine the result.

For Mike, this is $\boxed{32}$ $\boxed{\div}$ $\boxed{38}$ $\boxed{\%}$, which is 84% (rounded). You could ask: "Does this produce the same ranking? Does it help to separate Mike and Mark?" (It does. Mike ends up with 84% strength and Mark with 89%.)

Page 10 Integer Zap

Achievement Objectives

- explain the meaning of negative numbers (Number, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also subtract positive and negative integers.

GAME

Do not use this game to introduce your students to the subtraction of negative numbers, which is quite a difficult concept to grasp. The game has more value as a reinforcement activity.

Before they begin this game, the students may need to review and investigate some situations involving the subtraction of negative numbers. Subtracting an integer is the same as adding its opposite.
For example:

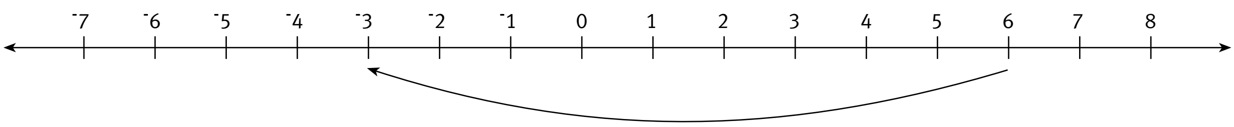
$7 - 3 = 4 \longrightarrow 7 + {}^{-}3 = 4$	
$7 - {}^{-}3 = 10 \longrightarrow 7 + 3 = 10$	

Therefore, instead of subtracting a negative number, we can add the corresponding positive number.

Small groups may explore a representative set of integer subtraction problems.
For example (answers are given in brackets):

${}^{-}2 - {}^{-}4 = (2)$	${}^{-}3 - 6 = ({}^{-}9)$	$3 - {}^{-}6 = (9)$
${}^{-}4 - 2 = ({}^{-}6)$	$6 - {}^{-}3 = (9)$	
$6 - 3 = (3)$	$3 - 6 = ({}^{-}3)$	

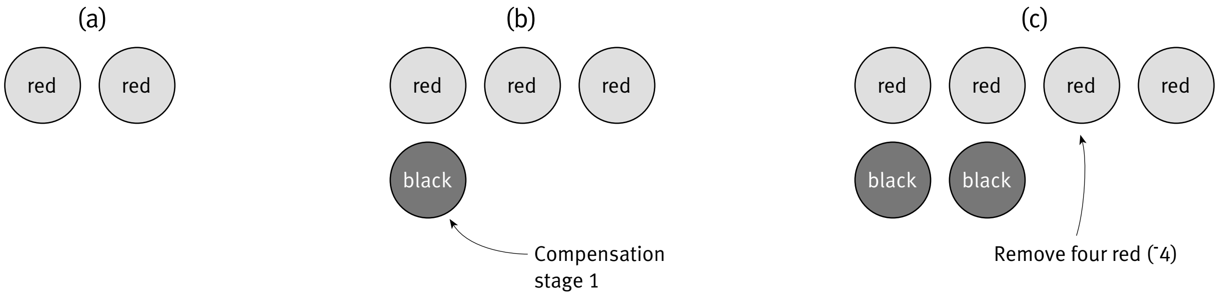
Another way the students can subtract integers is to apply an additive approach using inverse operations. For example, in the equation ${}^{-}3 - 6 = \square$, the ${}^{-}3$ is the sum and 6 is the addend. So, what number when added to 6 will give a sum of ${}^{-}3$, or $6 + \square = {}^{-}3$? As 6 would need a movement of ${}^{-}9$ to reach the position ${}^{-}3$, the answer in the original equation must be ${}^{-}9$.



$6 + {}^{-}9 = {}^{-}3$

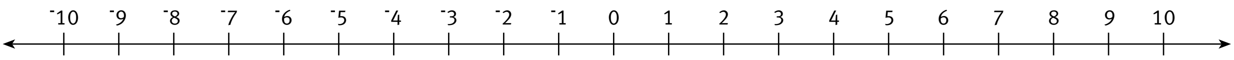
While the equations listed on the previous page cannot all be modelled directly using actual materials, it is possible to explore the problems using counters and compensation strategies. For example, using $^{-}2 - {}^{-}4 = \square$, ask: “What does this equation say?” (Begin with negative 2 and subtract negative 4.) “How can we do this?” Give the students two sets of coloured counters, for example, red and black. Invite discussion, perhaps using bank balances. Red could represent negative numbers, and black could represent positive numbers.

Ask: “How can you take away four negative counters when you have only two negative counters?”
Have the students add on a negative red counter and a positive black counter, which will keep the relationship equivalent to the original amount. They now have three red and one black.
“Can we now subtract four negative counters?” The answer is still no.
“Add another negative red counter and a positive black counter. You now have four red counters and two black counters. Take away the four negative red counters. What answer remains?” (Two positive black counters.) This is one way of modelling $^{-}2 - {}^{-}4 = 2$.



The students could explore the equations above, using strategies such as reverse additive, number line, or models with calculator checking. To play the game successfully, they need to become proficient at knowing what to do when thinking about:
positive – positive
positive – negative
negative – positive
negative – negative

A possible number line could be:



The students could use it by following these steps:

- Circle the starting number.
- Add on a positive number by moving to the right (for example, $1 + 2 = 3$). Subtract a positive number by moving to the left (for example, $1 - 2 = {}^{-}1$).
- Add on a negative number by moving to the left (for example, $1 + {}^{-}2 = {}^{-}1$). Subtract a negative number by moving to the right (for example, $1 - {}^{-}2 = 3$).

The process of subtracting negative integers may stretch some students, but the more they play the game, the better and more proficient they will become at mental subtraction with integers.

Check that the students are selecting the correct higher value card, especially when both cards are negative numbers. For example, $^{-}5$ is a higher value than $^{-}9$. While you should encourage mental subtraction, some students may need the support of being able to jot down their workings on scrap paper, use a number line, or even use a calculator until they feel able to do the subtraction mentally.

Page 11 Integer Slam

Achievement Objective

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

Other mathematical ideas and processes

Students will also add positive and negative integers.

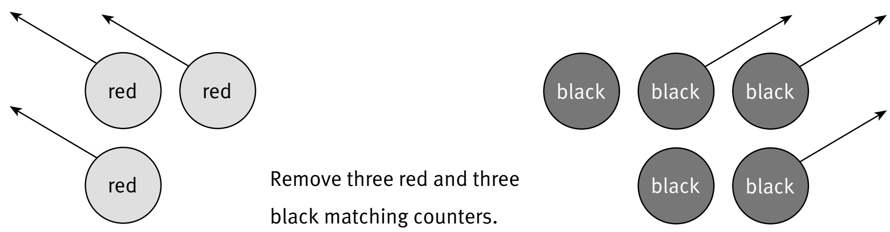
GAMES ONE AND TWO

Both games are similar to Integer Zap but also give practice with adding integers and working with three or more digits to achieve a set sum of 12 or $^-12$. The students will need to have developed some understanding of and confidence in adding small integers to cope with the calculations required. They need to be able to work with:

- positive + positive (for example, $5 + 3 = 8$)
- negative + positive (for example, $^-3 + 5 = 2$)
- positive + negative (for example, $5 + {}^-3 = 2$)
- negative + negative (for example, $^-5 + {}^-3 = {}^-8$)

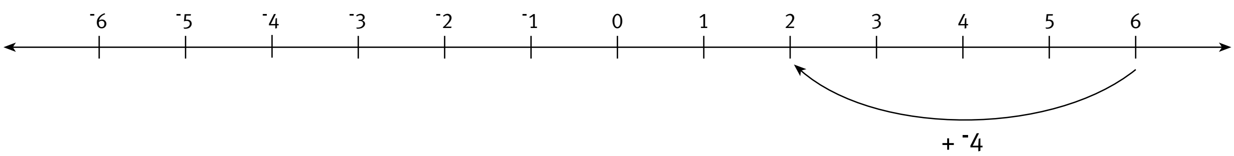
Students may benefit from some practice at solving problems such as those listed above. Challenge them to think out the answers in their head. Discuss and share responses to the question “Can you tell the class the answer and how you got it?”

Some students may need to model the problems using a two-coloured counter set. For example, for $^-3 + 5 = \square$:



Each positive and negative matching pair of counters cancel out because together they equal zero, which leaves two positive counters.

You could ask the students to explore related addition and subtraction facts using integers on a number line. The number line below shows that $6 + {}^-4 = 2$.



From this, the students can derive the rest of the family of facts.
For example:

$6 + {}^-4 = 2$	$2 - {}^-4 = 6$
$^-4 + 6 = 2$	$2 - 6 = {}^-4$

The students should also be able to solve these problems on a calculator once they are reasonably proficient at mental imaging. They can use calculators to check their thinking about adding and subtracting integers. For example, to add $^-3 + 6$ on the calculator, enter 3 $\boxed{+/-}\ \boxed{+}\ \boxed{6}\ \boxed{=}$. (On a DAL calculator, enter $\boxed{+/}\ \boxed{3}\ \boxed{+}\ \boxed{6}\ \boxed{+}\ \boxed{=}$.) The $\boxed{+/-}$ key will change the calculator from positive to negative. Check that the students press this key after they have entered the number. They can press the operation key in the usual way straight afterwards without overriding the negative integer sign.

While the students are playing either of the games on this page, you can watch to see who is able to carry out integer addition mentally and who needs further assistance. **Game Two** is an extension of **Game One.** It may be extended further by allowing both addition and subtraction to be used or, alternatively, by allowing bonus points to be scored for other combinations that can be found to give 12 or $^{-}12$ in the same hand. One student may be assigned the job of verifying answers on the calculator. As players get better, you could set a time limit for each turn.

Page 12 Integer Links

Achievement Objectives

- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

Other mathematical ideas and processes
Students will also add integers.

GAME

A copymaster of four different game boards is provided at the end of these notes.

This game will help the students to practise adding up integers and at the same time encourage thinking ahead to work out the best moves. Players will quickly learn to avoid closing off the higher negative numbers.

The students can either keep a running total on a calculator or on paper or wait until the game has finished to add up the integer value of their shapes. You could ask them: "Does it matter in what order the numbers are added together?" "Which way do you move on the number line when adding a negative number?" Then review the rules for adding integers. Another interesting discussion could focus on the strategies used by the students who worked out their totals mentally.

You can extend this game by having the students create a slightly bigger grid version that could be used by three to four players.

Page 13 Family Trees

Achievement Objective

- explain the meaning and evaluate powers of whole numbers (Number, level 4)

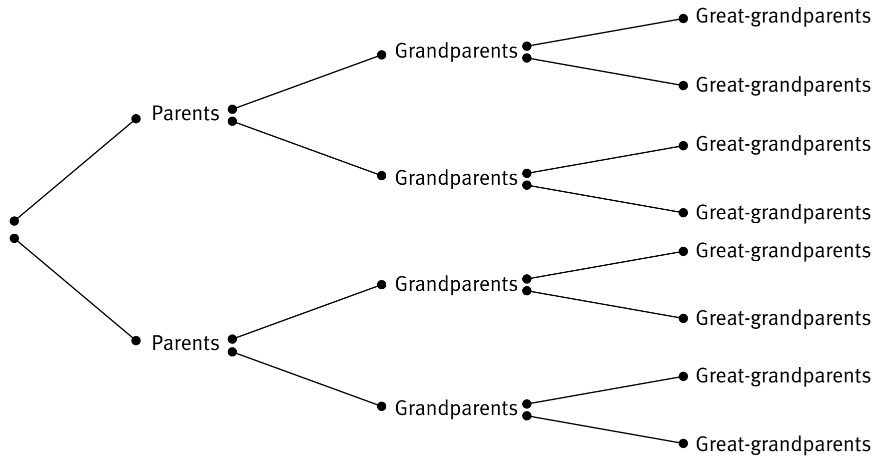
Other mathematical ideas and processes
Students will also add integers.

ACTIVITY

Be aware when using this activity that family trees can get quite complicated if the students want to include relationships and second or third marriages. It is important not to make assumptions and to handle this activity with sensitivity.

An exponent indicates how many times a factor is repeated. For example, the small "5" in 2^5 is an exponent telling how often 2 is used as a factor in naming the product. Products expressed using exponents, for example, 2^5, are called powers. The fifth power of 2 is 2^5, which is $2 \times 2 \times 2 \times 2 \times 2 = 32$. Exponents are used to save time and space when writing out mathematical equations or large numbers.

In question **1** of this activity, a tree diagram makes it easy to visualise how the number of grandparents progresses.



The students need to be aware that they must calculate a cumulative total of all three generations and not just the last in the sequence. To this end, they may find it useful to develop a table, such as the one below, as a problem-solving strategy for the more complex calculations required in questions **2–4**.

Generation level	Power of 2	Total	Cumulative totals
(1) Parents	$2^1 = 2$	2	2
(2) Grandparents	$2^2 = 2 \times 2$	4	6
(3) Great-grandparents	$2^3 = 2 \times 2 \times 2$	8	14

For question **2**, it is a matter of establishing the time span for one generation by dividing the years by the number of generations involved (75 ÷ 3). Then the students need to work out the number of generations altogether by dividing the total number of years by the time span for one generation (175 ÷ 25). Finally, they must extend the table created earlier so that they can include this seventh generation of grandparents.

After students working in small groups have attempted question **2**, they should be able to share strategies with the class. You should also be able to ask: “How could you use your calculators with this investigation? What patterns have you seen?”

The total researched for each generation progresses in a doubling sequence that is easy to generate using the calculator constant: $\boxed{2}\ \boxed{x}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{128}$. Note that there is always one more power of 2 than the number of $\boxed{=}$ pressed because the 2 entered initially before the $\boxed{x}$ needs to be included as a power. On a calculator with a y^x key, you can enter $\boxed{2}\ \boxed{y^x}\ \boxed{7}\ \boxed{=}$.

For the number researched, the cumulative total will always be 2 less than the total number of the next generation. The students can choose between keeping a cumulative frequency or waiting until they have all the generations listed and simply adding each generation, as outlined in the Answers.

The students can calculate question **3** by pressing the constant keys for multiplying by 2 and counting the sequence until they reach 1 000. Question **4** builds further on the strategies for questions **2** and **3**. By now, the students will probably have realised the advantages of using a calculator. As an extension, some students could write and solve their own exponential stories. You should check that their stories make sense.

Building Squares

Achievement Objective

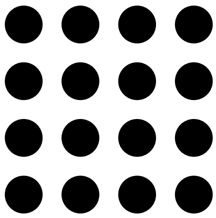
- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also investigate the properties of square numbers.

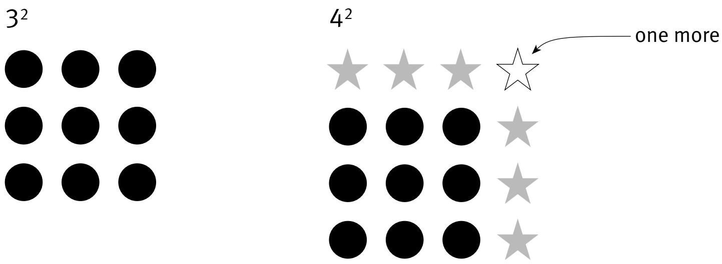
ACTIVITY

Square numbers are one type of polygonal number. They are called square numbers because when a pattern of evenly spaced dots is drawn to represent that number, the dots form a perfect square. This activity will enable students to investigate the square pattern and become familiar with how squares can be represented using exponents. For example, the “ ^{2} ” is always used to mean squared.

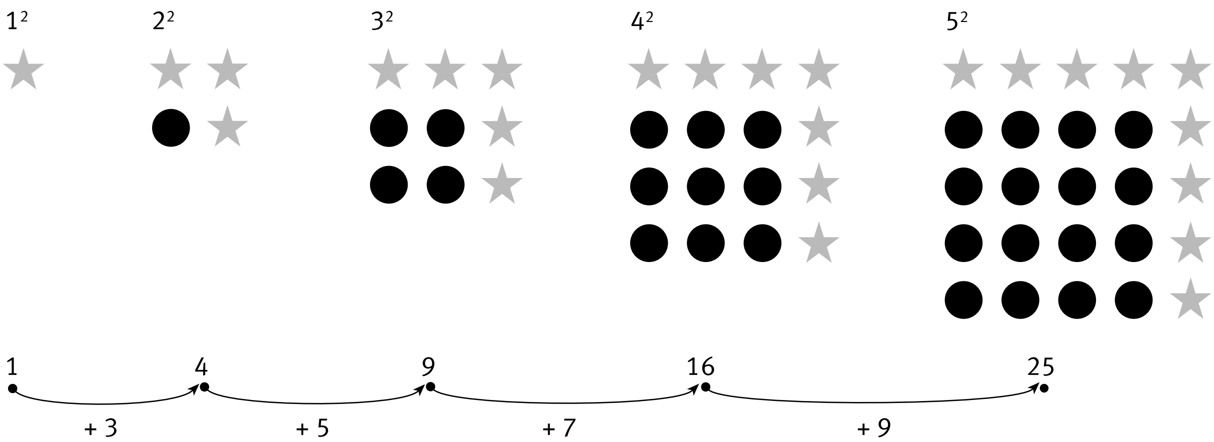


Four squared or 4^2 or 4 x 4

By building patterns with counters or square tiles or by shading squares up to 5^2 on grid paper, the students should be able to see a pattern developing. They can extend the sequence by building onto a smaller square to create the next largest square. They will thereby see that each larger square is formed by adding two side lengths and one more to the previous square total.



To look at the bigger picture in question **2**, a table would be useful alongside the models.



(Side length)^2	1^2	2^2	3^2	4^2	5^2
Square number	1	4	9	16	25
Difference	+ 3	+ 5	+ 7	+ 9	

In question **3**, encourage your students to use additive and multiplicative strategies as they think out their answers.

Question **3a i** can be reasoned using the formula "add two sides and one more to the previous square total", for example, $10^2 + (10 \times 2 + 1) = 11^2$, or from the progression of odd-numbered differences (3, 5, 7, … 21).

For question **3a ii**, the students could look for two consecutive numbers that total 19. However, you could encourage the students to look at it in terms of a similar patterning response to question **3a i** or to use the additive inverse to subtraction. You could ask: "What square number added to 19 will give the next highest square number in this sequence?" and so on.

$19 + \boxed{}^2 = (\boxed{} + 1)^2$

$19 + \boxed{}^2 = (9 + 1)^2$

$19 + 9^2 = 10^2$

The inverse of this is $10^2 - 9^2 = 19$.

Encourage students who wish to extend this investigation further to look for other patterns in consecutive square numbers. For example, they could observe that the square numbers alternate between even and odd. You could ask them questions such as:
"Is there a pattern in the ones digits?"
"Are there more square numbers in the second hundred (101–200) than in the first hundred (1–100)?"
"What patterns can you find between consecutive triangular numbers?" (1, 3, 6, 10, …)
"What triangular numbers can you combine to make square numbers?" (For example, 3 + 6 = 9)

Page 15 Shifty Subtraction

Achievement Objective

- explain satisfactory algorithms for addition, subtraction, and multiplication (Number, level 4)

Other mathematical ideas and processes
Students will also use negative numbers in subtraction.

ACTIVITY ONE
Questions **b** and **c** in this activity are open ended so that the students can explore the process of subtracting a larger number from a smaller number. Some students may choose to use the additive reverse to solve question **a**, that is, $\boxed{^-3} + \boxed{} = 17$.

ACTIVITY TWO
Students who have been encouraged to think through subtraction using part-whole knowledge of numbers and place value should find the novel method of subtraction in this activity an interesting investigation. Your challenge is to help your students make sense of this method so that they can apply it to the expressions that they are asked to find answers for.

Anyone who says "You can't do 4 take away 7" is actually wrong. So the overall discussion needs to centre round the question: "What is happening here?" This is a method of subtraction that doesn't adjust the sum above the answer line. If used in a more traditional working form, the algorithm would look like this:

$$\begin{array}{rcl} 64 & = & 60 + 4 \\ \underline{-\ 37} & = & \underline{-\ 30 + 7\quad} \\ & & 30 + {}^-3 = 27 \end{array}$$

You could put this question up on the board: "Lefu has used this method to find the answer, but he cannot explain how he did it. How can Lefu explain this method of subtraction so that the teacher understands what he did?" The students could discuss it in small groups.

Another possible strategy to help your students make sense of and use this method of subtraction is to cover part of the working and see if they can complete the missing part.
For example, with “80 = $^{-}40$” hidden:
142 – 83
2 – 3 = $^{-}1$
40 – 
100 – 40 – 1 = 59

Page 16 Calculator Power

Achievement Objective

- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also:

- explore number patterns, including powers
- use calculators to investigate powers.

ACTIVITY

In this activity, the students use a calculator to find powers of numbers and to investigate patterns.

You could introduce the activity by asking the students how they could work out 2^4 on the calculator. They need to realise that 2^4 is really 2 (x 2) three times. On most basic calculators, you press $\boxed{2}\boxed{x}\boxed{=}\boxed{=}\boxed{=}$ to get 2^4. (On some models, as pointed out on the students’ page, you may need to press $\boxed{2}\boxed{x}\boxed{x}\boxed{=}\boxed{=}\boxed{=}$).

Point out that the pattern on the wheel is symmetrical. Later, the students can compare it with patterns produced by the other powers during the investigation. The ones digits of powers of 2 repeat the sequence 2, 4, 8, 6, … For question **2a i**, the students may be able to think out that 2^9 is two cycles and one more number. For **2 a ii**, they could use similar reasoning or perhaps observe that the ones digit 8 can be skip-counted in fours from 2^3, that is, 2^3, 2^7, 2^{11}, 2^{15}. Likewise in **2 a iii**, 2^{20} can be skip-counted in fours from 2^4, 2^8, 2^{12}, 2^{16}, 2^{20}.

For question **3**, if the students realise that powers of 2 have a doubling pattern, they should quickly reason that 2^{20} is not twice 2^{10}. If they don’t, use a simpler example, such as comparing 2^2, which is 4, and 2^3, which is 8, and ask if their answer makes sense.

INVESTIGATION

The students can develop other powers in much the same way as they did in the activity. A spreadsheet could be used to compare all these powers. Ask the students to do the following: “Type 3 in cell A1. In cell A2, type A1*3. Drag down the A column to list all the powers of 3. Repeat this process in columns B, C, D, and E for the other powers and compare the columns.”

When the students are using the calculator constant, ask them to estimate: “Which power of 7 will fill up the calculator display?” (The tenth power of 7.)

Page 17 Cubic Capacity

Achievement Objective

- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also explore cubic numbers using powers of 3.

ACTIVITY

In this activity, the students explore cubic capacity using powers of 3. They will need to model cubic shapes and also (for question **3**) visualise hollow cubic shapes, with the cubes that do not form part of the faces removed from the inside. Cubic capacity is significant because it forms the basic structure for measuring the volume of a shape as well as being a special type of number that has a form that is a consistent shape.

2^3 is read as "two cubed" or "two to the power of three", meaning the same as 2 x 2 x 2 in extended notation.

In question **1**, you could use group discussion as an alternative to pairs. If necessary, prompt the students to recall how they work out the area of a square and to think out how the number of cubes in the whole shape **i** can be generated from their workings for the area of a square. You could use an imprinted decimetre cube as a model to demonstrate or verify answers or to review the dimensions of the length, width, and height. Multilink cubes can be used to represent smaller cubes.

Rather than the students counting individual cubes, encourage them to think of each model as one layer times the number of layers. For example, 4 x 2 = 8 is 2 x 2 x 2 (2^3).

Question **2** introduces a more formal recording process. This can be summarised in a brief table with the exponents fully expanded.

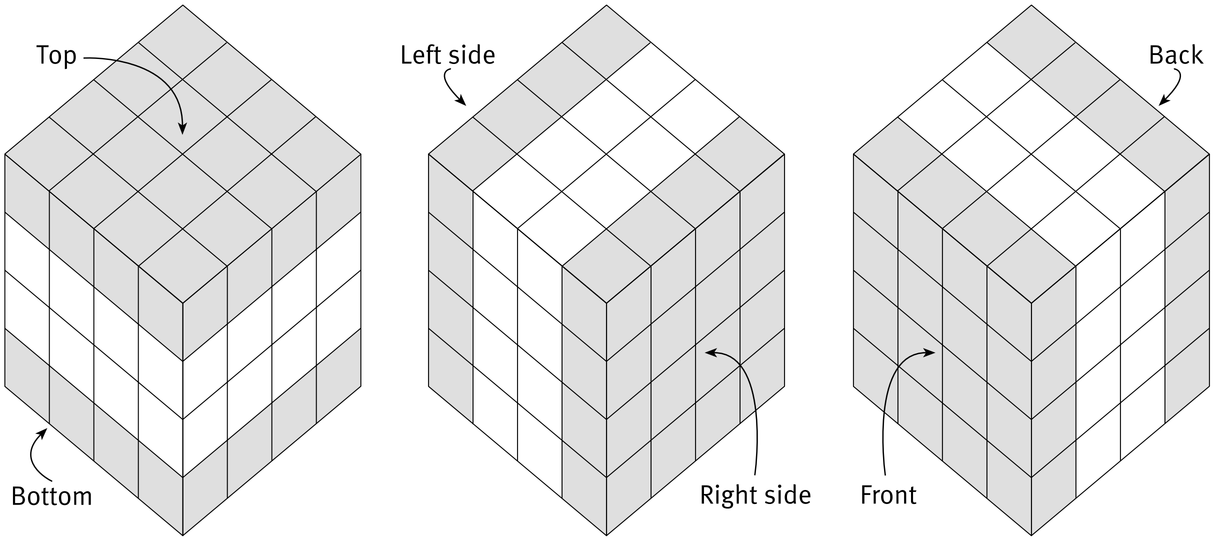
Exponent	Meaning	Value
5^3	5 x 5 x 5	125
3^3	3 x 3 x 3	27
10^3	10 x 10 x 10	1 000
4^3	4 x 4 x 4	64
8^3	8 x 8 x 8	512

As an extension, you could challenge the students to find the number that, when cubed, has a value of 343 or to find other similar numbers. For example, "What is the largest number that, when cubed, could be shown on your calculator display?"

In question **3**, the students may first try building a model to solve the simplest case or they could visualise and then check their thinking by building a model. For the bigger sized cubes, organising data systematically in a table may enable the students to predict the dimensions of the models and to see a pattern without building them. They could observe that the hollow space from any solid cube will be 2^3 less than the original cube. For example:
A solid shape of 5^3 = 125 cubes. To find the hollow space in a 5^3 cube:
$(5 - 2)^3 = 3^3$
$= 27$
Cubes in walls: 125 – 27 = 98 cubes

A Cubic capacity (n^3)	B Missing cubes in hollow space $(n - 2)^3$	C Total cubes (A – B) or $n^3 - (n - 2)^3$
3^3 (27)	$1^3 = 1$	26
4^3 (64)	$2^3 = 8$	56
5^3 (125)	$3^3 = 27$	98
10^3 (1 000)	$8^3 = 512$	488

Some students may need to visualise the overlap between the adjacent faces of each cube pictorially before they calculate the inner space.



Page 18 **Growing Pains**

Achievement Objective

- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also use exponents to solve problems.

ACTIVITY

The questions in this activity are based on sequences of powers, which increase at a constant rate. Students will gain an appreciation of what is meant by an amount increasing "exponentially". Encourage your students to think about the strategies they might use as they solve these problems.

The students are asked to use a table to organise the data. They could also use a systematic tree diagram, although it may start to become too congested to be manageable. Nevertheless, a tree diagram could be used in the initial exploration for question **1**, especially if it is done on a large piece of paper.

Some students will be able to use the knowledge they have gained from previous activities to make the connection between the months and the appropriate power. This may prompt them to use the calculator constant in conjunction with the table, for example, 2^3 = $\boxed{2}$ $\boxed{x}$ $\boxed{=}$ $\boxed{=}$ for the after-3-months growth by a two-split shrub. Students who are not calling on their previous knowledge to solve the question efficiently should be prompted by questions such as: "Is there a quicker way to calculate the tree growth?" "Can you remember a shorter way of writing that equation?" This will help them to recall the links between addition, multiplication, and exponents.

For their interest, the students may also compare the exponential sequences if the relationship between months and number of shoots is graphed on a number plane.

You can extend the activity by asking the students to apply the idea of exponential growth by writing their own similar problems for a different context. For example: "A rumour is being spread around our city, which has a population of X. If each person who heard the rumour passed it on to three other people the following day, how long would it take for everyone in our city to hear the rumour?"

Page 19 Fold and Crease

Achievement Objective

- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also use exponents in problem solving.

INVESTIGATION ONE

This is a practical investigation using strips of paper. The students will find it easier to solve the problems by first working out some simpler problems and then setting up a table and analysing the relationship they see from that. Encourage the students to use powers when they are describing patterns.

For example, **Investigation One** can be summarised as:

Number of folds	1	2	3	4	5	6	7	8	9	10
Number of creases	1	3	7	15	31	63	127	255	511	1 023
Number of sections	2	4	8	16	32	64	128	256	512	1 024

The number of sections doubles progressively. Sections can be expressed as 2n where n = the number of previous sections. Likewise, the number of creases is 2n – 1. It will be physically impossible to fold the paper more than about six or seven times, so encourage the students to extend their tables into the hypothetical before they communicate their findings.

INVESTIGATION TWO

This is a similar investigation, but this time it is based on sections following the pattern 3n, so a piece of paper folded in thirds three times will be 3 x 9 = 27 sections.

INVESTIGATION THREE

This investigation gives the students a good opportunity to summarise expressions using powers to save space. You can use 2 x 3 x 2 x 3 = 2^2 x 3^2 to begin the process of gathering up terms to simplify expressions. The students should find it rewarding and challenging to discover and explain the relationships they have found and to learn to express the patterns.

Page 20 Pip's Pay

Achievement Objective

- explain the meaning and evaluate powers of whole numbers (Number, level 4)

Other mathematical ideas and processes

Students will also use exponents in problem solving.

ACTIVITY

Pip's pay can be calculated in various ways or even acted out with paper money. However, in the ensuing shared discussion, although Kirsty incorrectly used 2^6, her method should be examined and the relevant exponent notation consolidated and encouraged (that is, 100 x 2^5).

The question involves doubling or using powers of 2, except that compared to previous activities, there is now an extra variable involved, namely $100.

You can help your students reflect on the solution and strategies used by writing the problem up on the board and having them work in small groups before they examine the options offered in the students' book. They can then compare these options with their own thinking. Some groups may use the same methods as in the students' book, or they may come up with different methods.

Students having difficulty could first solve a simpler problem using a smaller amount, such as $10. They could also act out the problem with play money or set out the problem in a table.

Year of earnings	Calculation	Total paid
First season	$100	$100
Second season	$100 x 2 or ($2^1$)	$200
Third season	$100 x 2 x 2 or ($2^2$)	$400
Fourth season	$100 x 2 x 2 x 2 or ($2^3$)	$800
Fifth season	$100 x 2 x 2 x 2 x 2 or ($2^4$)	$1,600
Sixth season	$100 x 2 x 2 x 2 x 2 x 2 or (2^5)	$32,000

After the students in each group have completed the problem themselves, you could put the four options as speech bubbles on a large chart and have each group indicate on the chart which method they think is best. Leave a space to display other ideas that the students have developed.

Some students may have used the constant function on their calculator. Be aware that some students will be confused by which key is being held constant by the calculator. Most calculators will have the first key as the constant, for example, the 2 in $\boxed{2}\boxed{\text{x}}\boxed{100}\boxed{=}\boxed{=}\boxed{=}$, but some students may think the reverse. If a scientific calculator is available in the maths corner, you could show the students how to calculate exponents with it as an extension. The exponent key is labelled x^y, so to calculate $100 x 2^5, the students need to press $\boxed{100}\boxed{\text{x}}\boxed{2}\boxed{\text{x}}\boxed{x^y}\boxed{5}\boxed{=}$. Some of the earlier activities in this student book can also use this process.

Page 21 Money Matters

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes
Students will also explore positives and negatives in a money context.

ACTIVITY
This activity gives students experience in sorting through relevant information and thinking out how to use it in a way that makes sense in terms of the questions being asked. Working through information systematically and using logic and reasoning will help the students to keep track and write appropriate equations. It may be beneficial to solve question **1** together co-operatively as a class or group, with you guiding the discussion.

A useful starting point would be to collate all the relevant information about Chris. We know that Chris:
- has $50
- owes $50
- buys a CD
- has $27 left with two bills to pay.

Ask the students: “How do we calculate the cost of the CD?” ($50 – $27)
For question **4**, the students will need to recognise that Chris will be disqualified because, even if he used his $27 to repay debt, he would still owe $23.

In question **2**, we know that Dion:
- has $100
- owes $100
- makes $35.

His balance could be found by either ($135 – $100) or ($100 – $65) or ($100 – $100 + $35).

Also in question **2**, we know that Maria:
- has $0
- owes $0
- earns $40
- buys three guitar strings
- finishes with $35.

So her guitar strings cost $40 – $35.

Ana in question **3**:
- has $80
- owes $80
- pays $104 for radio advertisements
- has $55 left. (She has $20 more than Dion or Maria.)

So she must have made $104 + $55 = $159 from her garage sale.

Another way to work through this activity is to summarise each person's story so that it can be traced in a table. In the second column, put the amount each contestant had to begin with and in the third, put the debts they have to work with. The fourth and fifth columns can be for purchases (losses) or earnings (gains) and the sixth column for the new balance.

The students will still need to read each person's situation carefully and summarise it with appropriate equations.

* * * * * Make Your Fortune * * * * *					
Contestant	Starting cash	Debts	Purchases (losses)	Earnings (gains)	New balance
Chris	$50	$50	?	–	$27, with $50 debts
Ana	$80	$80	$104	?	?
Maria	$0	$0	?	$40	$35
Dion	$100	$100	–	$35	?

Pages 22–23 Starting with Stamps

Achievement Objectives

- explain the meaning and evaluate powers of whole numbers (Number, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)

Other mathematical ideas and processes

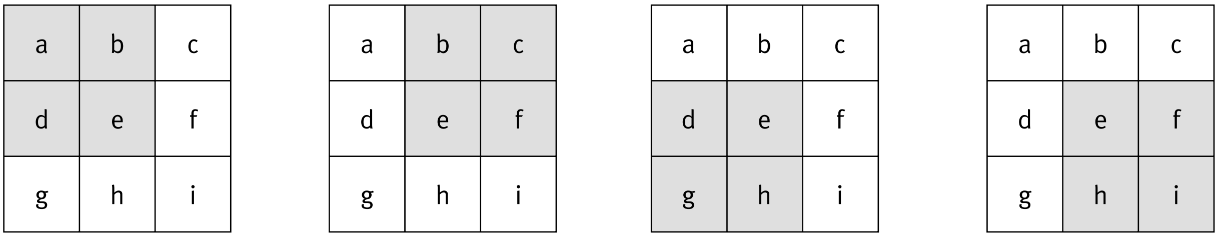
Students will also:
- investigate square numbers
- explore number patterns.

ACTIVITY ONE

In this activity, students are able to use their knowledge of squares and number patterns and use spatial visualisation to solve the puzzle. Some students may think that three ways must be the answer to question **1** because 1^2, 2^2, and 3^2 are the types of square shapes that can be made. Ask those students if there are other ways a 2 by 2 square can be divided up from the block of stamps.

A possible introduction could be to cut a 3 by 3 square block into 1 by 1 and 2 by 2 parts to show the progression of squares. The students can then use the 2 by 2 square to work out where to put their pieces. Discuss what they are doing to check the possibilities.

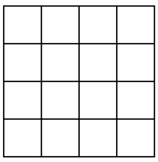
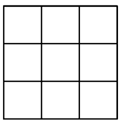
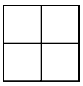
Another approach is draw the larger blocks on grid paper and shade in the possible squares. For example, for 2 by 2 squares from a 3 by 3 block:



Using this system, the students could then record their answers using symbols, for example:
1 x 1 stamp block: a, b, c, d, e, f, g, h, i = 9
2 x 2 stamp block: a b d e, b c e f, e f h i, degh = 4
3 x 3 stamp block: a b c d e f g h i = 1
9 + 4 + 1 = 14.
Coincidentally, all are square numbers and also add up to a square of a number.

Using question **2** as a starter, the students may now begin to investigate forming squares within larger square areas. Encourage some initial conjecturing:
"What area would the next square be?"
"What would the length of the sides be?"
"How many ways altogether do you think squares of stamps could be ripped off?"

Have the students record the patterns they make in a table. For example, for a 4 by 4 block:

Diagram of square	Square sides	Number of blocks	Increases in the number of ways
	4 x 4 (4^2)	1	
	3 x 3 (3^2)	4	+ 3
	2 x 2 (2^2)	9	+ 5
	1 x 1 (1^2)	16	+ 7

The students could extend the investigation to see how stamp blocks of 5^2 (5 x 5) would develop and then go on to predict a 20^2 and 50^2 block of stamps.

Encourage the students to find patterns in the square numbers and share their discoveries. They may notice that the number of blocks increases by one square number each time. A 3 by 3 block would have 1 + 4 + 9 = 14 ways it could be divided, and a 4 by 4 block would have 1 + 4 + 9 + 16 = 30 ways.

ACTIVITY TWO

In this activity, the students can explore further sequences of odd and square numbers but without reference to models of square numbers to give visual links. They may discover that no matter how many markers are put out, as long as they are extended by a consecutive sequence of odd numbers beginning from 1, the distance one way will always add up to a square number, as in:

$1 + 3 = 4$ (2^2)
$1 + 3 + 5 = 9$ (3^2)
$1 + 3 + 5 + 7 = 16$ (4^2).

The distance one way will also:

- relate to the previous distance
- get increasingly longer
- alternate between even and odd.

Extending the number of markers further should enable the students to investigate and discuss how the pattern of square numbers develops.

ACTIVITY THREE

This is another problem involving square blocks and numbers. This time, the largest shape has to be divided evenly into squares, and overlapping sections are not relevant as they were in **Activity One**. Encourage the students to use the power of 2 and to remember that a square number is formed when a number is multiplied by itself. Remind them that a number varied to the second power, such as 4^2, can be described as 4 to the power of 2 or 4 squared. (See also the notes for page **14** of the students' book.)

Advanced-counting students may prefer to use cut-out templates measuring 1^2, 2^2, 3^2, and 4^2.

Extending or reducing the size of the square region should enable the students to investigate if the number of sprinklers needed would also be a square number.

Page 24 Superior Side Lengths

Achievement Objective

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning)

Other mathematical ideas and processes

Students will also:

- explore number patterns involving square roots
- find square roots.

ACTIVITY

Question **1** introduces students to a visual notion of the square root as the length of one side of a square shape as well as to the procedure for finding the square root on a calculator. After the students have answered question **1a**, ask them for a number that when multiplied by itself gives 49. Explain that the square root of 49 is 7 or that $\sqrt{49}$ represents a number that, when multiplied by itself, is equal to 49. Discuss other examples using perfect squares, such as 9, 25, 64, 100, and so on.

Question **2** continues to explore the square root (length of the side of a square) using the calculator and the square root key. This time, the students will discover that, for many squares, the square root is not necessarily a whole number. Make sure that the students follow the correct procedure for their type of calculator.

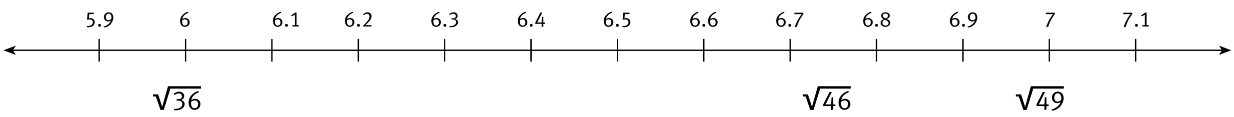
The students can draw up a table to record the measurement of the squares:

Area	$\sqrt{area}$ = length of side
4 x 4 = 16	4
9 x 9 = 81	9
78 x 78 = 6084	78
2.5 x 2.5 = 6.25	2.5
4.76 x 4.76 = 22.6576	4.46

Question **3** involves a larger square with a whole-number side. It can be drawn on grid paper with the tiles on one side shaded and counted. However, when your students are working on this, encourage them to first guess and then check their guesses, with successive approximations being made to find the square root. For example: "I know 10 x 10 = 100, which is too low. I'll try 11 x 11 next", and so on. Using the calculator to check mental approximations will help the students to gain a better understanding of the concept of square roots. You could use some other three-digit perfect-square numbers to extend and consolidate this experience, such as 196, 529, 9, and so on.

The students should draw the graph for question **4** on grid paper, taking care to plot the points accurately to help achieve a good result at question **5**. After discussing the graph pattern, continue to encourage prior thinking and estimation before the students find an approximate answer from the graph and an exact answer, as a check, on the calculator.

Discuss what the students think is the square root of 46. Then encourage them to narrow down the possibilities by guessing and checking. You could ask:
"What two square numbers is this number between?" (36 and 49)
"What two numbers will the square root be between?" (6 and 7)
"Is it a whole number?" (No)
"Will it be above or below 6.5?" (Above)
"What number could you press into your calculator for the sides? Is it above 6.6?" (Yes)
The students may be able to suggest where the side lengths (square root) will fit on a number line before they check the graph.

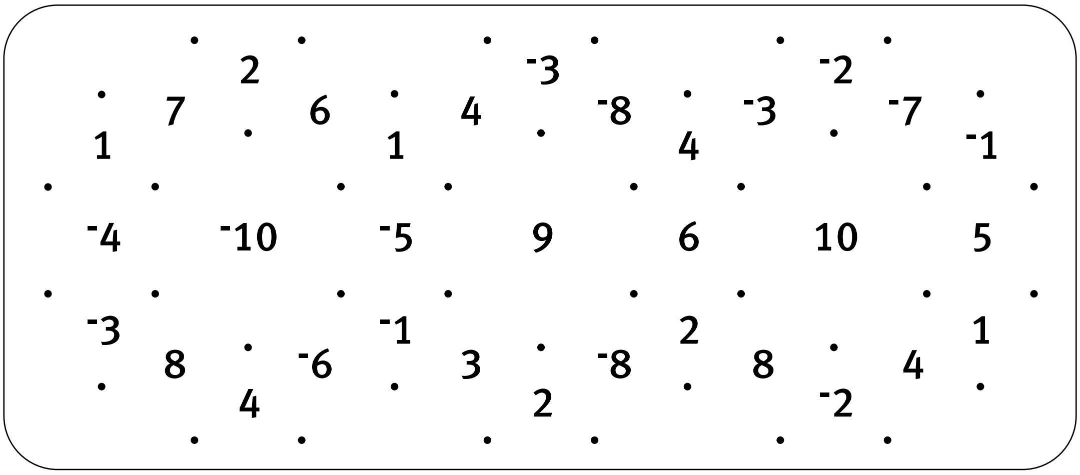


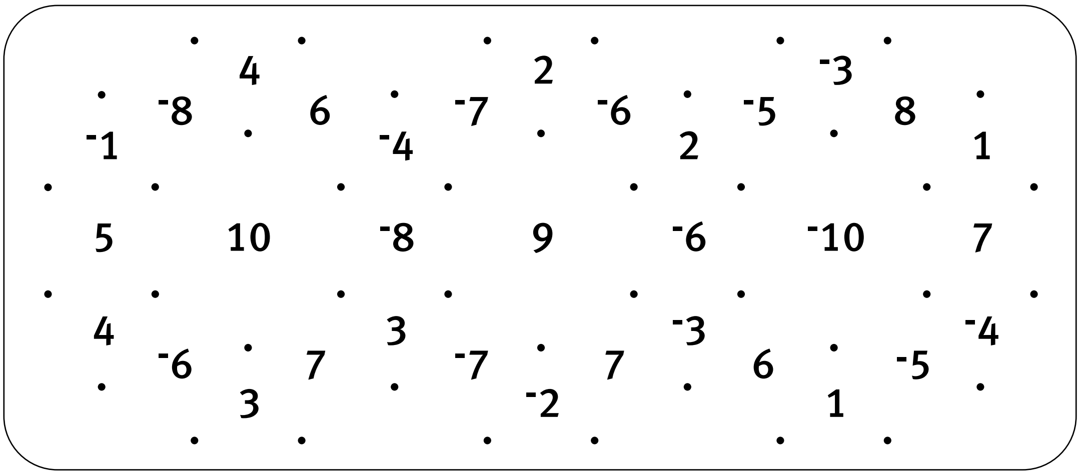
You may like to use an enlarged number line for follow-up discussion for all the question **5** examples and also a graph outline for shared recording and checking of students' own results on the overhead projector. The students can verify their answers by using the $\boxed{\sqrt{}}$ key on the calculator.

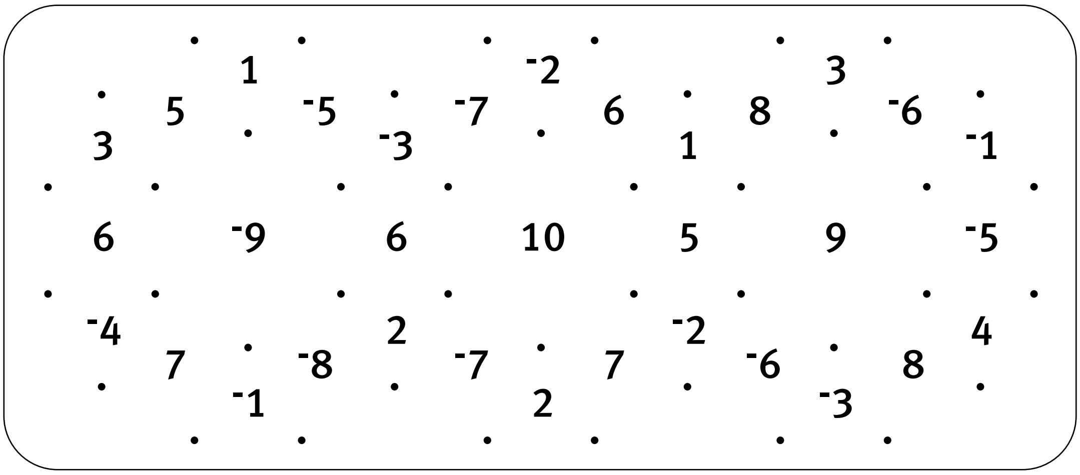
The students should read the line graph first from the horizontal axis and then from the vertical axis. A ruler can help link the number from the graphed line to the axis. The value that the students interpolate is an estimate that need not go beyond one decimal place.

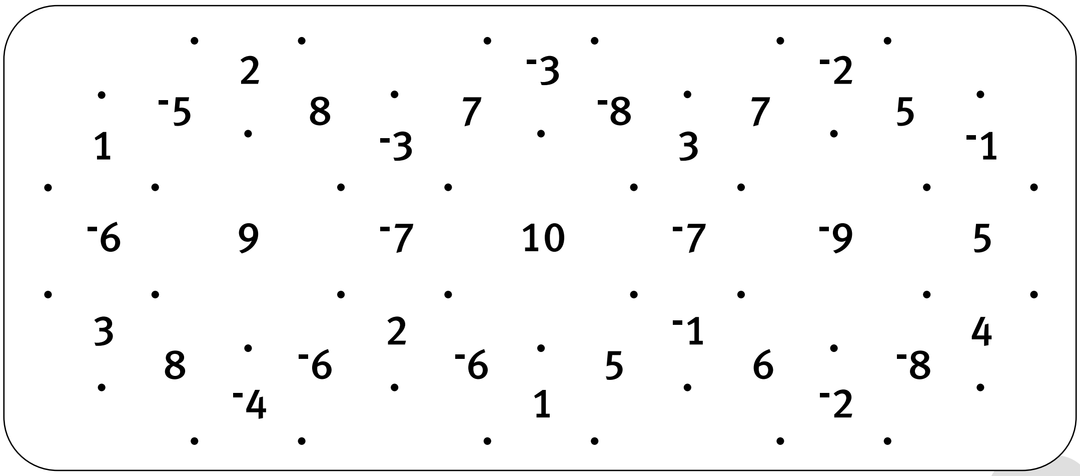
This overall progression from perfect-square, whole-number shapes to imaging, number lines, and graphs, all supported by mental estimation, will provide a solid basis for students to progress to using calculators for symbolic calculations.

Integer Links









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