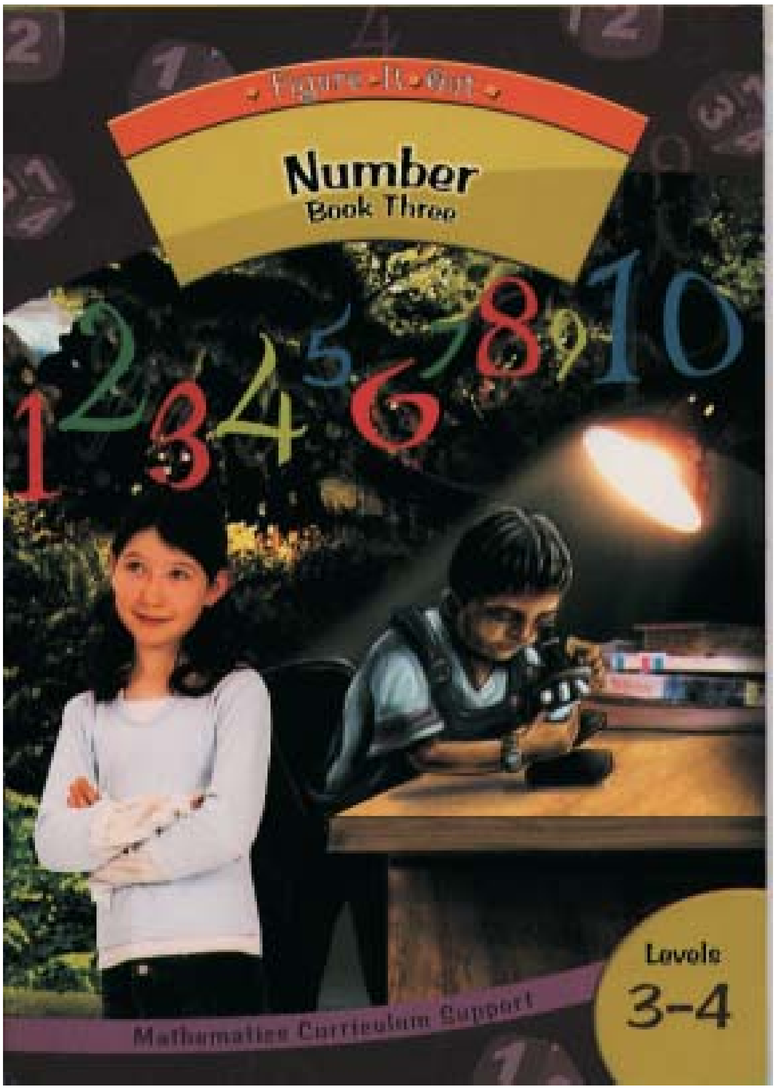
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for levels 3–4 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. *Number: Book Two* and *Number: Book Three* have been developed to support teachers involved in the Numeracy Project. These books are most suitable for students in year 6, but you should use your judgment as to whether to use the books with older or younger students who are also working at levels 3–4.

Student books
The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 6.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hotspots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes
The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom
Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number: Book Three Answers

Page 1: Place the Digits

Game

A game using place value with decimals

Activity

1. **a.** Your opponent has to score higher than you because your 4 in the hundredths place is higher than your opponent's 3.

   b. You will win outright if you score 8 or 9 on your next throw (which will beat your opponent's 7 in the ones place). If you score 7, you will still win if your opponent throws 7 or less. If you throw less than 7, you will lose no matter what your opponent throws.

2. Saulo. He needs to get a 4 or more to win the round. There are 36 possible combinations of the 2 dice, and Saulo has a $^{22}/_{36}$ chance of getting a 4 or more. Mele's chances of winning are $^{14}/_{36}$. (That is, there is a $^{14}/_{36}$ chance of Saulo not getting a 4 or more, in which case, Mele would win.)

Pages 2–3: Riding the Waves

Activity

1. **a.** Anaru, Eli, and Josh respectively

   b. Max would have to score more than 8.9 in his final surf to better Anaru's score of 22.2. This seems unlikely, based on his other scores, because his best 2 scores so far total 13.3. The result of Eli's fifth turn is also relevant. If he scores 7.4 or more, he will beat Anaru, which in turn would mean Max would have to score 9.1 or more to win.

2. Kelly: 6.4; Sue: 7.4; Margot: 6.7; Janice: 4.1

3. Answers may vary. Thomas's scores added up to 143.9, and Anaru's scores added up to 137.4. (Each judge scored Thomas higher than Anaru.) For Anaru to beat Thomas, the judges' scores for Anaru's "one great ride" would have to have added up to an extra 6.6 (or more) overall or an average of 1.32 (or more) per judge.

4.

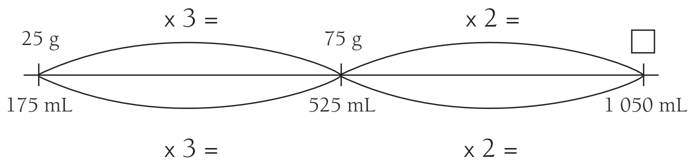
	Judges' mean scores							
Surfer	Wayne	Jared	Rachel	Ben	Ella	**Total**	**Mean**	**Place**
Kiri	6.2	6.0	6.5	6.3	6.7	31.7	6.34	3
Sue	7.4	7.8	7.1	8.3	7.3	37.9	7.58	2
Zoe	5.3	5.9	6.3	6.7	6.4	30.6	6.12	5
Hine	5.9	7.5	6.1	7.1	4.9	31.5	6.3	4
Eseta	5.8	6.1	5.9	6.2	5.4	29.4	5.88	6
Trish	7.5	7.6	8.1	8.4	8.3	39.9	7.98	1

Pages 4–5: Challenge Time

Activity

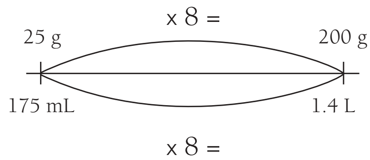
1. 45 L
2. 20 pieces
3. **a.** 75 g

   b. 150 g. To get this answer, you could extend the number line shown for **3a**:

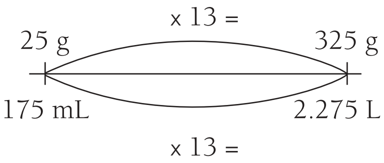


   c. Answers and possible number lines are:

   i. 1.4 L

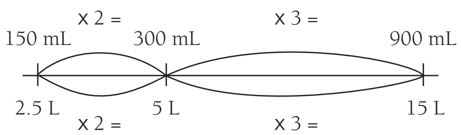


ii. 2.275 L

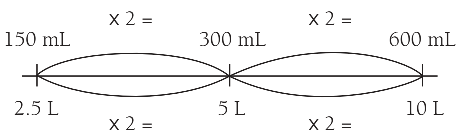


4. 15 days. (9 ÷ 3 = 3. 3 x 5 = 15)
5. 24 lollies. ($\frac{1}{2}$ + $\frac{1}{3}$ is the same as $\frac{3}{6}$ + $\frac{2}{6}$, which is $\frac{5}{6}$. So Mele's 4 lollies is $\frac{1}{6}$. Therefore, Ken gets $\frac{2}{6}$ or 8, and Hineata gets $\frac{3}{6}$ or 12.)
6. 7. (You would have 3 wheels and 3 trucks left over.)
7. $14
8. 9 pieces of spicy baked chicken; 18 pieces of regular chicken. (27 ÷ 3 = 9. 9 x 2 = 18)
9. Answers and possible number lines are:

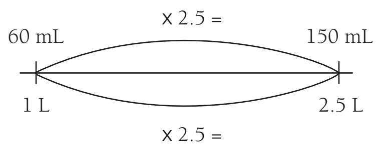
a. 900 mL



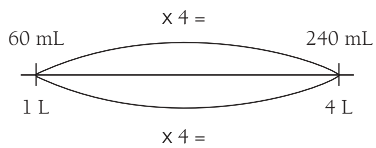
b. 10 L



c. 1 L



d. 240 mL



Page 6: Paddle On

Activity

1.

Waka	A	B	C	D	E	F
Speed (m/s)	1.6	1.8	1.5	1.7	1.3	1.6

2. **a.** 7 s

b. 7 sets

c. 6 sets. 48 ÷ 7 = 6, plus 5 of the next set of 6. They crossed the line before they completed the last stroke of the 7th set.

3. **a.** 6. (72 ÷ 12)

b. 11. (132 ÷ 12)

c. 18 (with 2 strokes on the last side because 218 ÷ 12 is 18 and 2 remainder)

4.

Waka	A	B	C	D	E	F
Speed (m/min)	114.3	100	92.3	109.1	80	104.3

Page 7: Choosing Fruit

Activity

1. Methods will vary. Some possible methods are:

i. 10c each would be $1.50; 5c each would be $0.75; 15c each would be $2.25; so 1 apple must be a bit more: about 16c.

ii. 15 at 20c equals $3.00: too big; 15 at 10c equals $1.50: too small; so 1 apple should be just over halfway between 16c and 17c.

iii. $2.39 is about $2.40, so you'd get 5 for about 80c, which means that 1 would be about 16c.

iv. 10 apples is $\frac{2}{3}$ of 15; $\frac{2}{3}$ of $2.40 is $1.60; $1.60 ÷ 10 = 16c.

2. **a.** Estimates will vary. For example:

i. 16c. ($7.95 is about $8.00, so 5 would be about 80c; 10 would be about $1.60, so 1 apple would be about 16c.)

ii. 15c. (If they were 10c, 30 would be $3.00. If they were 20c, 30 would be $6.00. $4.50 is halfway between, so they are about 15c each.)

b. 30 for $4.50 is the cheaper of the 2 bags.

3. **a.** Sensible estimates would be:

i. 25c each

ii. 25c each. (If you were calculating this exactly, without rounding, they would be slightly cheaper than **i**.)

iii. 23c each. (One way to estimate this is to work on the cost of 3 mandarins. Another way is to reason that you would pay 78c for the extra 2 mandarins in bag **ii**, so the mandarins in bag **iii** must be less than 25c each, and 23c would be a fair estimate.)

b. 18 for $4.14 is the cheapest bag because each mandarin is 23c. The dearest is 10 for $2.49 because each mandarin in this bag is nearly 25c.

Pages 8–10: Number Patterns

Activity

1. a. He repeats the first digit of the number being multiplied by 11. For example, 9 x 11 = 99.

b.

Number to multiply by 11	Product
10	110
11	121
12	132
13	143
14	154
15	165
16	176
17	187
18	198
19	209
20	220
21	231
22	242
23	253
24	264
25	275
26	286
27	297
28	308
29	319
30	330
31	341

c. Answers may vary. One way is to write (or visualise) the two digits that are being multiplied by 11 with a space between them and then add them together to get the middle digit. For example, 24 x 11 = 2 (2 + 4) 4
= 264.
(The pattern is that the 2 digits in the original number become the hundreds and ones digits respectively of the new number, and the tens digit is the sum of the 2 original digits.)

2. a. 1 500. (60 ÷ 4 = 15, so 60 x 25 = 15 hundreds.)

b. 20 000. (200 hundreds)

c. 2 100. (21 hundreds)

d. 4 400. (44 hundreds)

e. 9 125. (91 hundreds and one 25)

f. 16 175. (161 hundreds and three 25s)

g. 9 000. (36 ÷ 4 = 9, so 36 x 25 = 9 hundreds; 900 x 10 = 9 000. Or: 4 x 250 = 1 000. 36 ÷ 4 = 9, so 9 x 1 000 = 9 thousands or 9 000.)

h. 30 000. (12 ÷ 4 = 3, so 12 x 25 = 3 hundreds; 300 x 100 = 30 000. Or: 4 x 2 500 = 10 000. 12 ÷ 4 = 3, so 12 x 2 500 = 30 000.)

i. 120. (48 ÷ 4 = 12, so 48 x 25 = 12 hundreds; 1 200 ÷ 10 = 120. Or: 4 x 2.5 = 10; 48 ÷ 4 = 12; 12 x 10 = 120)

j. 5. (20 ÷ 4 = 5, so 20 x 25 = 5 hundreds; 500 ÷ 100 = 5. Or: 4 x 0.25 is 1, so 20 ÷ 4 is 5 ones.)

3. a. 3 000. (24 ÷ 8 = 3, so 24 x 125 = 3 thousands.)

b. 10 000. (10 thousands)

c. 5 500. (5 thousands and 4 lots of 125)

d. 12 125. (12 thousands and 1 lot of 125)

e. 21 000. (21 thousands)

f. 43 250. (43 thousands and 2 lots of 125)

4. Answers may vary. You could multiply by 10 or 100 and double your answer (for example, 23 x 10 = 230; 230 x 2 = 460) or multiply by 2 and use place value to multiply by 10 or 100 (for example, 23 x 2 = 46; 46 x 10 = 460).

a. 460

b. 8 640

c. 10 200

d. 7 200

e. 5 060

f. 63 800

5. a. Strategies will vary. You could adapt the strategies for 20 and 200 (for example, multiply by 10 or 100 and then multiply your answer by 5), or you could multiply by 100 or 1 000 and halve the answer (26 x 100 = 2 600; 2 600 ÷ 2 = 1 300; so 26 x 50 = 1 300, or, since 50 is half of 100, halve the 26 and call it hundreds, that is, 1 300).

b. i. 1 300
ii. 3 200
iii. 24 000
iv. 60 000
v. 784. (16 x [50 – 1] = 800 – 16, which is 784.)
vi. 11 904. (24 x 1 000 ÷ 2 – 4 x 24 or 24 x [500 – 4] = 12 000 – 96, which is 11 904.)

6. a. 297

b. 3 100

c. 2 250

d. 6 840

e. 40 800

f. 3 400

g. 10 375

h. 216 000

Page 11: Feel the Beat

Activity One

1.–2. Answers will vary. An average heartbeat rate for a 10-year-old is about 100 beats per min, and a maximum heartbeat rate for a 10-year-old is about 210 beats per min. The number of your heartbeats per minute, hour, day, and year will depend on your pulse count.

3. Answers will vary, but your heartbeat rate should be much higher after exercise.

Activity Two

1.

Animal	Heartbeats per minute	Heartbeats per hour	Heartbeats per day	Heartbeats per year
Hummingbird	1 000	60 000	1 440 000	525 600 000
Mouse	500	30 000	720 000	262 800 000
Human baby	120	7 200	172 800	63 072 000
Elephant	28	1 680	40 320	14 716 800
Blue whale	5	300	7 200	2 628 000

2. a. Answers will vary.

b. Answers will vary.

c. Heartbeat rates tend to be faster the smaller the animal is.

Pages 12–13: How Many?

Activity

1. Answers will vary. For example: 11 classes of 27 is approximately 10 classes of 30 students = 300. If 75% come, that's $^3/_4$ of 300; $^1/_2$ would be 150, and $^1/_4$ must be 75, so $^3/_4$ would be 225. Most would have a drink and chips, so they'll need about 225 of each. 200 is 10 lots of 20, so 225 is 11 or 12 boxes of chips. 24 is about 25, so that's 9 or 10 cartons of drinks. (It's probably better to go for the higher figure in each case.)

2. Answers will vary. For example, 10 rows is about 180, and 5 rows is about 90, so that's about 270. Add 30 for the one or more at the ends of rows, so that's now about 300. Not all the rows were full, but there could have been 10 standing by the door. So the total estimate could be between 300 and 315.

3. Answers will vary.

4. Answers will vary. For example:

a. Multiply the average number of people in each row by the number of rows.

b. Count the number of entries in a column. Multiply by the number of columns in a page and then multiply by the number of pages in the phone book.

c. Count the number of cars going past in 1 minute during the rush hour of 8 a.m. to 9 a.m. and multiply this number by 60 to get the number that could go by in the hour. Then count the number of cars going past in 1 minute after 9 a.m. and extrapolate this for the rest of the school day. The number of cars between 3 p.m. and 4 p.m. could increase significantly, so you may decide to regard this period as another rush hour.

d. Time how long it takes to read 1 page with a reasonable number of words on it. Multiply this by the number of pages.

5. Answers will vary.

Page 14: Dogs' Dinner

Activity

1. A quicker way is to multiply by 20, which you might choose to rename as 2 x 10. (Multiplying by 20 is the same as ÷ 5 x 100.)

2. a. 560 g

   b. 50 g

   c. 380 g

   d. 160 g

   e. 460 g

   f. 790 g

   g. 860 g

3. 4 460 g or 4.46 kg

Page 15: Upgrading the Hall

Activity

1. a. $30.25

   b. $49

   c. $29.25

   d. $59

2. a. $1,080

   b. $540

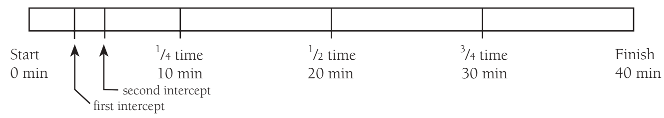
   c. $2,160

3. $1,458

Pages 16–17: Sporting Fractions

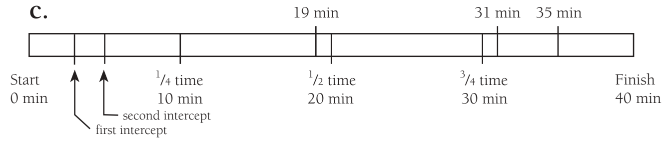
Activity

1. a.



b. First intercept: 3 min; second intercept: 5 min

c.



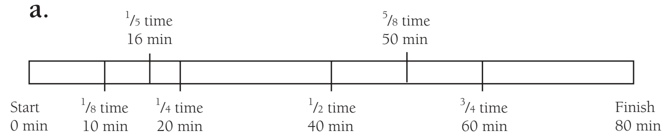
Fractions of the game completed:

i. just less than $1/2$

ii. just over $3/4$

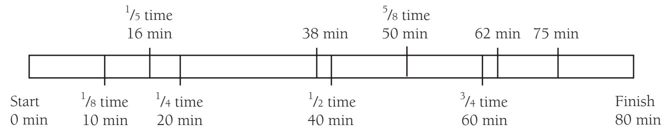
iii. $7/8$, midway between $3/4$ and the finish

2. a.



b. $7/8$

c.



Fractions should be close to:

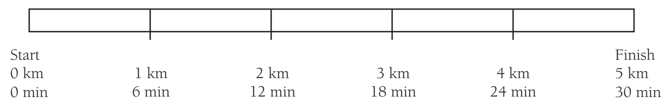
i. $1/8$

ii. $1/2$

iii. $3/4$

iv. Either $15/16$ or $9/10$

3.



(You could also show this as a double number line.)

4. $5/50$ or $1/10$. This is $1/2$ of $1/5$. Another explanation is: $1/5$ is $2/10$. Half of $2/10$ is $1/10$.

Pages 18–19: Water World

Activity One

1.

Name	Mass (kilograms)	Amount of water (litres)
Tamahou	30	21
Rangi	40	28
Jessica	35	24.5
David	42	29.4
Kate	34	23.8

2.

Name	Amount of water to drink
Tamahou	9% of 21 L = 1.89 L
Rangi	8% of 28 L = 2.24 L
Jessica	9% of 24.5 L = 2.205 L
David	9% of 29.4 L = 2.646 L
Kate	8% of 23.8 L = 1.904 L

3. Methods may vary.
   28% of 500 million = 140 million square km

Activity Two

1. **a.** Possible estimates are:
   kitchen: 40 L
   outside: 80 L (19% is close to 20%)
   laundry: 80 L (21% is close to 20%, too)
   bathroom: 100 L (24% is close to 25%)
   toilet: 100 L (26% is close to 25%)

   b. 430 L

   c. 372 L

2. **a.** 320 L

   b. Possible estimates, based on the same strategies as **1a**, are:
   kitchen: 32 L, outside: 64 L, laundry: 64 L, bathroom: 80 L, toilet: 80 L

3. 1%

Pages 20–21: The Volcanoes Erupt

Activity

1. 28 years
2. 440 years
3. **a.** 1 300 years
   b. 4 000 years
4. **a.** 2 800 years
   b. 7 200 years
5. **a.** 2 940 years
   b. 4 495 years
   c. 1 736 years
6. **a.** Haroharo
   b. Rotokawau
7. **a.** Maungarei
   b. Taupō
   c. Taranaki

Investigation

Results will vary. They may include the eruptions that eventually formed the Lyttelton, Akaroa, and Otago harbours, some of the mountains in the western Waikato, some of the cones in the vicinity of Auckland City, and Mount Edgecombe, Mount Ngāuruhoe, Mount Tongariro, and White Island.

Pages 22–23: Chilly Heights

Activity

1. $^-10$°C
2. $^-14$°C
3. $^-13$°C
4. About 16°C. (The difference in height is 3 754 – 700 = 3 054 m. 3 054 ÷ 150 = 20.36. 20.36° + $^-4$° = 16.36°)
5. 8°C. (4° – $^-4$° = 8)
6. 4°C

Page 24: Hidden Help

Activity

1.

Numbers between 0 and 1		
Fraction	**Decimal**	**Percentage**
2/10	0.2	20%
1/3	$0.\dot{3}$	33.3%
25/100	0.25	25%
2/3	$0.\dot{6}$	66.7%
6/10	0.6	60%
1/8	0.125	12.5%
3/8	0.375	37.5%
1/6	$0.1\dot{6}$	16.7%
1/2	0.5	50%
75/100	0.75	75%
1/10	0.1	10%
4/10	0.4	40%

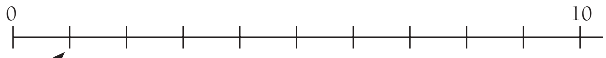
2. Answers will vary. Fractions and decimals between 0 and 1 are fractions of 1, whereas percentages are fractions of 100.

Possible rules:

To change a:	
fraction into a decimal	divide the numerator (top number) by the denominator (bottom number). For example, for 25/100: 25 ÷ 100 = 0.25.
decimal into a percentage	multiply it by 100 (that is, shift all digits 2 places to the left). For example, 0.25 = 25%.
fraction into a percentage	combine the above 2 steps.
percentage into a decimal	divide it by 100 (that is, shift all digits 2 places to the right). For example, 25% = 0.25.
decimal into a fraction (except for repeating decimals)	convert it to fractional form and simplify if possible. For example, 0.2 = 2/10 or 1/5 0.75 = 75/100 or 3/4.
percentage into a fraction	write the percentage as a fraction of 100 and simplify if possible. For example, 25% = 25/100 or 1/4 12.5% = 12.5/100 or 1/8.

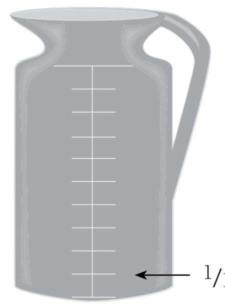
Possible diagrams:

A number line



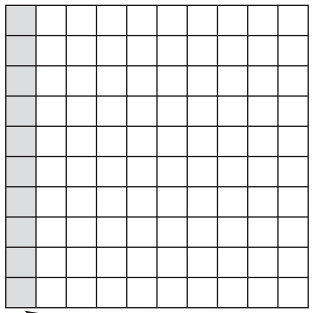
1/10 or 0.1 or 10%

A jug



1/10 or 0.1 or 10%

Ceramic tiles



1/10 (10/100) or 0.1 or 10%

A possible picture is a chocolate bar:

1/10 or 0.1 or 10%

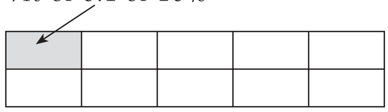


Figure It Out

Number: Book Three

Teachers' Notes

Overview of Number: Book Three

Title	Content	Page in students' book	Page in teachers' book
Place the Digits	Using place value with decimals	1	13
Riding the Waves	Working with decimals	2–3	14
Challenge Time	Using proportions	4–5	15
Paddle On	Solving problems involving whole numbers and decimals	6	17
Choosing Fruit	Estimating multiplication	7	18
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Feel the Beat	Solving multiplication problems involving rate	11	20
How Many?	Making sensible estimates	12–13	21
Dogs' Dinner	Solving problems involving decimal multiplication and division	14	22
Upgrading the Hall	Solving problems with fractions	15	22
Sporting Fractions	Solving problems involving fractions of time and distance	16–17	23
Water World	Solving problems involving percentages	18–19	25
The Volcanoes Erupt	Working with positive and negative numbers	20–21	27
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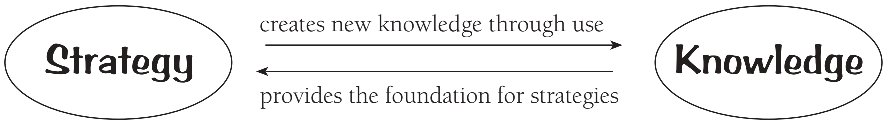
Introduction to Number

There is a remarkable commonality in the way many countries around the world are now teaching arithmetic. Changes in the approaches reflect the evolving demands of everyday life, a greater volume of classroom-based research about how students learn, and a desire to improve general levels of numeracy.

In the past, arithmetic teaching has focused on preparing students to be reliable human calculators. The prevalence in society of machines that calculate everything from supermarket bills to bank balances has meant that students now require a wider range of skills so that they can solve other problems flexibly and creatively.

The Figure It Out series aims to reflect these trends in modern mathematics education. A range of books is provided at different levels to develop both number skills and number sense. The *Number* books are aimed at developing students' understanding of the number system and their ability to apply efficient methods of calculation.

The development of the Figure It Out series has occurred against the backdrop of a strong drive for improved standards of numeracy among primary-aged and intermediate students. A key element of this drive has been the creation of the Number Framework as part of the Numeracy Strategy. The framework highlights this significant connection between students' ability to apply mental strategies to solving number problems and the knowledge they acquire.



Learning activities in the series are aimed at both the development of efficient and effective mental strategies and increasing the students' knowledge base.

Links to the Number Framework

There are strong links to the stages of development in the Number Framework in *Number,* Books Two and Three for level 3 and levels 3–4. The knowledge- and strategy-based activities in the level 3 books are at the early additive part–whole to advanced additive/early multiplicative stages of the Number Framework. Those in the level 3–4 books also link to the advanced multiplicative (early proportional) part–whole stage of the Number Framework.

Information about the Number Framework and the Numeracy Project is available on the NZMaths website (www.nzmaths.co.nz/Numeracy/Index.htm). The introduction includes a summary of the eight stages of the Number Framework. Some of the Numeracy Project material masters are relevant to activities in the Figure It Out student books and can be downloaded from
www.nzmaths.co.nz/Numeracy/materialmasters.htm
Books for the Numeracy Development Project can be downloaded from
www.nzmaths.co.nz/Numeracy/2004numPDFs/pdfs.htm

Terms

Teachers who are not familiar with the Numeracy Project may find the following explanations of terms useful.

The Number Framework: This is a framework showing the way students acquire concepts about number. It comprises eight stages of strategy and knowledge development.

Knowledge: These are the key items of knowledge that students need to learn. Knowledge is divided into five categories: number identification, number sequence and order, grouping and place value, basic facts, and written recording.

Strategies: Strategies are the mental processes that students use to estimate answers and solve operational problems with numbers. The strategies are identified in the eight stages of the Number Framework.

Counting strategies: Students using counting strategies will solve problems by counting. They may count in ones, or they may skip-count in other units such as fives or tens. They may count forwards or backwards.

Part–whole thinking or part–whole strategies: Part–whole thinking is thinking of numbers as abstract units that can be treated as wholes or can be partitioned and recombined. Part–whole strategies are mental strategies that use this thinking.

Partitioning: Partitioning is dividing a number into parts to make calculation easier. For example, 43 can be partitioned into 40 and 3, or 19 can be partitioned into 10 and 9 or thought of as 20 minus 1.

Relevant stages of the Number Framework for students using the level 3 and 3–4 *Number* books:
Stage five: early additive part–whole: At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and recombined. This is called part–whole thinking.

A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or "teen" numbers. For example, students at this stage might solve 7 + 8 by recalling that 7 + 7 = 14, so 7 + 8 = 15. They might solve 9 + 6 by knowing that 10 + 6 = 16, so 9 + 6 = 15. They might solve 43 + 35 as (40 + 30) + (3 + 5), which is 70 + 8 = 78.

Stage six: advanced additive/early multiplicative part–whole: At this stage, students are learning to choose appropriately from a repertoire of part–whole strategies to estimate answers and solve addition and subtraction problems.

Addition and subtraction strategies used by students at this stage include:
- standard place value with compensation (63 – 29 as 63 – 30 + 1)
- reversibility (53 – 26 = □ as 26 + □ = 53)
- doubling (3 x 4 = 12 so 6 x 4 = 12 + 12 = 24)
- compensation (5 x 3 = 15 so 6 x 3 = 18 [3 more])

Students at this stage are also able to derive multiplication answers from known facts and can solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6 x 6 as (5 x 6) + 6; or $^3/_4$ of 24 as $^1/_4$ of 20 is 5 because 4 x 5 = 20, so $^3/_4$ of 20 is 15, so $^3/_4$ of 24 is 18 because $^3/_4$ of the extra 4 is 3.

Stage seven: advanced multiplicative part–whole: Students who are at this stage are learning to choose appropriately from a range of part–whole strategies to estimate answers and solve problems involving multiplication and division. For example, they may use halving and doubling (16 x 4 can be seen as 8 x 8) and trebling and dividing by 3 (3 x 27 = 9 x 9).

Students at this stage also apply mental strategies based on multiplication and division to solve problems involving fractions, decimals, proportions, ratios, and percentages. Many of these strategies involve using equivalent fractions.

Page 1: Place the Digits

Achievement Objectives

- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- order decimals with up to 3 decimal places (Number, level 3)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- use a systematic approach to count a set of possible outcomes (Statistics, level 3)
- predict the likelihood of outcomes on the basis of a set of observations (Statistics, level 3)

Game

This game involves placing numbers to 3 decimal places (that is, to thousandths) and requires the students to engage in strategic thinking about place value. The development of decimal knowledge begins at the advanced additive stage of the Number Framework. In each round, the students try to create the largest possible number in 4 throws of the 2 dice, so they have to decide in which column to record the result of each throw. Each decision will probably depend on the student's idea about how likely it is that they will throw higher or lower numbers (with both dice) in their remaining throws.

The key learning outcomes of this game are understanding place value and simple probability, in this case, using decimals. There is a lot of scope here for discussion and investigation regarding the effect of placing a digit in different columns, for example, the tenths column rather than the hundredths column. The students need to understand that, in decimals, as with whole numbers, each column to the left is 10 times greater than the previous column and each column to the right is 10 times less than the previous one. For example, 0.6 is 10 times greater than 0.06, and 0.006 is 10 times less than 0.06.

Once the students understand the importance of place value in this game, they could investigate the likelihood of rolling digits that add up to each of the numbers 0–9. The chances could be set out in a table, as below:

Number	Chances with 2 dice (numbered as described in the game)	
0	2	(0, 0; 0, 0)
1	3	(0, 1; 0, 1; 1, 0)
2	4	(0, 2; 0, 2; 2, 0; 1, 1)
3	5	(0, 3; 0, 3; 3, 0; 1, 2; 2, 1)
4	6	(0, 4; 0, 4; 4, 0; 1, 3; 3, 1; 2, 2)
5	6	(0, 5; 0, 5; 4, 1; 1, 4; 3, 2; 2, 3)
6	4	(1, 5; 4, 2; 2, 4; 3, 3)
7	3	(2, 5; 4, 3; 3, 4)
8	2	(3, 5; 4, 4)
9	1	(4, 5)

There are 36 possible combinations of the 2 dice. Once these combinations have been worked out, the students may then be able to see that they have 20 chances of scoring a total less than 5 but only half that number of chances (10) of scoring a total greater than 5. (They have 6 chances of scoring 5.) This may help them to think strategically as they play the game. (A variation of the game could be to create the smallest possible number.)

Activity

This activity draws on the students' understanding of statistics as well as place value. Working out the chances of throwing each number from 0 to 9, as described above, will help the students to answer these questions.

The throws needed for question **1** are explained in the Answers. The focus of this question is on place value rather than on probability.

To win the game described in question **2**, Saulo has to throw a 4 or more. You may want to discuss this with the students to make sure they understand that 6.410 (or 6.510, 6.610, and so on) is higher than 6.409. (This is particularly relevant if Saulo throws a 4.) The students therefore need to work out the chance of Saulo throwing a 4 or more. The table above shows that Saulo has a $^{(6 + 6 + 4 + 3 + 2 + 1)}/_{36}$ chance, that is, a $^{22}/_{36}$ or $^{11}/_{18}$ chance of winning.

Pages 2–3: Riding the Waves

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- order decimals with up to 3 decimal places (Number, level 3)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

In this activity, students add, subtract, and divide numbers to 1 decimal place. They could use a calculator for some questions because the focus of the task is on logic and reasoning rather than on calculation. However, encourage them to use mental strategies (such as compatible numbers) for the more straightforward calculations, especially those in questions **1** and **2**. The development of mental strategies for adding and subtracting decimals is appropriate for students who have become fluent at the advanced additive stage of the Number Framework. They should be able to apply their strategies for adding and subtracting whole numbers to similar problems with decimals.

Question **1** requires the students to identify the 3 best scores for each of the 4 contestants and add them up. Make sure they realise that the numerals in the ones column must be considered before the numerals in the tenths column (for example, that 6.8 is smaller than 7.1). Then they can work out, for question **1b**, what Max would need to score on his final surf to win and use their judgment based on Max's pattern of scoring. The rationale for why Max is unlikely to win is given in the Answers.

In question **2**, the students need to devise a strategy to work out the missing numbers given the total of the best 3 waves. A possible strategy is to subtract the highest 2 existing scores from the total in each case to determine whether the missing score is one of the best 3. As it turns out, the missing score in each case is one of the best 3 for each surfer, but the students shouldn't presume this to be so.

Question **3** involves comparing Thomas's and Anaru's scores to determine the overall difference and hence the *extra* points Anaru would have needed to score from each judge in a single ride to beat Thomas. Again, encourage the students to devise their own mental strategy or strategies for investigating this issue.

Some may decide to mentally calculate the difference on each scorecard and add these together. For example, the scores given by Ben can be added in two parts using standard place value. First, the whole numbers:

18 + 25 + 27 + 17

= 35 + 52 (using doubles)

= 87.

Secondly, the decimals: 0.9 + 0.3 + 0.2 + 0.6 = 2.0. So the total is 87 + 2 = 89. This involves knowledge of place value with decimals, which develops at the advanced additive stage of the Number Framework. Other students may total each surfer's 5 scores and then subtract Anaru's score from Thomas's score. Whichever method they use to find the 6.5 point difference, they need to realise that this is over all the judges' scores so, on average, Anaru is 1.3 points behind Thomas. A fuller explanation for this is given in the Answers.

Question **4** uses *mean* scores as a way of ranking the surfers. The mean is an average derived from adding up a set of numbers (data) and dividing the total by the number of items in the data set. (This also applies in the explanation for question **3**.) The students may need to discuss what the mean is and how it can be calculated. (In this case, it involves dividing the total of the judges' scores by 5.) They also need to consider what strategies to use to find the missing numbers. For example, they could use trial and improvement to find Sue's missing wave score (that is, finding what number needs to be added to the other 4 numbers to give a mean of 7.58), but a more efficient strategy is to multiply the mean by 5 to find her total score and then subtract the total of the other 4 scores to find her missing wave score.

In question **1**, the students needed to consider the numerals in the ones column before the numerals in the tenths column. In question **4**, they need to take this further and look at the numerals in the tenths column before they look at the numerals in the hundredths column. For example, Sue's mean of 7.58 is less than Trish's mean of 7.98, and Hine's 6.3 (which is, in effect, 6.30) is less than Kiri's 6.34. This is important when it comes to ranking the surfers.

Pages 4–5: Challenge Time

Achievement Objectives

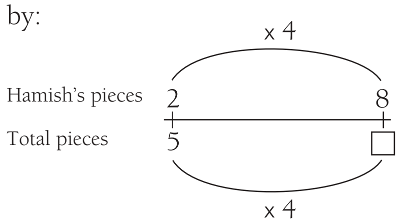
- find fractions equivalent to one given (Number, level 4)
- write and solve problems involving decimal multiplication and division (Number, level 4)

Activity

All of the problems in this activity involve proportions and require students to have efficient multiplicative strategies at the advanced additive stage or beyond. Many of them can be efficiently solved using double number lines, as suggested for question **1**. Using double number lines is one way of encouraging a systematic approach towards problem solving. However, students also need to realise that this is not the only way of solving proportion problems, and in some cases, other strategies are more effective. For example, question **5** is best approached using equivalent fractions, and the odd numbers in question **6** call for a different sort of reasoning. Ratio tables are also an efficient scaffolding method.

In question **1**, the task is to determine what the 100 has to be multiplied by to make it 450. The 10 must then be multiplied by the same number (4.5), so it becomes 45.

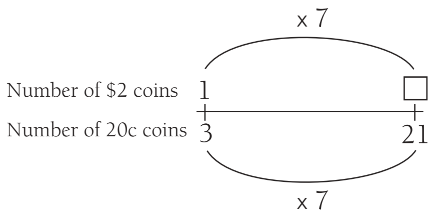
Encourage the students to draw up a double number line for question **2**. If they are unsure where to start, ask them "How many pieces of toast are eaten each day?" "How many of these does Hamish eat?" "Where will you put the 2 and the 5 on your double number line?" Finding the required answer becomes simple if the students show on their double number line what they are multiplying by:



The students should be able to apply the same systematic approach to all the problems in this activity. To make sure the students are on the right track, have them compare their double number line with a classmate's.

For question **3b**, you need to check that the students have recognised the change from millilitres to litres.

In question **7**, the students need to realise that the 10 cent coin is really irrelevant for solving the problem. A possible double number line for this problem is:



The 1 represents one $2 coin, and the 3 represents three 20 cent coins in change.

Questions **4**, **5**, **6**, and **8** are a bit different from the others and require the students to exercise some logical thinking and reasoning.

For question **4**, the students need to reason that the food should last Josh, Ani, and Philip 3 times as long as it would the 9 people, so it should last for 15 days.

In question **5**, the students need to work out what fraction of the total bag the 4 remaining lollies represent (see the explanation in the Answers). To do this, they first need to convert the half and the third into sixths, the lowest common denominator.

In question **6**, the 17 trucks and 31 wheels are not multiples of 2 and 4 respectively, so it is apparent that there are going to be some spare trucks and wheels when more skateboards are assembled. The students may find it helpful to make a simple sketch of a skateboard with an outline of the board itself, the 2 trucks, and the 4 wheels.

Question **8** involves *ratio*. This type of problem is more appropriate for students who have developed multiplicative reasoning skills and who are at the advanced additive stage or beyond. A ratio is a fraction that compares 2 quantities of the same item. Once the students realise that the 2:1 ratio means there are 3 pieces of chicken in a special combo and 1 of them is spicy baked, they should be able to think in terms of $3 \times \square = 27$. There are 9 combos, which means 9 pieces of spicy baked chicken and 18 pieces of regular chicken.

Two units of measurement (millilitres and litres) are used in any given part of the problem in question **9**. Some students may feel they need to convert one unit into the other, but this is not necessary when you are focusing on rates and the units used remain constant.

Many of the problems in this activity also lend themselves to exploring equivalent fractions. For earlier work on equivalent fractions, see *Number*, Figure It Out, Level 3, page 9, and *Number: Book Three*, Figure It Out, Level 3, pages 22–23.

Page 6: Paddle On

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- write and solve problems involving decimal multiplication and division (Number, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

Rates are difficult to model with materials and require students to have good, sound multiplicative strategies. The idea of average (mean) is established in the transition from advanced additive to advanced multiplicative.

Question **1**, which asks students to calculate metres per second, involves the idea of *rate*, that is, a fraction used to compare 2 quantities of different measures. To answer this question, the students need to divide 80 metres in turn by each of the 6 waka ama times. This part of the question can easily be done on a calculator, especially if 80 is put into memory and recalled for each calculation. If the calculation results in a decimal number that includes hundredths or thousandths, you may need to help the students to use the procedure for rounding to 1 decimal place:

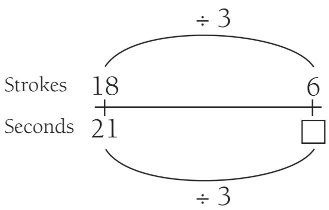
i. go up to the next figure if the result in the hundredths column is 5 or greater (for example, 180 ÷ 45 = 1.7777, which rounds to 1.8);

ii. leave the same figure if the result in the hundredths column is 4 or less (for example, 80 ÷ 49 = 1.632653, which rounds to 1.6).

An interesting extension to question **1** would be to work out:

- the *mean time* taken by the combined 6 waka ama to do the 80 metre sprint (by adding the 6 paddling times and then dividing by 6; rounded to 1 decimal place, this is 51.2 seconds).
- the *mean speed* of the 6 waka ama taken together (80 ÷ 51.2 = 1.6 metres per second, rounded to 1 decimal place).

In question **2a**, the students need to think logically to work out that 6 strokes is 1/3 of 18 strokes, so the time for 6 strokes must be 1/3 of 21 seconds, namely, 7 seconds. A double number line is a useful strategy here:



For question **2b**, the students can use their answer from **1a**: 6 strokes (1 set) takes 7 seconds. They can then use their knowledge of basic multiplication facts to answer 7 x ☐ = 49. The thinking that the students do for this question is relevant to question **2c** (see the Answers).

Question **4**, like question **1**, focuses on rate, but this time calculating metres per minute. To work out the average (mean) speed in metres per minute of the paddling times, the students need to recognise that 12 kilometres is 12 000 metres. Then they can use the same approach as in question **1**, namely, dividing the 12 000 by the time in minutes taken by each waka ama and rounding where necessary to 1 decimal place.

Possible extensions to question **4** are similar to those outlined above for question **1**:

- calculating the *mean time* of the 6 waka ama over the 12 kilometre distance (121.7 minutes)
- calculating the *mean speed* of the 6 waka ama (98.6 metres per minute)

Page 7: Choosing Fruit

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- recall the basic multiplication facts (Number, level 3)

Activity

The problems in this activity also involve rates and require students to have strong multiplicative reasoning skills (advanced multiplicative stage of the Number Framework).

An estimate is a number that seems reasonable when it is not possible or necessary in the circumstances to know the exact amount. The estimation required in this activity involves strategies such as sensible rounding of numbers to those that can be manipulated mentally (for example, $7.95 can be rounded to $8.00) and recognising relationships between numbers (for example, 10 mandarins is half the quantity of 20 mandarins). Jody also needs to compare the same fruit in different-sized bags. Reducing prices to their unit value allows valid comparison (for example, 10 at $2.49 gives a *unit value* of just under 25 cents per piece of fruit, whereas 5 at $1.50 gives a unit value of 30 cents per piece). Sometimes comparisons other than unit values are more efficient. For example, in question **2**, the price of 10 apples could be calculated to compare the two deals.

For question **1**, four alternative ways of estimating the price per apple are provided in the Answers. The fourth method requires a reasonable degree of number sense.

Some different ways of estimating the cost of an apple in each bag in question **2** are also provided in the Answers. Again, mentally calculating the cost of 10 apples is a good strategy to use. As an extension, ask the students to calculate what a bag with 50 apples would cost at this price and what the saving would be compared with the price of the first lot of apples.

Note that, in the mental calculations for 10 apples, we are using the strategy for dividing by 10 that was discussed in Place the Digits (page **1** of the students' book). If 10 apples cost $1.60, the price of 1 apple can be found by shifting each of the digits one column to the right (so that $1.60 becomes $0.16). This strategy can also be used with question **3**.

In question **3**, the students should be able to estimate the cost per mandarin in the first 2 bags. Estimating the cost per mandarin in the bag containing 18 mandarins for $4.14 poses a bigger challenge. One possibility would be to say that $4.14 is close to $4.20, so 3 mandarins would be $\frac{1}{6}$ of $4.20 (70 cents) and 1 mandarin would be $\frac{1}{3}$ of 70 cents, which is about 23 cents. This estimation is based on a number of mental calculations. The second method outlined in the Answers (using logical reasoning) is just as valid in terms of estimation. It is important that students are not afraid to give an estimate in case it is "wrong". The objective is to learn to give sensible estimates that can be justified.

As an extension, have the students calculate the cost of a bag of 10 mandarins at this cheapest rate of 23 cents per mandarin and a 20-mandarin bag at the same rate. They could also calculate the cost of a box of 100 mandarins at this rate and the amount they would save in comparison with the most expensive option.

Pages 8–10: Number Patterns

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- recall the basic multiplication facts (Number, level 3)
- explain satisfactory algorithms for addition, subtraction, and multiplication (Number, level 4)

Activity

This activity focuses on strategies that students develop in transition from advanced additive to advanced multiplicative.

Students usually enjoy investigating number patterns and are happy to use them once they realise that it's often quicker to use mental strategies than a calculator. Encourage the students to also investigate why the patterns work out as they do. In other words, what is the mathematics behind the patterns?

The tasks in question **1** all have to do with the 11 times table. The 11 times table results in a rather nice pattern (11, 22, 33, 44, and so on), which the students could explore to identify the underlying mathematics. Some may work out for themselves, and be able to share with others, that, for example, 3 x 11 is really 3 x (10 + 1). This in turn is the same as (3 x 10) + (3 x 1), which is 33. The mathematical principle being used here is the *distributive* principle. The 11 is being renamed as 10 + 1, and the multiplication is being distributed across the addition. This idea has its beginnings in the junior school, where students are helped to realise (perhaps with the use of Cuisenaire™ rods) that 2 lots of, say, 6 is the same as 2 lots of 5 and 2 lots of 1.

In question **1**, the students have to work out how Aimee gets her answers after 9 x 11. (See the explanation in the Answers.) Aimee's table goes only to 31, but you could explore some higher 2-digit multiples with the students. For example, with 11 x 86 = 946, the total of 8 and 6 is 14, so the 8 that would normally be in the hundreds column has a extra 1 added to it and becomes 9.

Question **2** also supports the students in developing number sense. Recognising that there are four 25s in 100 (or that 25 is $^1/_4$ of 100) is very helpful when it comes to multiplying numbers by 25.

Challenge the students to find other ways of mentally calculating the 40 x 25 mentioned at the beginning of question **2**. For instance, they may rename the 40 as 4 x 10, so the example becomes 4 x 10 x 25 (or 4 x 250) or 10 x 4 x 25 (or 10 x 100). This strategy is known as proportional adjustment or "reunitising", where the total is "repackaged" into easier units.

Questions **2a–f** can all be calculated mentally using the strategy outlined in the question itself or the other strategies described above. Questions **2g–j** involve multiplying by numbers that are related to 25 rather than by 25 itself. Thus, 250 is 25 x 10, 2 500 is 25 x 100, 2.5 is 25 ÷ 10, and 0.25 is 25 ÷ 100. The method in the Answers is based on Aimee's strategy, but your students could also look at other similar strategies. For example, 36 x 250 is the same as 36 x 100 ÷ 4 x 10 or, more conveniently, 36 ÷ 4 x 100 x 10, which is 9 x 1 000 or 9 000. Alternatively, if the students recognised that there are 4 lots of 250 in 1 000 (or 250 is $^1/_4$ of 1 000), the process could be shortened to 36 x 1 000 ÷ 4, or 36 ÷ 4 x 1 000, which again is 9 x 1 000.

In question **3**, Giulio's method for multiplying by 125 is based on there being 8 lots of 125 in 1 000, which is the same as $^1/_8$ of 1 000. That is, the 125 can be renamed as 1 000 ÷ 8 and used in this form to multiply with the numbers in questions **3a–f**.

Question **4** is about finding a pattern relating to the 20 times table and the 200 times table. The most obvious way is explained in the Answers. A similar strategy can be used for question **5**, or the students can rename the 50 as half of 100 (100 ÷ 2) and the 500 as half of 1 000 (1 000 ÷ 2). Reunitising is a critical strategy for calculating multiplication and division problems mentally.

The students will probably notice that, when multiplying by 50, the result is always half of the number being multiplied but expressed in hundreds (for example, 50 x 4 is 2 hundreds), and when multiplying by 500, it is again half of the number being multiplied but expressed in thousands (for example, 500 x 4 is 2 thousands). This pattern can then be used to work out the examples in question **5b**. Questions **5b v** and **vi** require a little more thought. These problems could be approached by rounding the 49 to 50 and the 496 to 500 (see the Answers).

Page 11: Feel the Beat

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

Activity One

Many students find the concept of heartbeats interesting, and it provides a good context for mathematical investigation. Suggested learning experiences for the Measurement strand of the curriculum include exploring the concept of change in relation to heartbeats and pulse rates. The activities involve calculating heartbeats per minute, hour, and so on and therefore introduce the idea of *rate*, which is comparing quantities of two different measures (for example, 100 heartbeats in 1 minute).

In question **1**, the students are asked to take their pulse to work out the number of times their heart beats in 15 seconds. If the students have not already seen this page, you could introduce the activity by asking them to devise ways of finding and measuring their heartbeats so they get a measure of the number of beats per minute. It is likely that some students will know where they can take a pulse from and will be able to tell the class. The students may then decide to count the total number of beats in a minute before someone works out that it would be more efficient to count the number of beats in half a minute and double them or the number of beats in 15 seconds and multiply them by 4. A stopwatch is not essential for this task; a watch that shows seconds is adequate.

Question **2** involves multiplying the answer in question **1** by 4 (for a minute), then that answer by 60 (for an hour), then that answer by 24 (for a day), and finally, that answer by 365 (for a year). Rather than the students automatically using their calculators, have them explore number strategies for working out the answers. For example, the rate per hour can be found by multiplying by 10 and then by 6 and the rate per day by (rate per hour x 10 x 2) + (rate per hour x 4). The rate per year can be found in a similar way. Some students may suggest rounding and compensating as a strategy. For example, to find the rate per day: (rate per hour x 10 x 3) – (rate per hour x 6).

Activity Two

Most students will be fascinated by the investigations in this activity. Suggest that they add their own pulse counts from **Activity One** to the table and see how these compare with those of the various animals. They should be able to see that the larger the animal, the slower the heartbeat and, conversely, the smaller the animal, the faster the heartbeat.

The students may not know that a person's heart rate decreases as they get older. Interested students could investigate why this happens.

Pages 12–13: How Many?

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)

Activity

The questions in this activity are all designed to help students develop their estimation skills using a range of data and taking into account various possibilities. Estimation strategies with respect to number include rounding up or down to reasonably close numbers that can be manipulated mentally. The estimates required in this activity involve multiplication and division. The activity is therefore suitable for students at the advanced additive stage or beyond of the Number Framework.

You could discuss with the students when and why they might estimate rather than calculate exactly. Estimating is useful when an exact calculation isn't required or, as in the case of the food for the school disco in question **1**, when it's not possible to calculate exactly because there is no exact data to work with. (The two students don't know how many people will come to the disco or how much they will eat and drink.) Estimating is often quicker than calculating, especially when you don't have a calculator handy. You could ask the students to compile a class list of situations in which estimating is useful.

See the Answers section for the thinking behind the possible answers for question **1**.

Question **2** is a problem that can be tackled initially in a similar way to finding the area of a rectangle, that is, by multiplying the number of rows (if you like, the length) by the approximate number of people in each row (if you like, the width). In this case, it means multiplying 15 by 18. This can be calculated mentally in several different ways, for example, $(10 \times 18) + 5 \times (20 - 2)$, which is $180 + 100 - 10$, giving 270. A different strategy is given in the Answers. Another possible strategy is to use doubling and halving ($15 \times 18 = 30 \times 9$), then known multiplication facts (3×9) and multiplication by 10. Obviously, several valid estimates are possible for this problem, depending on how the data is interpreted.

The students could use a variety of advanced multiplicative part–whole strategies to estimate the number of words on a page in question **3**. For example, if the page had 30 lines with an average of 17 words per line, they could use:

- standard place value: $$\begin{aligned} 30 \times 17 &= (30 \times 10) + (30 \times 7) \\ &= 300 + 210 \\ &= 510 \end{aligned}$$
- tidy numbers: $$\begin{aligned} 30 \times 17 &= (30 \times 20) - (30 \times 3) \\ &= 600 - 90 \\ &= 510 \end{aligned}$$

  or $$\begin{aligned} 30 \times 17 &= (30 \times 15) + (30 \times 2) \\ &= 450 + 60 \\ &= 510 \end{aligned}$$

Some possible estimation strategies for questions **4a–d** are given in the Answers. The students could discuss other strategies that they think of with a classmate.

In question **5b**, the students are asked to collect data and compare their own estimates with a classmate's. There are various reasons why estimates can vary. For example, different students could well read the same *School Journal* story at different rates and thus arrive at different estimates, all of which could be valid.

Page 14: Dogs' Dinner

Achievement Objectives

- write and solve problems involving decimal multiplication and division (Number, level 4)
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)

Activity

This activity involves decimal multiplication and division in the context of the ratio of grams of dog roll to grams (kilograms) of dog. The activity is therefore best suited for students at the advanced multiplicative stage or beyond of the Number Framework.

Question **1** asks the students to think of a short-cut method for calculating the mass of dog roll needed to feed various-sized dogs given that 100 grams is required for every 5 kilograms of a dog's mass.

To work out the amount of dog roll needed daily by each of the 7 dogs in question **2**, the students can make use of the strategy for multiplying by 20 that they have hopefully come up with for question **1**. A variation on renaming 20 as 2 x 10 or 10 x 2 is the use of doubling and halving, which, in this question, ties in with multiplying by 10. For example, for Brock, the border collie that weighs 23 kilograms, 23 x 20 can be renamed as 46 x 10 to give an answer of 460 grams. Encourage the students to calculate all the answers for this question mentally. If they use the 20 = 10 x 2 strategy with the decimal numbers in particular, they will be applying their knowledge of place value.

In question **3**, the students need to add the number of grams of dog roll required daily for all 8 dogs, not just the 7 in question **2**. Encourage them to use number strategies such as rounding and compensation to add these up without using a calculator.

The students should be aware that amounts larger than 999 grams are usually given in kilograms and that there are 1 000 grams in a kilogram. To convert 4 460 grams to kilograms, they just have to shift all the digits 3 columns to the right, so 4 460 grams becomes 4.460 or simply 4.46 kilograms.

Page 15: Upgrading the Hall

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

Activity

Fund-raising is a realistic context that will be familiar to many students. This activity involves finding fractions of amounts or quantities. The multiplicative reasoning required for this makes this activity suitable for students who are advanced additive or beyond.

Most of the questions in this activity can be solved mentally, although the students may need to jot down figures from time to time to assist their memories. They can probably suggest, through discussion, some good mental strategies that they can use.

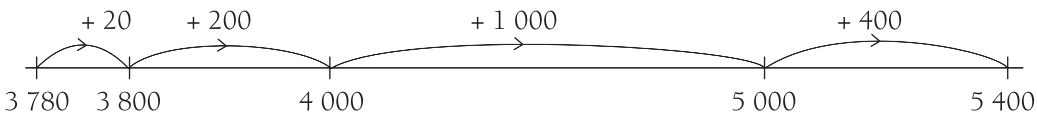
Cameron's contribution to the fund-raising can be worked out by calculating (5 x \$5) + (5 x 50c) + $\frac{1}{2}$ of \$5.50. That is, \$25.00 + \$2.50 + \$2.75, which is \$30.25. Another strategy is rounding and compensation: (6 x \$5 + 6 x 50c) – \$2.50 – 25c = \$33 – \$2.75
= \$33 – \$3 + 25c
= \$30.25

Catherine suggests a strategy in question **1b** for working out her contribution. An alternative approach would be to calculate (28 x 1) + $\frac{3}{4}$ of 28 (because 75 cents is $\frac{3}{4}$ of \$1.00, and it is easy to find $\frac{3}{4}$ of 28: $\frac{1}{4}$ is 7, so $\frac{3}{4}$ is 21, found by 3 x 7 or 28 – 7). This amounts to \$28 + \$21, which is \$49.

In Latham's case (question **1c**), the students need to find $\frac{3}{4}$ of 12 and multiply that by \$2. They could use rounding and compensation to work out the remaining 3 hours: 3 x \$4 – 75c.

Bradley's paper delivering yielded (12 x \$4.50) + \$5.00, which is \$48 + \$6 + \$5 or \$59.00 in total.

Question **2** requires the students to work out some simple fractions relating to the \$5,400 raised. They need to be accurate because their calculations on this activity are also critical to solving the problem in question **3**. For the painting cost of $\frac{1}{5}$, the students can divide the \$5,400 by 5 or use doubling: 10 800 ÷ 10. They will probably realise that they simply have to double this amount to calculate the $\frac{2}{5}$ cost of replacing the roof. Finding the cost of repairing the kitchen at $\frac{1}{10}$ of \$5,400 is even simpler: to divide by 10, shift the digits 1 place to the right. The students now need to add \$1,080 + \$2,160 + \$540 before they can work out how much money is left. Encourage them to do this mentally. One strategy is to use compatible numbers: add the \$40 to 2,160 to get \$2,200, then add the \$500 to get \$2,700. It should be easy then to add \$1,080 to get \$3,780. To subtract this total from \$5,400, they could rewrite this as a missing addends problem, \$3,780 + □ = \$5,400, and solve it using an empty number line:



Solving the problem in question **3** involves calculating $\frac{1}{10}$ of \$1,620 and subtracting this to work out the amount of money available for new seating.

Pages 16–17: Sporting Fractions

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)
- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- express quantities as fractions or percentages of a whole (Number, level 4)
- read and interpret everyday statements involving time (Measurement, level 3)

Activity

This activity involves the proportional concepts of fraction and rate. It is suitable for students at the advanced additive stage or beyond of the Number Framework.

Sporting commentaries (on radio and television) that include reference to various game statistics are fairly commonplace these days. This activity investigates statistics related to netball, rugby, and distance running and uses estimation involving fractions.

In questions **1a** and **1b**, the times of the first and second intercepts can be estimated fairly easily once the $^1/_4$ or 10 minute mark is inserted into the timeline. The second intercept appears to be halfway between the start and the 10 minute mark, so it is likely that it occurred at 5 minutes. Once the students have established this, they will see that the first intercept occurred at about the 3 minute mark. The students should use the $^1/_4$, $^1/_2$, and $^3/_4$ marks as benchmarks to solve the other problems.

With the $^3/_4$ or 30 minute mark placed on the timeline midway between half-time and the finish, the 3 times in question **1c** can now be marked on. The 19 minute intercept will be marked just before half-time, the 31 minute intercept just after $^3/_4$ time, and the 35 minute intercept midway between the $^3/_4$ time and the end of the game. Midway between $^3/_4$ time and the game's end, in fact, is the $^7/_8$ time mark. If the students are unsure of this, encourage them to draw up a fraction wall, for example:

<table>
<tr><td colspan="120">1/1</td></tr>
<tr><td colspan="60">1/2</td><td colspan="60">1/2</td></tr>
<tr><td colspan="30">1/4</td><td colspan="30">1/4</td><td colspan="30">1/4</td><td colspan="30">1/4</td></tr>
<tr><td colspan="15">1/8</td><td colspan="15">1/8</td><td colspan="15">1/8</td><td colspan="15">1/8</td><td colspan="15">1/8</td><td colspan="15">1/8</td><td colspan="15">1/8</td><td colspan="15">1/8</td></tr>
<tr><td colspan="40">1/3</td><td colspan="40">1/3</td><td colspan="40">1/3</td></tr>
<tr><td colspan="24">1/5</td><td colspan="24">1/5</td><td colspan="24">1/5</td><td colspan="24">1/5</td><td colspan="24">1/5</td></tr>
</table>

An activity involving a fraction wall can be found in Fraction Frenzy, *Number: Book Three*, Figure It Out, Level 3.

When a fraction is halved, the students may think that the halved fraction is bigger, when it is actually the opposite. For example, $^1/_2 \times {}^1/_5 = {}^1/_{10}$. You could show them this on a number line.

The fraction wall or the Cuisenaire™ rods activity suggested below for question **4** will also help the students with question **2a**, in which the fractions can be progressively marked on the timeline, beginning with the halfway or 40 minute mark. This can be followed by inserting the $^1/_4$ or 20 minute mark midway between the start and half-time and the $^3/_4$ or 60 minute mark midway between half-time and the finish. Now the $^1/_8$ (10 minute) and $^5/_8$ (50 minute) marks can be inserted midway between the start and $^1/_4$ time and the half-time and $^3/_4$ time respectively. The $^1/_5$ or 16 minute mark presents a slightly greater challenge, but it can be located just over midway between the $^1/_8$ (10 minute) and $^1/_4$ (20 minute) marks.

Having established that 10 minutes represents $^1/_8$, the students should not find it too hard to work out in question **2b** that, with 10 minutes to go, there is $^1/_8$ of the game remaining, which means that $^7/_8$ of the game has been played.

In question **2c**, the students should see at a glance that the try scored in the 10th minute occurred when $^1/_8$ of the game had been played. The try scored in the 38th minute would have occurred when just less than half the game had been played because the 40th minute was the halfway point. The try scored in the 62nd minute, on the other hand, would have occurred when just over $^3/_4$ of the game had been played (since the 60th minute was the $^3/_4$ point). Finally, the try scored in the 75th minute occurred midway between the $^7/_8$ point and full-time, so that must have been when $^{15}/_{16}$ of the game had been played. If, however, the students look at their timeline and estimate that this was after approximately $^9/_{10}$ of the game has been played, this is an acceptable alternative.

In question **3**, if Philip runs consistently and covers 5 kilometres in 30 minutes, this means that he takes 6 minutes to run each kilometre. Consequently, the timeline for his run will be divided into fifths, with the marks indicating 1 kilometre (6 minutes), 2 kilometres (12 minutes), 3 kilometres (18 minutes), 4 kilometres (24 minutes), and 5 kilometres (30 minutes).

Question **4** is similar to the task in question **2** of working out the midway point between $^7/_8$ and 1. In this case, the students may be able to work out that the clue to solving the problem is to convert the $^1/_5$ into smaller fractions. Their earlier work with decimals may suggest to them that fifths can be changed into tenths without too much difficulty. This can actually be done very effectively in a hands-on fashion with Cuisenaire™ rods (which are really a solid number line). Let the orange rod represent 1. Then the red rods represent fifths (because 5 of them fit exactly along 1 orange rod) and the white rods represent tenths (because 10 of them fit exactly along 1 orange rod). The students can then observe that there are 2 tenths for every fifth. From this, they should be able to work out that halfway between 0 and $^1/_5$ on the number line is $^1/_{10}$.

Pages 18–19: Water World

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- find a given fraction or percentage of a quantity (Number, level 4)
- make sensible estimates and check the reasonableness of answers (Number, levels 3–4)
- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

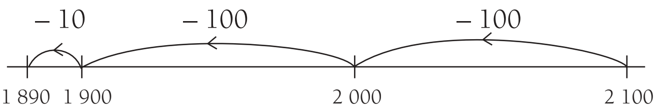
Activity One

This activity focuses on percentages in the context of water. Students will need to have strong multiplicative strategies to answer the questions in this activity, which is suitable for students at the advanced multiplicative stage or beyond of the Number Framework. Make sure your students understand that percent (%) means out of 100, with 100 as the whole. So 70% is $^{70}/_{100}$.

Students are often surprised to learn that water makes up a considerable percentage of the human body and the bodies of other mammals and of fruit and vegetables. They may not realise, either, just how much water or moisture we lose from our body in a day. Some long-distance athletes find out the hard way (through experiencing the debilitating effects of dehydration) that it is vital to continuously replace water lost through activity. Water World is an excellent activity for highlighting the importance of water in our lives.

Activity One provides the information that 1 litre of water weighs 1 kilogram. This is true of pure fresh water (although not of salty sea water), so it is relevant to the water in our bodies. Tamahou suggests that 70% be calculated by first finding 10% ($^1/_{10}$) and then multiplying this amount by 7. This is a good strategy, and although it will involve decimals in most cases, it shouldn't be too difficult. For example, in Jessica's case, $^1/_{10}$ of her mass is 3.5 kilograms, so $^7/_{10}$ or 70% will be 3.5 x 7. Using a part–whole strategy, the students can work this out mentally: 7 x 3 = 21 and 7 x 0.5 = 3.5, so 70% is 24.5 kilograms. The students will need to use their knowledge of equivalence between fractions, decimals, and percentages, for example, 10% = $^1/_{10}$.

The students will need to use the data from question **1** to find the amount of water each student needs to replace in question **2**. Tamahou, Jessica, and David each need to replace 9% of their water content, and Rangi and Kate need to replace 8%. Thus, the task becomes one of calculating 9% of 21 litres, 8% of 28 litres, and so on. Encourage the students to use number sense and mental strategies where possible. For example, 9% is 1% less than 10%. Tamahou's water content is 21 litres, 10% of that is 2.1 litres, and 1% is 0.21 litres. (The students' understanding of place value is important here.) So the problem becomes 2.1 – 0.21, which the students would be able do more easily if they work in millilitres and use a number line:



1 890 millilitres is 1.89 litres. All the problems in this question can be worked out in the same way, with 8% being seen as 10% – 1% – 1%.

In question **3**, the easiest strategy for the students to use to work out the land area is to recognise that it comprises 28% of the Earth's 500 million square kilometre surface area. Finding 28% of a large number like 500 000 000 need not be daunting if the students recognise that they only need to find 28% of 500 and call their answer millions. Encourage them to look for a number strategy rather than use a calculator. For example, using an earlier strategy, they could find 10% of 500, which is 50, and multiply that by 2 to get 20%. To find the 8%, they could use the same method suggested for question **3**, which in this case would be 50 – 5 – 5 = 40. So 28% of 500 million is 50 + 50 + 40 = 140 million.

Activity Two

Question **1** in this activity gives various percentages of water used in parts of the household. In question **1a**, the students are asked to estimate the percentages of 400 litres. To do this, they could think of the percentages as fractions or approximate fractions. Thus, 10% used in the kitchen is $^{1}/_{10}$, and $^{1}/_{10}$ of 400 is 40 litres. The 19% used outside is about 20% and so is the 21% used in the laundry, so the easiest calculation is to double the 10% calculated for the kitchen. The 24% used in the bathroom and the 26% used in the toilet are each about 25% or $^{1}/_{4}$, which is approximately 100 litres.

In question **1b**, the 43 litres is 10% (or $^{1}/_{10}$) of the total, so it follows that the total will be 10 times 43, which is 430 litres. In question **1c**, the bathroom and toilet combined account for 50% of water usage (24% + 26%), so the students need to realise that the figure of 186 litres has to be doubled (200 + 160 + 12 = 372 litres) to find the total household usage per day.

Question **2** requires the students to refer back to the average household of question **1a** and reduce the 400 litres by 20%, that is, to take off $^{1}/_{5}$ (80 litres). Alternatively, they could find 80% of 400. This can be done mentally by either subtracting 80 litres (20% of 400) from 400 litres or multiplying 80 litres by 4 (as the reduced amount will be $^{4}/_{5}$ of the original 400 litres). Either way, the result is 320 litres. To tackle question **2b**, the students can use the same estimation strategies they used in question **1a**.

Question **3** is useful for making sure that the students understand what percentages are and how to calculate them. The 10 litres is the whole (100%), so the students need to find what percentage 100 millilitres is of 10 litres. If they understood that the 70% in **Activity One** was $^{70}/_{100}$, they can transfer this to $^{100\text{ mL}}/_{10\text{ L}}$. However, to work this out, they first need to either convert the 100 millilitres to litres or the 10 litres to millilitres. In the first case, the 100 millilitres is 0.1 litres, so the problem becomes $^{0.1}/_{10} = {}^{\square}/_{100}$, which is 1%. In the second case, the 10 litres converts to 10 000 millilitres, so the problem becomes $^{100}/_{10\,000} = {}^{\square}/_{100}$, which again is 1%. This means that there is a 99% saving of water during teeth brushing if you take the wet-your-brush-and-rinse-quickly option.

Pages 20–21: The Volcanoes Erupt

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3–4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, levels 3–4)
- explain the meaning of negative numbers (Number, level 4)

Activity

Although negative numbers are introduced at level 4 of *Mathematics in the New Zealand Curriculum*, they are readily understood by students at the advanced additive stage or beyond of the Number Framework.

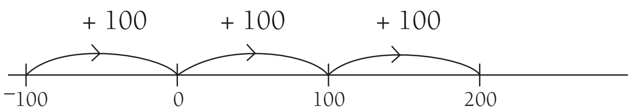
Volcanoes provide a meaningful context in New Zealand for mathematical investigations. The eruption times listed in this activity also serve to remind us that, in geological time, some of these eruptions have occurred very recently and, realistically, there is every likelihood that further eruptions will occur.

Before the students begin this activity, you may need to discuss with them the abbreviations BC and AD. (BC means before Christ and AD is the abbreviation of the Latin phrase *Anno Domini*, which means in the year of the Lord. Another term, used most commonly by non-Christians, is BCE, which means before the common era, indicating dates before the Christian era.)

The students may be interested to find out that there is, in fact, no year 0. Although Amrit uses 0 to divide BC and AD on his number line as a way of simplifying the mathematics involved, the sequence is actually ... 2 BC, 1 BC, 1 AD, 2 AD ... There is no zero in Roman numerals, which were used in the sixth century when the system we use to count years was invented.

The students also need to understand that years BC are effectively *negative* numbers. Thus, to find the difference between 100 BC and 200 AD, it is necessary to work out the difference between $^{-}100$ and $^{+}200$, which is 300 years.

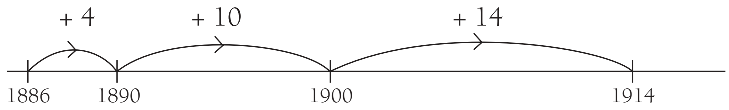
A number line is a useful model to use for working out and understanding the difference between years BC and years AD or between negative and positive integers. On a number line, the difference between 100 BC and 200 AD would be:



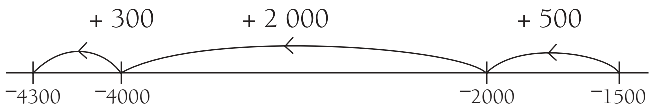
A good context for helping the students to understand positive and negative numbers is owing money. Being in debt means having a negative amount of money, regardless of whether it is a $100,000 mortgage on a house or borrowing $5 from an older brother or sister. A student who borrows $5 and spends it has to earn $5 before they get back to a nil balance. (In a mortgage, of course, there is also the issue of interest charged.)

One approach to the questions in this activity would be to draw on a chart or the whiteboard a number line from $^{-}10$ to 10. The students could use the number line to explore ways of finding the difference between a number on the positive side and a number on the negative side. Eventually, they may be able to see that finding the difference requires adding the two amounts. For example, the difference between $^{-}5$ and 4 requires adding 5 to get from $^{-}5$ to 0 and then another 4 to get to 4, that is, a total of 9. Mathematically, they may come to realise that $4 - {}^{-}5 = 4 + 5$, which is 9. Ask "What is the difference between 4 and $^{-}5$?"

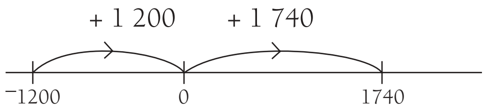
To calculate the answers to the various questions in this activity, the students need to employ a range of strategies. Questions **1** and **2** both refer to volcanoes that have erupted since the year 0. The students could solve this using a missing addends strategy (1886 + □ = 1914), which can be easily solved using an empty number line:



The answers to questions **3** and **4** can be worked out in similar ways. For example, the Rotokawau eruption was in 1500 BC and the Tūhua eruption was in 4300 BC so, counting back from 1500 BC, 1 500 plus 500 years takes it to 2000 BC and another 2 300 years takes it to 4300 BC, a total of 2 800 years. This could be shown on an empty number line like this:



The trickier calculations are the ones that cross the year 0. In these cases, the strategies described earlier can still be used. For example, in question **5a**, the first Taranaki eruption was in 1200 BC and the second in 1740 AD, so the students might say that it takes 1 200 years to reach the year 0 and another 1 740 to reach 1740 AD, a total of 2 940 years. This could be shown on an empty number line as:



To answer questions **7a** and **c**, encourage the students to think about what they will have to subtract from the years given in order to arrive at year 0. They may decide that subtracting 2 000 years would be about right, and this is quite reasonable. The next step, subtracting 2 000 from 10 700, would result in 8 700, which is approximately the time of the Maungarei eruption. Using approximation is perfectly valid in these circumstances because the eruption dates themselves are best approximations based on geological evidence. In the case of question **7b**, the students can simply subtract 1 800 from 2 000. This results in the year 200 AD, which is roughly the time of the last Taupō eruption.

Investigation

The students will need to do some book or online research to find the answers in this investigation.
A suitable book resource is:
Hicks, G. and Campbell, H., eds (1998). *Awesome Forces: The Natural Hazards That Threaten New Zealand.* Wellington: Te Papa Press.
A website that provides relevant information on New Zealand volcanoes is:
www.gns.cri.nz/what/earthact/volcanoes/nzvolcanoes

Pages 22–23: Chilly Heights

Achievement Objectives

- explain the meaning of negative numbers (Number, level 4)
- report the results of mathematical explorations concisely and coherently (Mathematical Processes, communicating mathematical ideas, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 3 and 4)

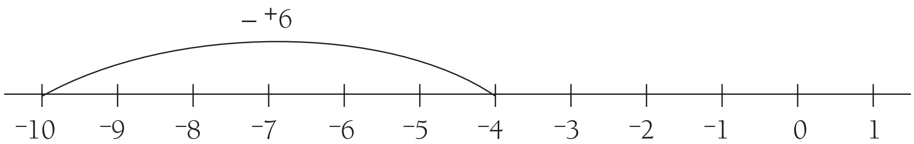
Activity

Like the activity on the previous page, this activity involves positive and negative numbers. The addition and subtraction of integers is very meaningful for students at the advanced additive stage or beyond of the Number Framework.

Temperature variations in relation to height and wind chill are certainly factors that need to be taken seriously when out tramping or engaging in other outdoor pursuits. Search and Rescue services regularly report finding people suffering from hypothermia in the outdoors, often as a result of being ill-equipped for the conditions. The activities in Chilly Heights are therefore very relevant.

Like the years BC in the previous activity, temperatures below 0°C are effectively negative numbers. However, some of the questions in Chilly Heights require two or more calculations involving negative numbers.

In question **1**, the students will have to calculate not only Jonathan and Penny's increase in altitude from Plateau Hut to the top of the Linda Glacier (3 100 – 2 200 = 900 metres) but also the drop in the temperature felt, assuming that the temperature decreases 1°C for every 150 metres in altitude. The first step is to determine how many lots of 150 metres there are in 900 metres. Encourage the students to look for a strategy to work this out. For example, 150 x 2 = 300, and there are 3 lots of 300 in 900 (because 3 x 3 = 9), so there must be 3 x 2 = 6 lots of 150 in 900. Therefore, 6° must be subtracted from $^{-}4$°C to allow for the further drop in temperature ($^{-}4° - {}^{+}6° = {}^{-}10°$). If the students are still unsure how to subtract a positive number from a negative number, you could show them this on a number line:



Question **2** is more straightforward. The sheltered temperature of $^{-}7$°C is already given. All the students need to do is to read off the temperature on the wind chill temperature chart. Questions **3** and **6** can be approached in the same way.

Question **5** is not too dissimilar. The wind chill temperature with the 80 kilometres per hour wind, given a temperature of 4°C in the sheltered spot, is $^{-}4$°C, a difference of 8°C. Again, a number line is a useful tool.

To calculate the temperature at Mt Cook Village (question **4**), the students will find it necessary to:

i. work out the difference in altitude between the village (700 metres) and the summit of Aoraki (3 754 metres);
ii. work out how many lots of 150 are in their answer to **i** (to find how many lots of 1°C the temperature is increasing with the drop in altitude);
iii. add this increase in temperature to the $^{-}4$°C registered on the thermometer at the summit.

To answer each part, the students can use the strategies suggested for earlier questions in this activity.

Possible extensions of Chilly Heights would be for the students to investigate:

- the temperature at the heights that different aeroplanes and gliders typically fly
- the difference in temperatures on the wind chill temperature chart from 7°C through to ⁻10°C . For example, at the 7°C mark, the perceived temperature drops by 7°C as the wind increases from 0 to 80 kilometres per hour, whereas at the ⁻10°C, the temperature drops twice this amount to register ⁻24°C.

A handy website relating to height, altitude, and temperature is:
http://earthsci.terc.edu/content/investigations/es1702/es1702page05.cfm

This website includes a graph of decreasing temperature with increasing altitude based on data collected from a balloon ascent in Wyoming on 20 August 2001. The graph shows a steady decrease in temperature from about 2 000 metres up to an altitude of about 12 500 metres. It also mentions that commercial jet aeroplanes typically fly at an altitude of about 12 000 metres, at which height the graph shows a temperature of approximately ⁻60°C. Within New Zealand, jet aircraft reach an altitude of about 9 000 metres, while turbo-prop aircraft typically fly at about 5 000 metres unless they are only flying short distances, in which case they tend to reach a height of about 2 000 to 2 500 metres.

Further teaching ideas for negative numbers can be found in the numeracy project materials section of the NZMaths website (www.nzmaths.co.nz/numeracy/materialmasters.htm).

Page 24: Hidden Help

Achievement Objectives

- express a fraction as a decimal, and vice versa (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)

Activity

Students who are progressing to the advanced multiplicative stage of the Number Framework are expected to convert between fractions, decimals, and percentages. Students need to use multiplicative strategies when they are naming equivalent fractions.

The questions in this activity are designed to help the students gain and use an understanding of the relationships between fractions, decimals, and percentages. To complete the table in question **1**, it is necessary to know, or work out, how fractions can be converted to decimals (and vice versa), how fractions can be converted to percentages (and vice versa), and how decimals can be converted to percentages (and vice versa). A summary of how these conversions can be done is set out in a chart in the Answers for question **2** and illustrates how students might respond to this question.

Further possible explanations regarding the relationships between fractions, decimals, and percentages (question **2**) include:

- percentages are fractions of 100 that can often be simplified (for example, 75% is $^{75}/_{100}$, which can be simplified to $^{3}/_{4}$ if it is divided by 1 in the form of $^{25}/_{25}$);
- decimals are also fractions: those in the first column to the right of the decimal point are tenths, those in the second column are hundredths, and so on. For example, 0.4 is really $^{4}/_{10}$, and 0.75 is really $^{75}/_{100}$.

Note that changing a repeating decimal into a fraction requires more understanding of place value, equation solving, and fraction manipulation than we expect students at this level to have.

Ideas for the physical modelling of fractions, decimals, and percentages can be found in *Book 7: Teaching Fractions, Decimals, and Percentages* (Numeracy series).

Copymaster: Place the Digits

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Ones	•	Tenths	Hundredths	Thousandths
	•			
	•			
	•			
	•			
	•			

Copymaster: Riding the Waves

Judge: Wayne						Total (best 3 waves)
Anaru	7.2	3.4	8.4	6.6	5.3	
Eli	3.6	7.4	6.1	7.5		
Josh	4.3	7.1	5.5	5.4	6.3	
Max	5.2	6.8	6.5	4.5		

Judge: Wayne						Total (best 3 waves)
Kelly	4.2	5.6	4.7	4.9		16.9
Sue	5.9		5.7	6.1	8.0	21.5
Margot	6.5	7.2	4.8	8.2		22.1
Janice	2.7	4.2		3.8	4.2	12.5

	Judges' mean scores							
Surfer	Wayne	Jared	Rachel	Ben	Ella	**Total**	**Mean**	**Place**
Kiri	6.2	6.0	6.5	6.3	6.7	31.7	6.34	
Sue	7.4	7.8	7.1	8.3			7.58	
Zoe	5.3	5.9	6.3	6.7	6.4			
Hine	5.9	7.5	6.1		4.9	31.5		
Eseta	5.8	6.1	5.9	6.2	5.4			
Trish	7.5	7.6	8.1	8.4	8.3			

Copymaster: Feel the Beat

Animal	Heartbeats per minute	Heartbeats per hour	Heartbeats per day	Heartbeats per year
Hummingbird	1 000			
Mouse	500			
Human baby	120			
Elephant	28			
Blue whale	5			

Copymaster: Hidden Help

Fraction	Decimal	Percentage
	0.2	20%
$\frac{1}{3}$	$0.\dot{3}$	
		25%
$\frac{2}{3}$		
		60%
$\frac{1}{8}$		12.5%
$\frac{3}{8}$	0.375	
	$0.1\dot{6}$	16.7%
$\frac{1}{2}$		
		75%
$\frac{1}{10}$		
$\frac{4}{10}$		40%

Fraction	Decimal	Percentage
	0.2	20%
$\frac{1}{3}$	$0.\dot{3}$	
		25%
$\frac{2}{3}$		
		60%
$\frac{1}{8}$		12.5%
$\frac{3}{8}$	0.375	
	$0.1\dot{6}$	16.7%
$\frac{1}{2}$		
		75%
$\frac{1}{10}$		
$\frac{4}{10}$		40%

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