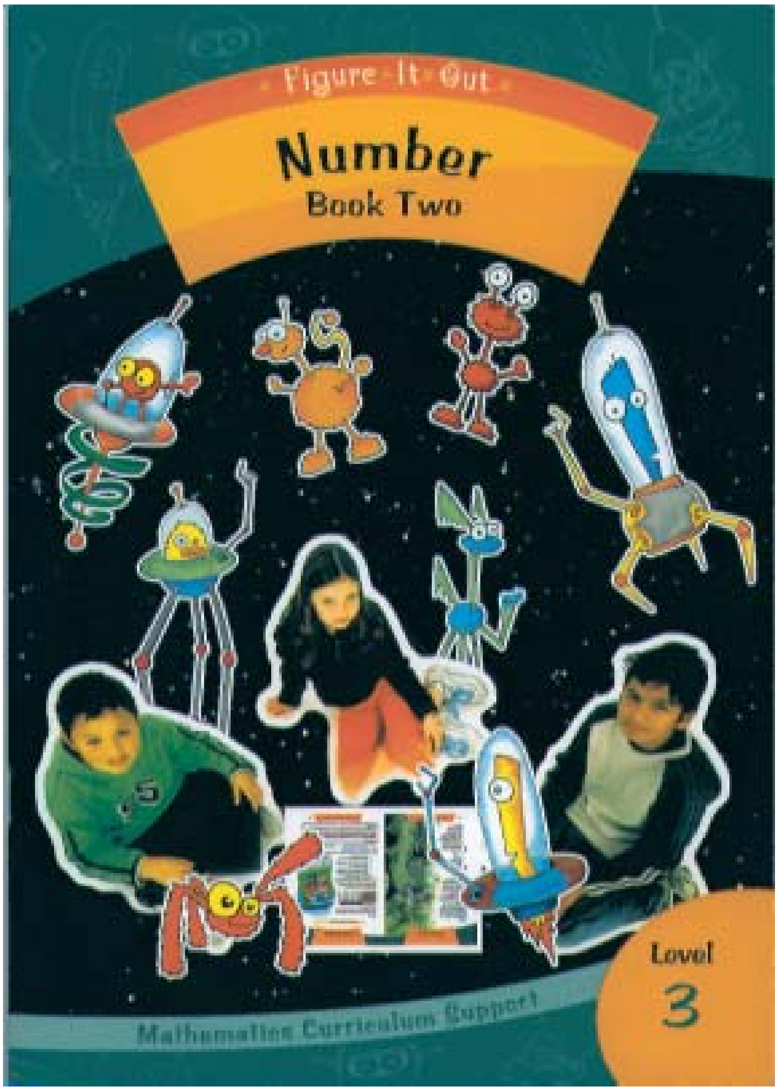
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for level 3 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. *Number: Book Two* and *Number: Book Three* have been developed to support teachers involved in the Numeracy Project. These books are most suitable for students in year 5, but you should use your judgment as to whether to use the books with older or younger students who are also working at level 3.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 5.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

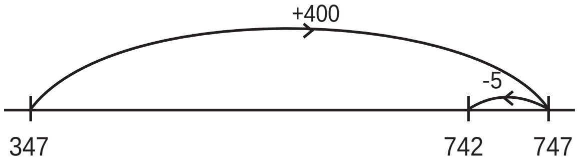
Number: Book Two Answers

Page 1: Tracking Toroa

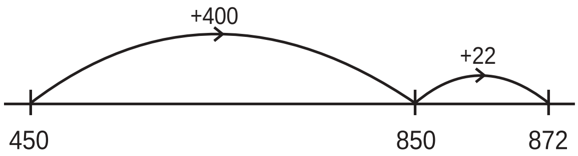
Activity

1. Answers and possible number lines are:

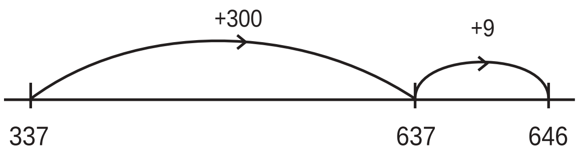
Tahi 742 km



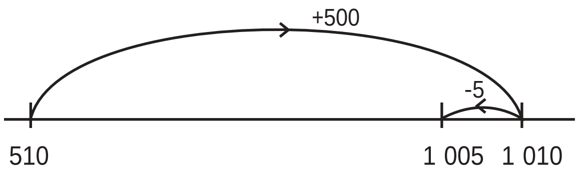
Rua 872 km



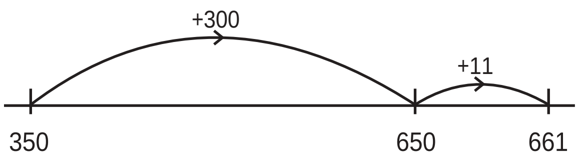
Toru 646 km



Whā 1 005 km



Rima 661 km

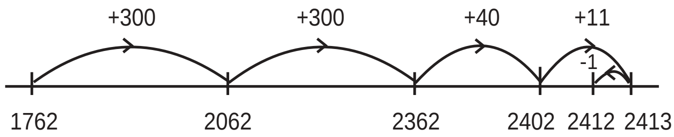


2. a. Rua. (Days 5 and 6)

   b. Toru. (Days 9 and 10)

   c. Whā. (Days 1–4)

3. 2 412 km. A possible number line is:



4. 1 047 km. (Days 3, 4, and 5)

Pages 2–3: Stepping Out

Activity

1. 54 min
2. First Piri's house and then Mira's house
3. To get to 2 friends' houses before the swimming pool, Mira must take one of the following routes:
   - Mira's to Tom's to Brad's to the swimming pool: 14 + 18 + 33 = 65 min
   - Mira's to Tom's to Emeli's to the swimming pool: 14 + 21 + 23 = 58 min
   - Mira's to Emeli's to Tom's to the swimming pool: 19 + 21 + 19 = 59 min
   - Mira's to Emeli's to Piri's to the swimming pool: 19 + 16 + 13 = 48 min
   - Mira's to Piri's to Emeli's to the swimming pool: 23 + 16 + 23 = 62 min
   - Mira's to Piri's to Hannah's to the swimming pool: 23 + 18 + 22 = 63 min
4. a. 113 min. (Tom's to Mira's to Emeli's to Piri's to Hannah's to Brad's and home or the same route in reverse)

   b. 161 min. (The slowest possible route, which can be done in reverse, is: Tom's to Emeli's to Mira's to Piri's to Hannah's to Brad's to the swimming pool and home.)
5. Answers will vary.

Page 4: Playing for Points

Activity

1. Jessica 636 194
   Hemi 635 293
   Shaun 627 194
   Tuvalu 591 999
   Adam 521 371
   Kelsi 521 299

2. Jessica 636 194
   Jamal 636 190
   Hemi 635 293
   Shaun 627 194
   Kent 610 102
   Tuvalu 591 999

Game

A game for ordering numbers

Page 5: Saving Up

Activity

1. **a.** $3
   b. $4
   c. $6
   d. $8

2. (Note: ⅖ of $1 is 40c, and ¼ of $1 is 25c.)

Abby	**a.**	$1.20
	b.	75c
	c.	$1.05. ($3 – $1.95)
Charlotte	**a.**	$1.60
	b.	$1
	c.	$1.40. ($4 – $2.60)
Jarrod	**a.**	$2.40
	b.	$1.50
	c.	$2.10. ($6 – $3.90)
Brendon	**a.**	$3.20
	b.	$2
	c.	$2.80. ($8 – $5.20)

3.

	Abby	Charlotte	Jarrod	Brendon
a.	$4.50	$6.00	$9.00	$12.00
b. i.	$1.80	$2.40	$3.60	$4.80
ii.	$1.15 (rounded)	$1.50	$2.25	$3.00
iii.	$1.55 (rounded)	$2.10	$3.15	$4.20

4. Strategies will vary. One possible strategy is given in the speech bubble on the page. Other strategies could be based on the fact that 75c is ¾ of a dollar or that 75c is 50c + 25c (so the pocket money is increased by half).

Page 6: Movie Maths

Activity

1. **a.** 4 different combinations.
   (1 x 24, 2 x 12, 3 x 8, 4 x 6)

   b. 3 different combinations.
   (1 x 32, 2 x 16, 4 x 8)

   c. 2 different combinations.
   (1 x 27, 3 x 9)

   d. 3 different combinations.
   (1 x 64, 2 x 32, 4 x 16)

   e. 3 different combinations.
   (1 x 50, 2 x 25, 5 x 10)

   f. 5 different combinations.
   (1 x 48, 2 x 24, 3 x 16, 4 x 12, 6 x 8)

2. 10 different combinations.
   (1 x 48, 2 x 24, 3 x 16, 4 x 12, 6 x 8, 8 x 6, 12 x 4, 16 x 3, 24 x 2, 48 x 1)

3. **a.–b.** Answers will vary.

4. Answers will vary.

Page 7: Singing up a Storm

Activity

1. 810. (An estimate could be: 18 + 9 is about 30. 30 x 30 = 900, but there are 3 less than 30 in each team, so a sensible estimate would be 800. 900 – [3 x 30] = 810)

2. a. 72. (An estimate could be: 20 x 4 = 80. 80 – [4 x 2] = 72)

   b. 2 160. (An estimate could be: 70 x 30 = 2 100. 2 100 + [30 x 2] = 2 160. Or you could work out the number of girls [18 x 30 = 540] and double that number twice: 540 x 2 x 2 = 2 160.)

3. a. 18. (An estimate could be: 10 x 2 = 20. 20 – 2 = 18. But you should probably know 9 x 2 = 18.)

   b. 540. (An estimate could be: 20 x 30 = 600. 600 – [2 x 30] = 540)

4. 2 430. (800 [the estimate] x 3 = 2 400. But the actual number is 810, so 2 400 + [3 x 10] = 2 430.)

5. Answers will vary.

6. There are 20 possible ways:
   1 x 810, 2 x 405, 3 x 270, 5 x 162, 6 x 135, 9 x 90, 10 x 81, 15 x 54, 18 x 45, 27 x 30, 30 x 27, 45 x 18, 54 x 15, 81 x 10, 90 x 9, 135 x 6, 162 x 5, 270 x 3, 405 x 2, 810 x 1

7. a. $2.00

   b. $1,620. ([50 x 30] + [4 x 30] = 1 620)

Pages 8–9: Booked!

Activity One

1. a. $3

   b. $4.50

   c. $6.90

2. Either 6 books overdue by 6 days, 4 books overdue by 9 days, 3 books overdue by 12 days, or 2 books overdue by 18 days

3. Discussion could include the cost to the library of replacing lost books, fairness to other readers who are waiting to read the overdue books, and helping to pay the library's running costs.

4. Practical activity

5. Answers will vary. Possible answers include: CDs usually cost more to replace than books; libraries have fewer CDs than books, and there is generally a long waiting list for the more popular CDs; and CDs are smaller than books and may get lost more easily, so the libraries want people to remember they have them and return them.

6. a. $7.50

   b. $13.50

   c. $19.50

7. 2 CDs and 3 books

Activity Two

Problems and solutions will vary.

Pages 10–11: Multiple Methods

Activity

1. Strategies will vary.

   a. Some possible strategies for 48 x 5 are:

      (48 x 10) ÷ 2

      (40 x 5) + (8 x 5)

      (50 x 5) – (2 x 5)

      24 x 10

      48 + 48 + 48 + 48 + 48

      50 + 50 + 50 + 50 + 50 – 10

   b. Some possible strategies for 16 x 4 are:

      8 x 8

      (4 x 10) + (4 x 6)

      (4 x 15) + 4

      (4 x 20) – (4 x 4)

   c. Some possible strategies for 92 x 4 are:

      (4 x 100) – (4 x 8)

      (92 x 2) then x 2 again. (92 x 2 x 2)

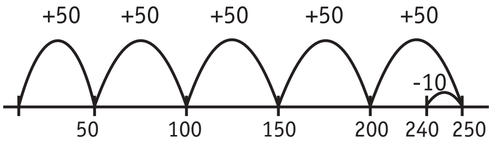
      92 + 92 + 92 + 92

      (90 x 4) + (2 x 4)

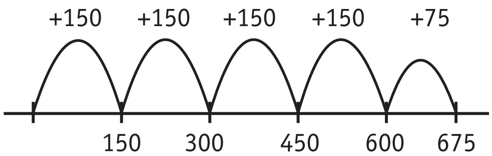
2. Answers will vary.

3. a. Answers may vary. For 48 x 5, the 6 methods could be:

i. Hamish: (50 x 5) – (2 x 5).
(Tidy numbers)
Fili: (40 x 5) + (8 x 5).
(Place value)
Wai Li: (48 x 10) ÷ 2.
(Known multiplication facts)
Keriata: 48 + 48 + 48 + 48 + 48.
(Repeated addition)
Nandan: 24 x 10.
(Doubling and halving)
Tamahou: A possible number line is:



ii. For 75 x 9, the 6 methods could be:
Hamish: 75 x 10 – 75. (Tidy numbers)
Fili: (70 x 9) + (5 x 9).
(Place value)
Wai Li: (80 x 9) – (5 x 9).
(Known multiplication facts)
Keriata: 75 + 75 + 75 = 225.
225 + 225 + 225 = 675.
(Repeated addition)
Nandan: (25 x 9) x 3 (dividing and multiplying by 3) or
(25 x 10) x 3 – (25 x 3)
(dividing and multiplying by 3 and using tidy numbers)
Tamahou: A possible number line is:



b. Answers will vary.

Pages 12–13: That Old?

Activity

1. a. i. 61 years old. (21 + [10 x 4])

ii. 73 years old. (21 + [13 x 4])

b. i. 45 years old. (21 + [7 x 4])

ii. 49 years old. (21 + [7 x 4])

2. a. 50 years old. (10 + [8 x 5])

b. 65 years old. (10 + [11 x 5])

c. 75 years old. (10 + [13 x 5])

Investigation

Year	Dog	Cat	
1	21	10	
2	25	15	
3	29	20	
4	33	25	
5	37	30	
6	41	35	
7	45	40	
8	49	45	
9	53	50	
10	57	55	
11	61	60	
12	65	65	← Same age
13	69	70	
14	73	75	
15	77	80	
16	81	85	
17	85	90	
18	89	95	
19	93	100	
20	97	105	

The table shows the + 4 pattern for dogs and the + 5 pattern for cats. Although the dog starts at a higher age (21), the cat gradually catches up until it is the same age in human years after 12 years of life. The cat's age in human years then gradually gets higher than the dog's age.

Page 14: Pītoitoi Pecks

Activity

1. a. About 1 400

b. About 6 000

c. About 73 000

2. a. About 4 000

b. About 28 000

3. 40

Page 15: Sweet Thoughts

Activity

1. a. Collected $188 ($160 + $28);
   profit $79.90 ($17 x 4 + 85c x 14)

   b. Collected $138 ($120 + $18);
   profit $58.65 ($17 x 3 + 85c x 9)

   c. Collected $30 (15 x $2);
   profit $12.75 (85c x 15)

   d. Collected $100 ($40 x 2 + $20);
   profit $42.50 ($17 x 2 + $8.50)

2. $3,111. ($17 x 183)
   One way to work this out is:
   183 x 10 = 1 830
   1 830 + 1 830 – (183 x 3)
   = 3 660 – (300 + 240 + 9)
   = 3 660 – 549
   = 3 111

3. a. 311 cartons. (5 287 ÷ 17)

   b. 6 220 bars. (5 287 ÷ 0.85 or 311 x 20)

Investigation

Answers will vary.

Page 16: Target Time

Game

A game for adding and subtracting decimals

Page 17: Dallying with Decimals

Game

A game for adding and subtracting decimals

Pages 18–19: Better Buy Bargains

Activity

1. a. A DVD would cost Caitlin $24. The most expensive cookbook is $15 at sale price, so she can buy any of the cookbooks as well as a DVD.

   b. $15

2. Answers will vary. For example, he could buy: 1 DVD and 2 pencils ($24.90), the Property Magnate game, *Dr Hoot*, and 1 biro ($24.70), or *The Zoo Zapparoo*, the Oops game, a pencil, and an eraser ($24.70).

3. Fili could buy *Pene's Adventures* or *The Zoo Zapparoo* (he can't buy both) as well as one of the other two books.

4. Answers will vary. For example, Simon can't buy a DVD, but he would spend his whole $18 if he bought the Property Magnate game or if he bought *Pene's Adventures* and *Dr Hoot*. He would spend $17.75 if he bought Oops or *No Worries Curries* at $15 each, plus a ruler, 1 biro, 2 pencils, and an eraser.

5. a.–c. Answers will vary.

Pages 20–21: Jumping Along

Activity

1. a. Round 1: 1st Tama, 2nd Ian, 3rd Elyse
   Round 2: 1st Elyse, 2nd Tama, 3rd Emma
   Round 3: 1st Morgan, 2nd Elyse, 3rd Abeba

   b. Elyse. She had the best jump overall. (If it were based on points for placings, she would still be the overall winner. She had a 1st, a 2nd, and a 3rd placing. Tama, the next best jumper, had a 1st and a 2nd placing.)

2. Answers will vary.

3. Practical activity

Game

A game for ordering decimals

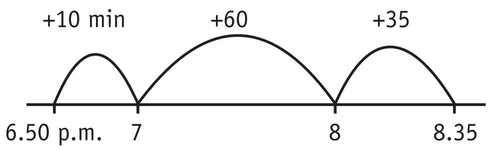
Page 22: Baffling Braids

Activity One

1. **a.** 16 red, 8 blue, 8 yellow, 16 white
   b. 28 red, 14 blue, 14 yellow, 28 white
   c. 40 red, 20 blue, 20 yellow, 40 white
2. **a.** 8 red, 12 yellow
   b. 20 red, 30 yellow
   c. 32 red, 48 yellow
   d. 60 red, 90 yellow
3. Practical activity. Designs will vary.

Activity Two

1. **a.** 2.5 min
   b. 54. (135 ÷ 2.5 or 12 x 4 + 6)
   c. 42. (105 ÷ 2.5). You could work out the time on a number line:



2. **a.** **i.** 4.55 p.m. (Ferila will have a 10 min break after 18 braids.)
   ii. 5.30 p.m. (She will need two 10 min breaks.)
   b. 23

Page 23: On the Trail

Activity

1. **a.** 20 g chocolate, 50 g raisins, 40 g sunflower seeds
   b. 26 g chocolate, 65 g raisins, 52 g sunflower seeds
   c. 38 g chocolate, 95 g raisins, 76 g sunflower seeds
   d. 72 g chocolate, 180 g raisins, 144 g sunflower seeds
2. 5 kg peanuts, 4 kg sunflower seeds, 5 kg raisins, 2 kg chocolate

Page 24: Heading for Home

Activity

1.

Spacecraft number	Alien number
24	3, 5, 9
3 600	4, 10
28	2, 8, 12
26	1, 6
3 375	7
3 450	11

Figure It Out

Number: Book Two

Teachers' Notes

Overview of Number: Book Two

Title	Content	Page in students' book	Page in teachers' book
Tracking Toroa	Solving addition and subtraction problems	1	12
Stepping Out	Using addition strategies	2–3	13
Playing for Points	Ordering numbers	4	14
Saving Up	Multiplying money and finding fractions	5	14
Movie Maths	Practising basic multiplication facts	6	15
Singing up a Storm	Estimating and multiplying large numbers	7	16
Booked!	Solving problems with multiplication	8–9	17
Multiple Methods	Investigating multiplication strategies	10–11	18
That Old?	Using multiplication and division	12–13	19
Pītoitoi Pecks	Multiplying and dividing with large numbers	14	20
Sweet Thoughts	Solving problems with multiplication and division	15	21
Target Time	Adding and subtracting decimals	16	22
Dallying with Decimals	Adding and subtracting decimals	17	23
Better Buy Bargains	Working with rounding and percentages	18–19	23
Jumping Along	Ordering decimals	20–21	24
Baffling Braids	Solving problems involving fractions	22	25
On the Trail	Finding fractions of numbers	23	26
Heading for Home	Solving problems using fractions and decimals	24	28

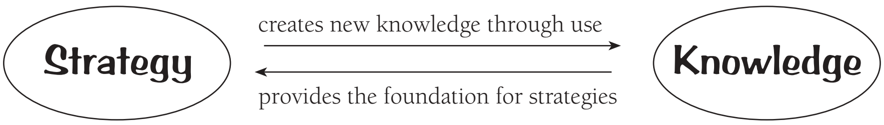
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence in society of machines that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve other problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The Number books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged and intermediate students. A key element of this drive has been the creation of the Number Framework as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



Learning activities in the series are aimed at both the development of efficient and effective mental strategies and increasing the students' knowledge base.

Links to the Number Framework

There are strong links to the stages of development in the Number Framework in *Number,* Books Two and Three for level 3 and levels 3–4. The knowledge- and strategy-based activities in the level 3 books are at the early additive part–whole to advanced additive/early multiplicative stages of the Number Framework. Those in the level 3–4 books also link to the advanced multiplicative (early proportional) part–whole stage of the Number Framework.

Information about the Number Framework and the Numeracy Project is available on the NZMaths website (www.nzmaths.co.nz/Numeracy/Index.htm). The introduction includes a summary of the eight stages of the Number Framework. Some of the Numeracy Project material masters are relevant to activities in the Figure It Out student books and can be downloaded from www.nzmaths.co.nz/Numeracy/materialmasters.htm
Books for the Numeracy Development Project can be downloaded from www.nzmaths.co.nz/Numeracy/2004numPDFs/pdfs.htm

Terms

Teachers who are not familiar with the Numeracy Project may find the following explanations of terms useful.

The Number Framework: This is a framework showing the way students acquire concepts about number. It comprises eight stages of strategy and knowledge development.

Knowledge: These are the key items of knowledge that students need to learn. Knowledge is divided into five categories: number identification, number sequence and order, grouping and place value, basic facts, and written recording.

Strategies: Strategies are the mental processes that students use to estimate answers and solve operational problems with numbers. The strategies are identified in the eight stages of the Number Framework.

Counting strategies: Students using counting strategies will solve problems by counting. They may count in ones, or they may skip-count in other units such as fives or tens. They may count forwards or backwards.

Part–whole thinking or part–whole strategies: Part–whole thinking is thinking of numbers as abstract units that can be treated as wholes or can be partitioned and recombined. Part–whole strategies are mental strategies that use this thinking.

Partitioning: Partitioning is dividing a number into parts to make calculation easier. For example, 43 can be partitioned into 40 and 3, or 19 can be partitioned into 10 and 9 or thought of as 20 minus 1.

Relevant stages of the Number Framework for students using the level 3 and 3–4 *Number* books:
Stage five: early additive part–whole: At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called part–whole thinking.

A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or "teen" numbers. For example, students at this stage might solve 7 + 8 by recalling that 7 + 7 = 14, so 7 + 8 = 15. They might solve 9 + 6 by knowing that 10 + 6 = 16, so 9 + 6 = 15. They might solve 43 + 35 as (40 + 30) + (3 + 5), which is 70 + 8 = 78.

Stage six: advanced additive/early multiplicative part–whole: At this stage, students are learning to choose appropriately from a repertoire of part–whole strategies to estimate answers and solve addition and subtraction problems.

Addition and subtraction strategies used by students at this stage include:

- standard place value with compensation (63 – 29 as 63 – 30 + 1)
- reversibility (53 – 26 = □ as 26 + □ = 53)
- doubling (3 x 4 = 12 so 6 x 4 = 12 + 12 = 24)
- compensation (5 x 3 = 15 so 6 x 3 = 18 [3 more])

Students at this stage are also able to derive multiplication answers from known facts and can solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6 x 6 as (5 x 6) + 6; or $^3/_4$ of 24 as $^1/_4$ of 20 is 5 because 4 x 5 = 20, so $^3/_4$ of 20 is 15, so $^3/_4$ of 24 is 18 because $^3/_4$ of the extra 4 is 3.

Stage seven: advanced multiplicative part–whole: Students who are at this stage are learning to choose appropriately from a range of part–whole strategies to estimate answers and solve problems involving multiplication and division. For example, they may use halving and doubling (16 x 4 can be seen as 8 x 8) and trebling and dividing by 3 (3 x 27 = 9 x 9).

Students at this stage also apply mental strategies based on multiplication and division to solve problems involving fractions, decimals, proportions, ratios, and percentages. Many of these strategies involve using equivalent fractions.

Page 1: Tracking Toroa

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

The toroa theme uses an interesting local context to apply addition and subtraction. This sort of context enables the students to understand the problem before deciding on a way to solve it.

The empty (or open) number line offers a diagrammatic way for students to devise and explain a strategy for solving the problems. This visual model also encourages the students to come up with the advanced additive stage strategies described in the Number Framework.

An empty number line marks the steps in thinking used by the students. Unlike the traditional number line, it is not intended to be a calculating tool that uses equal divisions.

Discuss the tidy number strategy that is used in the number line example of Tahi's flight distance for days 5 and 6. Make sure that the students notice the directional change in the arrows: the 299 is rounded up to 300 and added to 329, and then the arrow is reversed to subtract 1 to adjust the tidy number.

There are other possible number lines apart from those shown in the Answers that can be used to solve each question. The students' lines should match the thinking strategies they have used and result in the correct answer. Have some students share their ways of thinking.

The key to question **2** is the strategy the students choose to find the birds that fit the conditions. The easiest way, as given in the hint, would be to find 2 numbers in the ones place for each bird on Ōriwa's distance table that add up to the number required in that place and then test to see if the choice is correct. Some students may look at the hundreds place for 2 numbers that add up to either the same or 1 less than the required number, then check to confirm their choice.

Have the students discuss some of the strategies they used to confirm the exact answer.

The best strategy for question **3** is for the students to start from 1 762 kilometres. Students who add the first 5 days again will arrive at the correct answer, but they won't have used the information in the question to best advantage.

In question **4**, the students cannot presume that the question applies to the first 4 days, so they need to find out which 4 days add up to 1 429 kilometres. When they have done this, they may choose to add up the distances for the first 3 of those 4 days. This is fine, but the question is designed to encourage subtraction (1 429 – the fourth day = total of the first 3 of those days). Ask the students to find a way that makes use of all the information in the question. Discuss the strategies used and encourage the students to use empty number lines as part of their explanation.

Encourage the students to make up their own problems using the toroa context. They should be able to work out their own problem so that they can then explain it to others. Encouraging them to think about and explain the problems they make up will ensure that they do not choose numbers that are ridiculous.

Pages 2–3: Stepping Out

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

This activity focuses on addition strategies. A good place to start is to have the students compare Piri's and Tom's strategies. Both are trying to find an easy way to add. The difference is that Piri sees that 8 and 2 are compatible because they make 10, while Tom chooses to round to a tidy number and then do an adjustment to allow for the rounding. Hannah uses place value knowledge to split the addends and then adds the tens before the ones.

These examples of different strategies show part–whole thinking in addition that indicates the transition between stages 5 (early additive) and 6 (advanced additive) of the Number Framework.

The students should use a calculator only to check their answers, otherwise there will be no opportunity for strategy development.

It's important in teaching strategy development that you don't tell the students to learn a particular strategy and apply it. The strategies they choose will depend on the numbers that they are working with: there is no one strategy that is best for all situations. The students must "buy into" a strategy because they see it makes sense. You can encourage this by sharing your strategies and those of students without directing the students to use a particular one. Some suitable questions are: "Where did you begin?" "What was it about the numbers that helped you choose that strategy?" "Can you use a diagram (or equipment) to explain the steps in your thinking?" "Is there another efficient way to do it?"

For question **4**, the students need to realise that Tom does not have to visit the pool (although this may add to the time in **4b**) and he needs to return home. The students will probably use a trial-and-improvement strategy to test out all possibilities, so encourage them to work co-operatively, in problem-solving groups of four students, by dividing up the different routes to check between them. They should all check the "best" route for themselves before you ask them to share their calculation strategy.

Page 4: Playing for Points

Achievement Objective

- explain the meaning of the digits in any whole number (Number, level 3)

Activity

By ordering the high scores in this activity, the students will show and develop their knowledge of whole-number sequences and place value. The numbers on the Zone Ball high score table have been carefully chosen to ensure that the students show a robust understanding of the increasing value of digits as they are placed to the left-hand part of the numeral. The students who understand this structure will look for the largest digit on the left in the whole number that has the most digits. If this does not define the order, they then check the second-to-left digit for size, and so on.

Ask some of the students to explain how they decide on the order of the numbers. The students could produce a flow chart of the decision-making process needed to decide the largest number.

Do the numerals have the same number of digits?

↓ NO

Find the whole number with the most digits. This will be the largest.

Check the rest of the numbers.

↓ YES

Look for the number with the largest digit in the furthest left-hand column. This will be the largest.

If two or more numbers have the same largest digit, look at the next column to decide, and so on until the numbers are sorted.

Game

Insist that all 3 players agree who should win each counter before it is taken. After the students have used the bonus point idea a few times, have them discuss the best strategy for winning this extra point. For example, should they always remove the card with the largest digit? How can they best use a 0?

Page 5: Saving Up

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

This activity looks at finding a fraction of a set in a money context. The fractions involved are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{5}$, and possibly $\frac{1}{5}$ and $\frac{3}{4}$. Note the progression from unit fractions with a numerator of 1, starting with $\frac{1}{4}$, to $\frac{2}{5}$, which has a numerator of 2. The strategies that the students use to handle this progression are the key learning intention here. The strategy illustrated in the speech bubble shows the beginning of proportional thinking.

Question **1** may be solved by multiplying 50 cents by the number of years and then converting the cents to dollars. You could encourage the students to see the 50 cents as ½ a dollar by asking questions such as:
"Can you multiply by 50 cents without multiplying by 50?"
"Instead of multiplying by 50 cents, can you solve the problem by dividing by another number?"

Question **2** can be solved in at least two ways. The students can solve 2/5 by finding 1/5 of the total and then multiplying by 2, as shown in the speech bubble. Another way is to find 2/5 of \$1 or 100 cents and then see how many of these units make up the targeted amount. For example, 2/5 of \$1 is 40 cents, and as Abby gets \$3, she must put aside 3 lots of 40 cents, which is \$1.20.

There are various strategies that can be used to answer question **3a**. For example, the students could think of 75 cents as ¾ of a dollar and then multiply the ages by ¾. They could also solve it by multiplying the age by 75 and then converting that to dollars and cents. Note that Abby's answer in **3b ii** has to be rounded because the exact calculation is \$1.125 cents, which is not realistic. We suggest rounding up to \$1.15 because \$1.125 rounds to \$1.13, and the nearest 5 cent multiple is \$1.15. (This leaves her \$1.55 to spend.)

The students may realise that the children have had their pocket money increased by half because 75 cents is 50 cents plus 25 cents. So all the saving and banking will increase by half. For example, Abby's holiday savings are now \$1.20 + 60c = \$1.80.

Encourage the students to look for a proportional connection between the two rates of pocket money as opposed to an additive one, such as "the rate is 25 cents more than before".

As an extension, you could ask the students to find amounts of pocket money given after 5 weeks, 3 months, 1/6 of a year, and so on. Choose units that suit the level of the students.

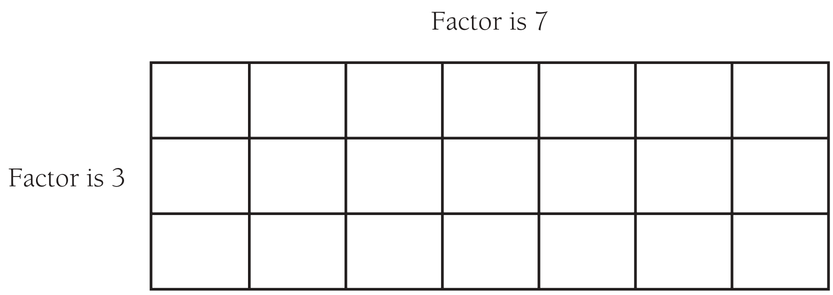
Page 6: Movie Maths

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- recall the basic multiplication facts (Number, level 3)

Activity

The context for this activity was chosen so that students would connect the array structure in the seating with multiplication. The sides of the array are the factors, while the total number of seats is the product.



A 3 by 7 array has a product of 21.

The ability to visualise multiplication as an array pattern is vital for interpreting many problems. It also enables the students to work out multiplication facts before they have committed them to memory.

Question **1** is based on the fact that Mikhail notices that there are more people in the row than there are rows. This reduces the choice of arrays. For example, they can sit in 3 lots of 6 people in a row but not in 6 lots of 3 in a row.

In question **2**, this restriction is removed, so all the combinations of 2 factors that make 48 need to be shown.

In question **3**, which is an open-ended question, the students practise seeing the arrays as factors and multiples. It may also present an opportunity to look at prime numbers. For example, if 29 children are in the class, the only array is 29 people in 1 row. This is a nice way to visualise a prime number.

Question **4** is important because it encourages the students to generalise the array notion of multiplication by choosing other contexts that have that structure. This generalisation is their key to recognising types of problems that can be solved by multiplication. Ensure that this point is made explicit at the conclusion of the activity.

Page 7: Singing up a Storm

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- make sensible estimates and check the reasonableness of answers (Number, level 3)

Activity

This activity encourages estimation strategies as well as exact multiplication. Ask the students to do the estimations mentally before they calculate the exact multiplications with pencil and paper. Use the speech bubble as a model and have the students discuss two or three estimation strategies before the exact answer is calculated.

The think, pair, and share technique is a good way of encouraging the mental strategies. The students could be given 30 seconds to estimate on their own, then they share their estimates and strategy with a classmate, and finally, some pairs share back with the whole group.

For question **5**, the students could use two colours of counters: 9 of one colour for the boys and 18 of another for the girls. They could then experiment with different ways of arranging the team of 27 using the counters. 1 and 27 and 3 and 9 are factors of 27, so a team could be arranged in rows of 1, 3, 9, or 27.

Have the students try to describe their arrangement using numbers as well as words. Encourage them to think about symmetry as they arrange their teams. For example, an arrangement such as:

B G G B G G B G G
B G G B G G B G G
B G G B G G B G G

has translation symmetry and could be described as columns of 1 x 3 boys + 2 x 3 girls, repeated 3 times.

An arrangement such as:
B G G G B G G G B
B G G G B G G G B
B G G G B G G G B
has reflective symmetry and could be described as columns of 1 x 3 boys and 3 x 3 girls, repeated twice, with a final column of 1 x 3 boys.

Question **6** explores all the combinations of 2 factors that make the product 810. Encourage the students to record systematically so that they do not miss some possible combinations. It may help to have the students examine the number 810 to see what hints it gives about the factors. Statements such as these would be useful:
"It's an even number, so it must be 2 x $\square$."
"It ends in 0, so it must have a 10 x as well as a 5 x $\square$."
"It's got 81 tens, so it must have 9 x as well as 3 x and 27 x $\square$."

Although all the combinations are asked for, the students may wish to eliminate some of the possibilities by using logical thinking. For example, it is unlikely that the venue could fit the 1 x 810 option or even the 2 x 405 option.

Pages 8–9: Booked!

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity One

This activity presents a multiplication scenario that combines 3 factors to get a result.

In their initial discussion, the students could work in problem-solving groups of four and focus on understanding the problem. Have them report back on the numbers involved in question **1a** and how they relate to each other (that is, the 3 books, the 10 cents, and the 10 days work together as factors, not addends).

After the students have solved the problem, discuss the strategies used. In question **1a**, the factors are 3 x 10 x 10 = 300 cents or $3. Some students would multiply left to right (3 x 10 = 30, then 30 x 10 = 300), while others would multiply right to left (10 x 10 = 100; this is $1, so 3 x 1 = $3).

It is good practice to ensure that the students change the 300 cents into $3 because that is the most economical form.

Questions **1b** and **1c** have only one factor of 10, whereas **1a** has two factors of 10. Encourage the students to see the effect of multiplying by 10 on the place value of the other factor. Question **1c**, where the equation is 3 x 23 x 10, can be solved by doing 3 x 23 = 69, then 69 x 10 = 690. Ask them to notice how the 69 moves one place to the left to become 690.

| 6 | 9 becomes

6 | 9 | 0

This effect can be shown on a calculator when the students enter 10 x 69 and watch the display as they press the equals button. They can then use the constant function to multiply by 10 repeatedly by pressing $\boxed{=}$ a number of times and watch the digits move one place to the left each time. (On some calculators, the students will need to enter 10 $\boxed{x}\boxed{x}$ 69 before they can use the constant function.)

If the students say "To multiply by 10, we add a 0", challenge this language by asking them to add 0 to 69. The answer, of course, is 69 (69 + 0 = 69). The digits remain where they are. This linguistic short cut can confuse students about the properties of adding 0 and multiplying by 10. The students show good understanding if they can say "Because the digits have moved 1 place to the left, we put a 0 in the vacant ones place."

You may wish to extend this concept to multiplying by a factor of 100 or 1 000.

An interesting strategy can be used in question **2** if the students realise that $3.60 is 36 x 10 cents. The combination of books and days are the only variables, and so the factors of 36 provide the solution.

Question **3** is a useful context for logical thinking. Ensure that the students discuss their reasoning and focus on the sensibleness of their conclusions.

Question **7** is a challenge because there are now two variables to consider. The overdue days are constant at 5, but it is the mix of books and CDs that needs to be explored.

You may need to help the students to work out a way to show all possible combinations of books and CDs that make a fine of $6.50 over 5 days. For example,

CDs	x 5 days	Books	x 5 days	Fine
1	= $2.50	8	= $4.00	= $6.50. (7 books maximum, as explained in question **2**)
2	= $5.00	3	= $1.50	= $6.50. This is correct.

An alternative strategy would be to find the fine per day: $6.50 for 5 days is therefore $1.30 for 1 day. $1.30 can be made up only of 1 CD and 8 books or 2 CDs and 3 books. A maximum of 7 books can be borrowed, so the solution is 2 CDs and 3 books.

Activity Two

In this activity, the challenge is for the students to make sure that the problems they make up do, in fact, have sensible answers. They also need to take account of other possible answers.

Pages 10–11: Multiple Methods

Achievement Objectives

- recall the basic multiplication facts (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 3)

Activity

This activity presents multiplication strategies from the early additive stage through to advanced multiplicative thinking.

Have the students attempt to solve the opening scenario mentally and then compare their strategies with those shown in the speech bubbles. Direct them to Bridget's chart in question **3** that classifies the strategies. This chart is a useful way to describe the strategies.

Note that Nandan's strategy, where 3 x 27 is seen as 9 x 9, may need special attention. This can be explained in two ways:

- Partitioning numbers into factors.
  3 x 27 = 3 x (9 x 3) can be regrouped as (3 x 3) x 9 = 9 x 9.
- Multiplying one factor (3) by 3 and dividing the other (27) by 3.
  This is an extension of the more common doubling and halving strategy.

Note the difference in thinking between the strategies that partition numbers additively and those that use factors to split numbers into easier steps. The efficiency of strategies can be looked at from two perspectives. Firstly, they are efficient if they provide a quick and easy way to get the correct answer. Secondly, there is a qualitative difference between those students who can split numbers only by adding or subtracting and those who can use factors as well as adding or subtracting. This second way provides more options for the problem solver, both with a particular problem and with the development of more advanced thinking.

Students in transition to the advanced multiplicative stage of thinking will be using factors to split multiplication and division for at least a part of the calculation.

Pages 12–13: That Old?

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)

Activity

This activity uses a fun context that often intrigues students and provides an opportunity for them to develop multiplicative strategies. The scenario also encourages the finding of rules to use a sequential number pattern.

Before the students start the activity, have them share their understanding of the relationship between cat or dog ages and human age. They may mention multiplying the animal's age by 7. (This is a common fallacy. For example, for dogs, it depends on the dog's breed and size, as implied in the vet's first comment.)

Refer the students to the speech bubbles and have them compare the way the equations are written and the thinking behind them. Chu's owner, Wai Li, uses addition by 4s to show the pattern in the expression 21 + 4 + 4 + 4 + 4, while the vet, who uses multiplication to describe Jack's age, substitutes 8 x 4 for the repeated addition of 4 in the equation 21 + (8 x 4) = 21 + 32.

Extend very able students by challenging them to find the general rule for this expression. In the vet's explanation, it is $4(n - 1) + 21$, where n is the number of years old for a dog. This can also be expressed as $4n + 17$. Hint: Because the pattern grows in 4s, the general rule must be a variation of the 4 times table.

In question **2**, the cat's "human" age progresses by adding 5, so the general rule here must be related to the 5 times table. For each year (n), the cat's human age is $5n + 5$ or $(5n - 1) + 10$.

Investigation

Ask the students to work in problem-solving groups of four students and discuss a way to start solving the problem. As an alternative to a trial-and-improvement strategy, the students may find a chart like the one below useful:

Year	Dog age	Cat age
1	21	10
2	25	15
3	29	20
4	33	25

Have the students estimate at what year the dog and cat ages are likely to coincide and then complete the chart to confirm the estimate.

Some students may like to contact a local pet expert to see if they have other ways of comparing the ages of cats and dogs or other animals to human ages.

Some very able students could be shown the two general rules for the cat and dog as described earlier. Explain to them that the investigation could be solved by seeing that the cat's rule must equal the dog's rule at the year we want to find. So $5n + 5 = 4n + 17$. They may see that the dog has 1 lot of n less than the cat but has 12 more years, so therefore the ages coincide when $n = 12$ years.

Page 14: Pītoitoi Pecks

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

The context in this activity uses personification to engage students in the problem.

The activity focuses on multiplying and dividing by multiples of 10. It builds on the understanding of multiplying by 10 developed earlier in Booked! The students need to be able to see how multiplying and dividing by multiples of 10 affects the place value of the digits in the factors involved. This is a vital strategy for students if they are to progress into the advanced multiplicative stage of strategy thinking.

The expression 7 x 200 in question **1a** can be thought of as 7 x 2 x 100, which becomes 14 x 100. Now the place value effect of x 100 can be used to move the 14 two places to the left and make 1 400.

Question **1b** involves the expression 30 x 200. Have the students explore splitting these numbers into factors of 10 or 100. The expression then becomes 3 x 10 x 2 x 100. This can be rearranged into 3 x 2 x 10 x 100, which equals 6 x 1 000 = 6 000. Guide the students to the realisation that they can reposition the factors to group the multiples of 10 together.

In question **1c**, the challenge is extended in the expression 365 x 200. This can become 365 x 2 x 100 and 730 x 100 = 73 000 as the digits move 2 places to the left.

Question **3** involves division by multiples of 10 in the equation 8 000 ÷ 200 = $\boxed{}$. Challenge the students to attempt a mental strategy for solving this. Some possibilities include:

- 80 hundreds ÷ 2 hundreds = 80 ÷ 2
  = 40
- 800 x 10 ÷ 200 = 800 ÷ 200 x 10
  = 4 x 10
  = 40

If the students do not come up with the strategies, show them some. Explain that the idea behind the strategies is to split the numbers into factors of 10 so that we can use the place value positions as we multiply or divide.

As an extension, the students could use the personification in this activity as a model for other number stories that they could research and present to a group of classmates. They must be able to solve the problems they produce and ensure that they are sensible and can be explained to the group.

Page 15: Sweet Thoughts

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

The quantities involved in this activity are challenging because they require the students to use the distributive property as they split numbers. You will have to decide to what extent you wish the students to use a calculator.

You may decide to use some of the questions to help the students develop their advanced multiplicative strategies, in which case, they should solve the problems without a calculator. (Some strategies are shown in the Answers.) They could use the calculator to check their work once they have a solution.

Alternatively, you may decide, for some of the questions, that you want the students to form a suitable equation by interpreting the problems. In this case, they could estimate the answers and then solve them on a calculator.

You can encourage both learning intentions by having the students do some of question **1** without a calculator and then allow them to use a calculator for the rest.

Estimation is an important skill that links clearly to the use of the calculator as a tool. The students must be able to estimate the answer so that they know if they have pushed the right buttons on the calculator. Estimation skills require quick recall of all basic facts, number sense, knowledge of place value, and multiplying and dividing by 10, 100, and so on.

Note that there is a mix of units in question **1**. The profit on cartons is in dollars, while that of single bars is in cents. Some students may have problems with this.

For question **3a**, most students will need to have access to a calculator because the question requires long division by a divisor that is a 2-digit prime number. Without the use of a calculator, it is beyond level 3 of the curriculum.

Investigation

Have the students explore two or three sources of bread and sausages to get the best profit. They also need to note the number of slices in particular loaves of bread. There may be a significant difference between the toast-sliced and the sandwich-sliced loaves. They may wish to discuss the provision of whole grain versus white bread in terms of cost and as a health issue.

Page 16: Target Time

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

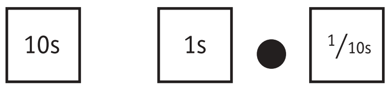
Game

This game provides a fun context to practise the addition and subtraction of decimals. It also presents opportunities for you to explore and share addition and subtraction strategies, logical thinking in selecting a winning strategy for the game play, and place value issues with decimal numbers.

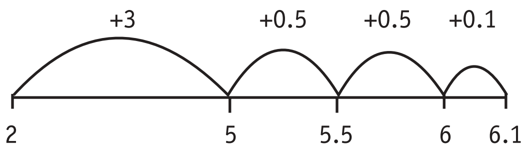
The Target game board can be photocopied (see the copymaster at the end of these notes), or the students can sketch out the boxes on draft paper for a quick copy.

This game is an ideal independent activity for pairs of students to practise decimals, but it must still be taught to the group so that they understand how to play. It can also be used as a class game.

When you use the game as a teaching activity, check that the students appreciate the place value in each of the boxes. Discuss the role of the decimal point as a device that indicates the ones column to the left. When this place is identified, the students should work out the place value of the other boxes.



Strategies for adding or subtracting the decimals may also be part of the teaching activity. For example, if the dice produce the expression 2.5 + 3.6, the students may choose to add the ones first. This could be recorded on an open number line:



Other strategies may be to make 2.5 up to 3 and so on, for example, 2.5 + 0.5 + 3 + 0.1 = 6.1. A third strategy could be 2.5 + 2.5 + 1.1 = 6.1.

These options show that the students are doing decimal addition at the advanced additive stage of thinking.

After the students have played the game a few times, have them discuss their strategies for winning. Ideas may include: "I put my first throw in the ones place unless it is a 5 or a 6 because these would be too high. I then have three throws to get a number in the other ones place that will make 5. The decimal bits should get me close to 6." "I put the numbers under 4 in the ones place and the numbers greater than 3 in the tenths place." Ask the students to test out the strategies to see which ones work best for them.

Page 17: Dallying with Decimals

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Game

This is another game that provides a fun context to practise addition and subtraction of decimals and opportunities for the students to explore and share additive strategies for decimals.

Note that the dice is restricted to the numbers 1, 2, and 3. This increases the likelihood of the students hitting a star, which is where the purpose of the game is fulfilled. If you cannot access a dice like this, use an ordinary one and tell the students to subtract 3 if they throw a 4, 5, or 6.

It would be a good idea to use two colours to make up the different sets of cards. This helps with the organisation of the game.

Once the game has been taught and practised, it will become a useful independent activity. You may also use it for strategy teaching by asking the students to reflect on how they did some of the star calculations.

If the students need assistance with the decimal calculations, have them use the blank decimal fraction number lines from the Numeracy Project material master 4-25 (available at www.nzmaths.co.nz/numeracy/materialmasters.htm).

Pages 18–19: Better Buy Bargains

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity

This activity provides a realistic context for rounding numbers and applying simple percentage calculations.

Check that the students can round to the nearest dollar. All the items except the stationery on this page round up before the discount is taken off because they are priced at 95 cents or 99 cents. The stationery items are rounded to the nearest 5 cents after the 10 percent discount. This would see the rulers cost 90 cents, erasers cost 25 cents, pencils 45 cents, and biros 70 cents.

Ask the students to explain the connection between the percentages and fractions shown in the conversion chart at the top of the page. You may wish to use the Numeracy Project material masters 4-28 and 7-4 to help the students with the conversions in this activity and to practise other conversions (available at www.nzmaths.co.nz/numeracy/materialmasters.htm).

Have the students discuss a starting strategy for the problems. A chart showing all the relationships may be useful, for example:

Item	List price	Rounded price	Discount price
No Worries Curries	$29.95	$30	$15
Tasty Treats with Taro	$17.99	$18	$9
Summer Salads	$21.95	$22	$11

The finished chart will show all the discount prices, so the students can concentrate on the list of purchases they need to do in questions **2** to **5**.

Question **5c** asks for a calculation of savings made. This could be worked out by adding a savings column to the chart.

Pages 20–21: Jumping Along

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- order decimals with up to 3 decimal places (Number, level 3)
- perform measuring tasks, using a range of units and scales (Measurement, level 3)

Activity

This activity develops understanding of decimal numbers through ordering and comparing them in a practical measurement context.

The key question to ask the students is: "How do we decide which of the decimal numbers is the largest?"

Note that students who read decimals like 1.97 as "one point ninety-seven" often develop misunderstandings about the place value of digits in decimal numbers. Ensure that the students read 1.97 as "one point nine seven". They should also be able to read decimal numbers as equivalent fractions, for example, "one and ninety-seven hundredths".

The students need to appreciate that the digit to the left of another is always greater in total value because of its place. This is true of decimal numbers as well as of whole numbers. Therefore, when the students order the jump distances, they should first look to the digit on the left that has the highest place value. If some digits are in the same column, as they are in this activity, the students then look at the face value of the digits to determine which is the largest. If some are still the same, the students should look at the next-to-left column and sort these digits. They continue this process until they have determined the largest number.

A useful summary sentence for this process is "Look left, then right to order the decimal numbers."

The students could use the Numeracy Project material master 4-15 to help them order the decimals (available at www.nzmaths.co.nz/numeracy/materialmasters.htm).

Game

Have the students discuss some winning strategies for placing digits in the boxes. For example, "I always put the first number I get that is greater than 6 in the ones place" or "When I'm using dice, I try to wait for an 8 or a 9 before filling in the ones place."

As a variation, instead of filling the numbers in sequence with each set of 3 digits, the students could put the first 3 digits in any of the 9 spaces and so on until all the spaces are filled.

Page 22: Baffling Braids

Achievement Objectives

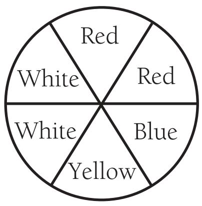
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity One

This activity poses problems involving solving fractions of a set. It can be used with students who are at the counting-on stage through to those at the advanced proportional stage of the Number Framework. It is vital that the students keep in mind the total number of beads that are the unit being used in each problem. You could ask:
"One-third of how many beads are red?"
"How many beads are in the whole set of beads being referred to here?"

Use the picture for question **1** to ensure that the students can relate the fraction of each colour to the whole set. You should ask them to find out what fraction of the set of beads is white.

A fraction circle showing sixths may be helpful to demonstrate the relationship between the thirds and sixths as well as the fraction of the set that is white. For example:



This shows that 2 sections out of 6 are white, so $\frac{2}{6}$ of the beads are white. This means that $\frac{1}{3}$ must be white.

The students should also compare the fraction for each colour. They should see that the red and white sections are the same and the blue and yellow sections are the same. Encourage them to see that the blue or yellow fraction is half the red or white fraction. That is, $\frac{1}{6}$ is half of $\frac{1}{3}$.

Students at the counting-on or advanced counting stages may need to use beads or counters and place them on the circle diagram. Once they have done this, help them record the appropriate number sentence, such as $\frac{1}{6} \times 12 = 2$ because $12 \div 6 = 2$.

So that the students can see how to work out numerators greater than 1, ask them "How can we use the fact that $\frac{1}{6}$ of 12 equals 2 to work out $\frac{1}{3}$ of 12?"

In question **1**, ask the students to find a number of strategies to solve the problems. They may write $\frac{1}{3} \times 48$ or $\frac{1}{3}$ of 48. To help them, you could show them that $\frac{1}{3} \times 48$ is the same as $48 \div 3$.

They may relate **1a**, **1b**, and **1c** to the example of 12 beads in the picture. So the answer to **1a** is four times the example because 48 is 12 x 4. The answer for question **1b** is 7 times the amount because 84 ÷ 12 = 7, and that for **1c** is 10 times bigger.

Question **2** extends the numerator to 2 in $^2/_5$. The students will need to appreciate that $^2/_5$ is 2 x $^1/_5$ and that if $^2/_5$ are red, $^3/_5$ are yellow because the whole set is $^5/_5$. Use a circle diagram showing fifths if necessary. The Numeracy Project material master 4-19 has these diagrams (available at www.nzmaths.co.nz/numeracy/materialmasters.htm).

You could encourage the students to make a chart to help them to see the patterns in the questions:

Number of braids	Number of beads	Red beads	Yellow beads
10	20	8	12
15	30	12	18

Activity Two

This activity uses fractions greater than 1. The students need to use these mixed fractions so that they do not have the common misconception that fractions are always smaller than 1.

In small groups, the students could discuss ways to begin to solve the problems. In question **1b**, they may see that they could use the relationship of 30 minutes for 12 braids as a unit. Then they can find out how many lots of 30 minutes there are in $2^1/_4$ hours. There are $4^1/_2$ periods of 30 minutes, so $4^1/_2$ x 12 = 54 braids. This may be an easier strategy for the students than the conventional way of dividing 135 minutes by 2.5 minutes.

Question **2** is intended to provide a challenge for students moving into the advanced proportional stage. Have the students discuss what is happening in the question and explain it to others in their own words. Check their understanding with questions like: "How many braids will Ferila have to do in question **2a i**?" (30) "How long does she take to do 1 braid? How about 10 braids?" "If she starts at 3:30 p.m., when will her first break be?"

The students may see that a good strategy could be to use the fact that 10 braids will take 25 minutes, so 30 braids will take 75 minutes. 75 minutes + a 10 minute break = 85 minutes.

Page 23: On the Trail

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

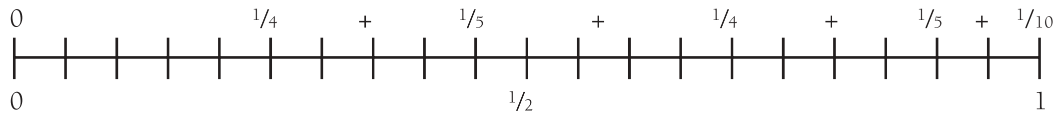
This activity allows the students a choice of strategies as they tackle these fraction problems. The keys to choosing a strategy are the relationships between the amounts of the ingredients and the connections between the different amounts of trail mix. The students could explore these connections in groups of four and report back to the class.

A ratio table would help the students to see the connection between the proportion of the ingredients and how these match the amounts in each mix.

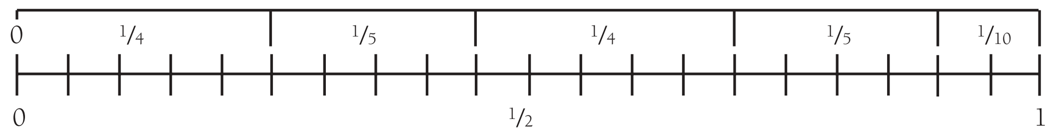
$\frac{1}{4}$ peanuts	$\frac{1}{5}$ sunflower seeds	$\frac{1}{4}$ raisins	$\frac{1}{5}$ dried apricots	$\frac{1}{10}$ chocolate
25 grams	20 grams	25 grams	20 grams	10 grams

Ask the students to find connections in the table. They could make statements such as: "Peanuts and raisins have the same proportion, which is a quarter." "Sunflower seeds and dried apricots have the same proportion, which is one-fifth." "The chocolate is half the amount of the dried apricots." "Peanuts and raisins make up half of the mix."

Have the students check that the amounts of the ingredients make up the whole. That is, that $\frac{1}{4} + \frac{1}{5} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10} = 1$. A double number line with 20 divisions may be useful to show this:



Another way of showing this is to have the amounts of each ingredient shown as blocks along the number line:



Have the students explore some strategies for the addition of the parts using numbers. They may see that $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ and $\frac{1}{5} + \frac{1}{5} + \frac{1}{10} = \frac{1}{2}$ and $\frac{1}{2} + \frac{1}{2} = 1$.

If they choose the common denominator of 20 and convert all the fractions to this, they will get $\frac{5}{20} + \frac{5}{20} + \frac{4}{20} + \frac{4}{20} + \frac{2}{20} = 1$, which is an equivalent expression for the relationships.

You can now ask the students to try and use these connections to help them solve the problems. Discuss the strategies they use.

Multiplicative thinkers may see that question **1a** uses an amount that is 2 times that of the sample Scooter has already made. Question **1b** can be seen as either 2.6 times greater than Scooter's 100 gram sample or 1.3 times the answer to **1a**. The students could continue these strategies for the other questions.

Additive thinkers may solve **1a** by doing repeated addition of each amount.

In question **2**, the students could relate the new quantities of apricots to the ratio table at the beginning. They may see that it is just double the number and a change of unit from grams to kilograms. They can then use this table or make another ratio table to solve the problem.

Page 24: Heading for Home

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Activity

This activity involves carefully chosen quantities that enable the students to choose a variety of strategies to use in the fraction and decimal calculations. The calculator could be used in a variety of ways, depending on the desired learning intention.

You could encourage the students to develop a sense of a reasonable answer by asking them to do a mental estimation, discuss the mental strategy with a classmate, and then check with a calculator to see how successful they were in estimating. It is important to ensure that the calculator is not used before the estimate is made and recorded.

If you want to encourage pencil and paper strategies, the calculator could be used as a checking device.

When the learning intention involves using a calculator to solve multiplication and division with decimals and fractions, the students should use a calculator from the start.

Some interesting strategies to share with students include:

- Alien 1: Divide 104 by 2 and 2 again.
- Alien 2: Divide 140 by 10 and multiply by 2.
- Alien 3: Divide 144 by 2 and that answer by 3.
- Alien 4: Divide 48 by 4, multiply by 3, and then multiply by 100.
- Alien 5: Find $^1/_3$ of 60 and then $^1/_3$ of 12 and add them together.
- Alien 6: Multiply 6.5 by 2 and by 2 again.
- Alien 7: Move the digits in 33.75 two places to the left of the decimal point so that you get 3 375.
- Alien 8: Find $^1/_4$ of 100 and $^1/_4$ of 12 and add them together.
- Alien 9: Think of 120 as 10 x 12. Divide 10 by 5 and then multiply by 12.
- Alien 10: Find $^1/_2$ of 72 and multiply by 100.
- Alien 11: Find $^1/_2$ of 6 000 and $^1/_2$ of 900 and add them together.
- Alien 12: Double and halve, that is, $(3.5 \times 2) \times (8 \div 2) = 7 \times 4$.

Once again, the Numeracy Project material masters 4-28 and 7-4 could be useful to help students with the conversions and to practise other conversions (available at www.nzmaths.co.nz/numeracy/materialmasters.htm).

Target Number **6**

☐.☐ + ☐.☐ = ☐☐.☐

Score → ☐.☐

Target Number **8**

☐.☐ + ☐.☐ = ☐☐.☐

Score → ☐.☐

Target Number **10**

☐.☐ + ☐.☐ = ☐☐.☐

Score → ☐.☐

Target Number **7.5**

☐.☐ + ☐.☐ = ☐☐.☐

Score → ☐.☐

My total score

Set A

− 1.1	+ 1.1	− 0.9	+ 0.9
− 1.1	+ 1.1	− 0.9	+ 0.9
− 1.1	+ 1.1	− 0.9	+ 0.9
− 1.1	+ 1.1	− 0.9	+ 0.9

Set B

6.9	4.8	13.5	17.9
3.4	7.2	14.6	18.1
2.1	8.3	15.7	19.2
5.6	2.4	16.8	12.3

JUMPING ALONG

PLAYER NAME	ROUND															SCORE
	1		.			>		.			>		.			
	2		.			>		.			>		.			
	3		.			>		.			>		.			
	4		.			>		.			>		.			

TOTAL SCORE:

Scoring: 5 points: fully correct 2 points: part correct 0 points: none correct

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