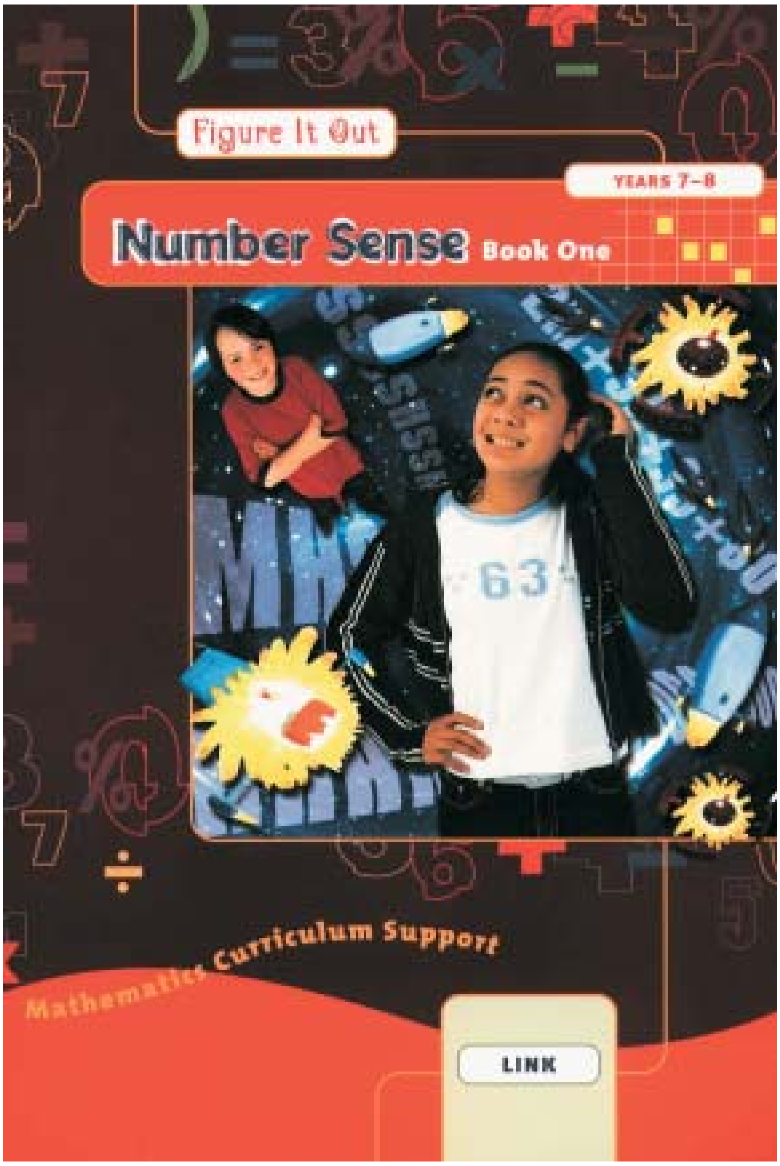
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:

Number (two linking, three level 4, one level 4+)
Algebra (one linking, two level 4, one level 4+)
Measurement (one level 4, one level 4+)
Number Sense (one linking, one level 4)
Geometry (one level 4, one level 4+)
Statistics (one level 4, one level 4+)
Themes (level 4): *Disasters Strike!, Getting Around*

These 20 books will be distributed to schools with year 7–8 students over a period of two years, starting with the six *Number* books that were distributed in October 2002.

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect the ethnic and cultural diversity and the life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/community

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Number Sense: Book One

Page 1 Choice Calculations

ACTIVITY ONE

1. a.–b. Answers will vary. Possible answers include:
   - i. 3 x 6 – 2 x 9 = 0, 3 – 6 x 2 + 9 = 0, or 3 x 6 ÷ 2 – 9 = 0
   - ii. 4 x 6 – 3 x 6 = 6
   - iii. 2 x 3 – 3 + 2 = 5, 2 + 3 ÷ 3 + 2 = 5, or 2 – 3 + 3 x 2 = 5
   - iv. 8 ÷ 4 ÷ 2 ÷ 1 = 1, 8 – 4 – 2 – 1 = 1, or 8 ÷ 4 – 2 + 1 = 1
2. Problems will vary.

ACTIVITY TWO

1. Solutions may vary depending on the type of calculator and the approach used. Possible calculations are:
   - a. 34 + 34 + 34 + 34 + 34 + 34 or 34 + 34 = = = = = (or on some calculators, 34 + + 34 = = = = =)
   - b. 444 – 333 ÷ 3 = or 333 ÷ 3 ÷ 3 =
   - c. 33 – 4 + 33 – 4 + 33 – 4 + 33 – 4 =
2. Expressions will vary.

Pages 2–3 Splitting Numbers

ACTIVITY ONE

1. a. 15 + 1, 14 + 2, 13 + 3, 12 + 4, 11 + 5, 10 + 6, 9 + 7, 8 + 8
   b. 8 and 8 (because 8 x 8 = 64)
2. a. 5 + 5 (because 5 x 5 = 25)
   b. 6 + 6 (because 6 x 6 = 36)
   c. 9 + 9 (because 9 x 9 = 81)
   d. 6 + 7 (because 6 x 7 = 42)

ACTIVITY TWO

1. 3 + 3 + 3 because 3 x 3 x 3 = 27. (The other sets are: 1 + 1 + 7, 1 + 2 + 6, 1 + 3 + 5, 1 + 4 + 4, 2 + 2 + 5, 2 + 3 + 4.)
2. a. 4 + 4 + 4 (because 4 x 4 x 4 = 64)
   b. 5 + 5 + 5 (because 5 x 5 x 5 = 125)
   c. 6 + 6 + 6 (because 6 x 6 x 6 = 216)
   d. 6 + 7 + 7 (because 6 x 7 x 7 = 294)
3. Yes. You get the greatest product when each set of two or three numbers consists of whole numbers that are the same or nearly the same. (If you were using fractions instead of whole numbers only, with 13 you would get $6\frac{1}{2}$ x $6\frac{1}{2}$ = $42\frac{1}{4}$ as the greatest product instead of 6 x 7 = 42, and with 20, you would get $6\frac{2}{3}$ x $6\frac{2}{3}$ x $6\frac{2}{3}$ = 296.296, instead of 6 x 7 x 7 = 294.)

Pages 4–5 Aiming High

GAME

A game using place value and addition

ACTIVITY ONE

1. a. Strategies will vary. One possible strategy is to use large numbers first (hundreds and tens) until your total is in the nine hundreds and then use tens and ones only.
   b.–2. Strategies will vary.

ACTIVITY TWO

1. Leila wins because she is closer to 1 000 than Tama. (Tama is 22 less than 1 000, and Leila is 21 more.)
2. If Mere puts her last throw in the correct column, she has a 6 out of 6 chance of winning the game. (To win, she needs to put 1 or 2 in the tens column or 3, 4, 5, or 6 in the ones column.)
3. a. Strategies will vary.
   b. It is possible to get exactly 500, but only if players get exactly the numbers they need at the end.

Page 6 Number Scavenge

ACTIVITY

1.–2. Practical activity

3. Results will vary.

Page 7 Hit the Target

GAME

A game using whole-number operations

Page 8 Pathways

ACTIVITY

1. Possible pathways include:

a. 20 x 5 ÷ 10 x 15 + 36 + 45 + 15

b. 20 – 15 + 25 ÷ 6 x 9 + 45 + 15

c. 20 x 5 – 70 + 36 ÷ 11 – 6

d. 20 ÷ 4 x 7 + 3 ÷ 2 – 6

e. 20 ÷ 4 x 15 + 36 + 45 + 15

f. 20 – 15 x 3 x 7 + 3 ÷ 2 – 6

g. 20 x 5 ÷ 10 x 7 + 11 + 45 + 15

(Note that the order of operations does not apply in this step-by-step process.)

2. Totals and pathways will vary.

3. Pathway diagrams will vary.

Page 9 Flying Feet

ACTIVITY

1. a. Eru: 12 strides, Matiu: 16 strides,
Api: 24 strides, Myra: 48 strides

b. Eru: 48 ÷ 4 = 12, 4 x 12 = 48
Matiu: 48 ÷ 3 = 16, 3 x 16 = 48
Api: 48 ÷ 2 = 24, 2 x 24 = 48
Myra: 48 ÷ 1 = 48, 1 x 48 = 48

2. a. Api

b. Matiu

3. Matiu: step 12, Api: step 24, Myra: step 36

Page 10 Space Marauders

ACTIVITY

1. a. 10S + 3M + 3H
= (10 x 10) + (3 x 100) + (3 x –20)
= 100 + 300 – 60
= 340

b. 14S + 4M + 2H
= (14 x 10) + (4 x 100) + (2 x –20)
= 140 + 400 – 40
= 500

c. 18S + 4M + H
= (18 x 10) + (4 x 100) + (1 x –20)
= 180 + 400 – 20
= 560

2. Records will vary. For example: SSSSMSSMSSSMHSSMSSSSO (15S + 4M + H); score: 530. Another way to get this score is an arrangement that includes 21S + 4M + 4H.

Page 11 Double and Halve

ACTIVITY ONE

1. a. 2 000
1 000
500
250
125
62.5

b. 2 500
1 250
625
312.5

c. 32
16
8
4
2
1
0.5

2. Numbers will vary. They are all odd numbers.

3. Numbers will vary. They all end up as an odd number after the second halving.

ACTIVITY TWO

1. 10 times. (On the tenth doubling, he got 1 024 000.)

2. 9 days (counting the $40.00 as 1 day)

Page 12 Find the Number

ACTIVITY

1. Set 1: 162
   Set 2: 175
   Set 3: 100
   Set 4: 66
   Set 5: 77
   Set 6: 144
2. Clues will vary.

Page 13 Ways to 43

ACTIVITY

1. Some possibilities are:
   8 x 3 + 9 + 8 + 2 = 43
   (8 + 2) x 6 – 9 – 8 = 43
   8 x 7 + 4 – 6 x 5 + 2 x 3 + 4 + 1 + 2 = 43
   (8 + 3) x 6 – 9 – 8 – 2 x 3 x 1 = 43
   (8 + 7) x 2 + 9 + 1 + 2 + 1 = 43
2. Many equations are possible. Some possibilities are:
   9 x 5 + 3 – 4 – 1 = 43
   (9 + 1) x 5 – 3 x 2 – 1 = 43
   9 x 2 + 3 x 6 + 1 + 3 + 2 + 1 = 43

Page 14 Writing 1 000

ACTIVITY

1. **a.** Results will vary. For example: 510 + 490, 156 + 844, and so on.
   b. Results will vary. For example: 100 + 300 + 600, 236 + 552 + 212, and so on.
2. 1 000 x 1 or 1 x 1 000
   500 x 2 or 2 x 500
   250 x 4 or 4 x 250
   200 x 5 or 5 x 200
   125 x 8 or 8 x 125
   100 x 10 or 10 x 100
   50 x 20 or 20 x 50
   25 x 40 or 40 x 25

   Discussion will vary. You will know if you have found all the possibilities when you have used all the factors of 1 000. (For example, 3, 6, 7, and 9 do not divide evenly into 1 000.)
3. **a.** Answers will vary.
   For example: 700 + 500 – 200 = 1 000
   2 100 – 1 500 + 400 = 1 000
   1 600 ÷ 2 + 200 = 1 000
   1 200 ÷ 2 + 400 = 1 000.
   b. Answers will vary. For example,
   700 + 500 – 200 = 1 000 could be expanded
   to (70 x 10) + (50 x 10) – (20 x 10) = 1 000
   (140 x 5) + (100 x 5) – (40 x 5) = 1 000.

Page 15 Up the Ladder

GAME

A game ordering numbers between 100 and 1 000

ACTIVITY

Vanna. She has a 50% chance of winning. (There are two cards left, a 0 and a 5, and she needs to draw the 5.) Logan has no chance of winning in this round because he cannot use the 1 or the 5.

Page 16 Playzone Discount

GAME

A game estimating percentages

Page 17 Different Approaches

ACTIVITY

1. **a.** Discussion will vary. See comments in **b** below.
   b. **i.** In the head is easy: 50 + 50 = 100, so 49 + 49 = 98.
   ii. You can do this in your head using standard place value: 900 + 30 + 4 = 934.
   iii. Easy on a calculator, but it can also be done easily in your head:
   63 – 40 = 23, so 63 – 37 = 23 + 3
   = 26
   or
   63 – 37 = ☐ is the same as 37 + ☐ = 63;
   37 + 3 + 23 = 63
   so 37 + 26 = 63
   and 63 – 37 = 26.
   iv. 704 – 300 = 404, so 704 – 299 = 405

v. 9 + 9 + 9 + 9 + 9 = 5 x 9
= 45
or 10 + 10 +10 +10 +10 = 50,
so 9 + 9 + 9 + 9 + 9 = 50 – 5
= 45

vi. 26 + 24 = 50
so 26 + 26 + 24 + 24 = 100

vii. 98 – 43 = 55 (standard place value)
8 – 3 = 5
90 – 40 = 50
50 + 5 = 55

viii. 423 – 286 is probably easier on a calculator, but it can be done in the head:
286 + 14 = 300
300 + 123 = 423
123 + 14 = 137

ix. 68 + 32 = 100 (8 + 2 = 10, 60 + 30 = 90, 10 + 90 = 100)

x. 498 is 2 fewer than 500.
500 + 302 = 802
so 498 + 302 = 800
or 498 + 302 = 500 + 300
= 800 (compensation)

2. Problems will vary.

Page 18 It Pays to Win!

ACTIVITY

1. a. $104,600
   b. $3,486.67
2. a. $112,000
   b. $7,400
3. a. $109,700
   b. $5,100 better off

Page 19 Drink Stall

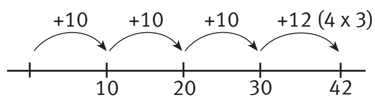
ACTIVITY

1. a. 2 L
   b. 3 L
2. a. 8 sachets. (A 4:5 ratio means 8 sachets for 10 litres.)
   b. 12 sachets. (15 litres is 3 times 5 litres, so she needs three times as much water. 3 x 5 = 15 litres, 3 x 4 = 12 sachets)
   c. 20 sachets. (5 x 5 = 25 litres, 5 x 4 = 20 sachets)
3. Cordial. (Jenny would spend $7.00, and Toline would spend $8.00.)
4. a. i. 3 L
      ii. 9 L
   b. i. 6 cups of sugar
      ii. $1\frac{1}{2}$ cups of sugar
   c. 50 lemons and 10 cups of sugar
5. a. 60 cups
   b. 30 L
6. Jenny gets $40.00, and Mere and Toline get $20.00 each.

Page 20 Grocery Grapplers

ACTIVITY

1. 42. Two possible methods are:
   i. 14 x 3 = 7 x 6
   = 42
   ii. 10 x 3 = 30, 4 x 3 = 12
   30 + 12 = 42
   A possible number line for this method is:



2. 13. Two possible methods are:
   i. 6 x 12 = 72, 72 + 6 = 78. That's 13.
   ii. 60 ÷ 6 = 10, 18 ÷ 6 = 3
   10 + 3 = 13
3. 81. Two possible methods are:
   i. 18 ÷ 2 = 9 lots of 2 kilograms (9 bananas)
   9 x 9 = 81
   ii. $4\frac{1}{2}$ bananas per kilogram
   18 x $4\frac{1}{2}$ = (18 x 4) + 9
   = 72 + 9
   = 81
4. $14.00. Two possible methods are:
   i. 10 x 4 = 40
   4 x $3.50 = 2 x $7.00
   = $14.00

ii. 10 x 4 = 40
4 x $3 = $12
4 x 50c = $2
= $14.00

5. 100. Two possible methods are:

i. 25 x 4 = (20 x 4) + (5 x 4)
= 80 + 20
= 100

ii. 4 lots of 25 = 100

6. Two possible methods are:

i. 78 ÷ 12 = 6 r6, so 7 bags are needed.

ii. 60 ÷ 12 = 5, 18 ÷ 12 = $1\frac{1}{2}$ (so they need 2 bags here); 5 + 2 = 7

Page 21 Birthday Treat

ACTIVITY

1. $27.50
2. $5.50
3. a. Earth meal: $5.00; mega moon meal: $5.30; super saturn meal: $5.90

   b. Super saturn meal, no upgrade

4. Answers will vary. Two possible answers are:

earth meal upgrade	$5.00
mega moon meal	$4.80
mega moon meal upgrade	$5.30
super saturn meal	$5.40
super saturn meal upgrade	$5.90
Total:	$26.40
Change:	$1.10
or	
earth meal upgrade	$5.00
two earth burgers, large liquid fuel	$5.00
mega moon meal upgrade	$5.30
super saturn meal	$5.40
two super saturn burgers and one medium chip booster	$6.80
Total:	$27.50
Change:	0.00

Page 22 Shopping Around

ACTIVITY

Methods will vary. Answers and some possible methods are:

1. $15.00. $19.95 rounds to $20.00.
$\frac{1}{4}$ of $20.00 is $5.00.
$20.00 – $5.00 = $15.00

2. $14.75. $2.95 rounds to $3.00. 5 x $3.00 = $15.00.
$15.00 – (5 x 5c) = $14.75

3. $119.60. There are two 5s in 10, so two CDs are free. 8 x $14.95 rounds to 8 x $15.00.
(8 x 10) + (8 x 5) = 80 + 40
= $120
or 8 x 15 = 4 x 30
= 120
$120.00 – 0.40 = $119.60

4. $18.50. 5 x $1.30 = $5.00 + (5 x 0.30)
= $6.50
2 x $1.80 = (2 x $2.00) – (2 x 0.20)
= $4.00 – $0.40
= $3.60
4 x $1.50 = 2 (2 x $1.50)
= 2 x $3.00
= $6.00
2 x $1.20 = $2.40
$6.50 + $3.60 + $6.00 + $2.40 = $18.50

5. $33. 6 x $5.50 = (6 x $5.00) + (6 x 0.50)
= $30.00 + $3.00
= $33.00

6. $17.55. Bananas: $1.45 rounds to $1.50.
6 x $1.50 = $9.00.

   Apples are almost twice the price but half the quantity, so 3 x $2.95 is also about $9.
$9 + $9 = $18.

   As a single purchase, this would be $17.55, with 30 cents off the bananas and approximately 15 cents off the apples.

Page 23 Keep Your Shirt On

ACTIVITY

1. Methods will vary. You could calculate your answer like this:
   16 x 25 = 4 x 100
   = 400
   so
   15.95 x 25 = 400 – (25 x 0.05)
   = 400 – 1.25
   = 398.75.
2. Answers may vary. Possible methods include:
   - **a.** Take the answer to question **1** and add 25 x 4 = $100 to make up for the price increase.
     398.75 + 100 = $498.75
   - **b.** Multiply the answer to question **1** by 3:
     398.75 x 3 = $1,196.25
   - **c.** Take 2 x $15.95 from the answer to question **1**:
     398.75 – 31.90 = $366.85
   - **d.** Take 25 x 8 = $200 from the price in question **1**:
     398.75 – 200 = $198.75
3. **a.** The exact answer is $598.13.
   (25 x 15.95 = 398.75. Half of $398.75 is $199.38. 398.75 + 199.38 = $598.13)
   - **b.** Approximately $12.00. (The exact answer is $11.96.) $598.13 is nearly $600.
     600 ÷ 50 = 12

Page 24 Division Dilemmas

ACTIVITY

1. Display methods will vary.
2. Use of strategies will vary. The solutions are:
   - **a.** 31 packets
   - **b.** 19 weeks
   - **c.** 37 dozen (A dozen is 12.)
   - **d.** 24 CDs
   - **e.** 26 teams, with three players left over

Teachers' Notes

Overview Number Sense: Book One

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Page 1

Choice Calculations

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY ONE

These problems are designed to encourage students to use flexible thinking with number operations to find a solution. The numbers involved have been kept small so that the students can focus on trying out different operations rather than being sidetracked by difficult calculations. The problems also reinforce the order of operations rules that apply to equations where multiplication or division is combined with addition or subtraction. There are no brackets or exponents used in this activity, so the BEDMAS rules are used only in a limited way. (The acronym BEDMAS signifies the order in which operations should be carried out in an equation: brackets, exponents, division and multiplication in the order that they occur, and then addition and subtraction in the order that they occur.) Draw the students' attention to the rule in question **1a**.

Present the problems as a challenge and look for opportunities to give feedback such as "Yes, Sam has found one that works!"

One approach is for the students to insert signs randomly into each equation and see what results they get. They can then use these results to give them clues for further attempts.

Encourage the students with prompts such as "This equation gave an answer that was too large, so what could you change to get closer to the target?"

One result for question **1a i** could be 3 – 6 x 2 + 9. For this answer, the students need to understand that it is possible to have a negative quantity. Students who have been taught ideas such as "You can't take 12 away from 3" will need to be shown that you can in fact do it using negative quantities.

In question **2**, insist that the students work out for themselves at least one result for each equation they make up. Otherwise they may produce some that are too difficult or tedious for their classmates to attempt.

ACTIVITY TWO

This is an example of a genre of problems known as "broken calculator problems". In these problems, the students need to rethink the numbers and operations used in a problem to give them an equivalent result. By doing this, they build on their understanding of the relationships between numbers and operations and develop their fluency with part-whole strategies.

Make sure that the students understand which keys can be used. It is also important for them to know the way basic calculators handle the order of operations. Basic calculators work out equations in the order in which the operations are entered. In terms of BEDMAS, you could say that basic calculators are programmed to automatically put brackets around numbers as they are entered. For example, if a student enters [33] [–] [4] [x] [4] [=] on a basic calculator, it will read this as (33 – 4) x 4 =. To encourage their awareness of this process when they are combining addition or subtraction with multiplication or division, it is good practice to have the students use brackets when recording on paper the sequence of keypad entries that they used on a basic calculator.

Scientific calculators have bracket keys so that the user can group numbers together anywhere in a sequence, as we do when we follow the order of operations rules with equations such as 33 – (4 x 4) = ☐. To solve this equation on a basic calculator, a student has two options: they could start by solving 4 x 4 and then entering [33] [–] [16] [=], or they could use the memory keys in the sequence [4] [x] [4] [M+] [33] [–] [MR] [=]. (On some calculators, this button is named MRC instead of MR.)

Pages 2–3 Splitting Numbers

Achievement Objectives

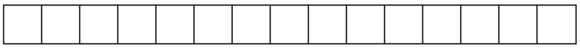
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY ONE

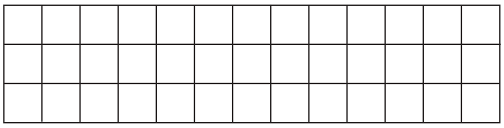
This is a step-by-step investigation that relates to an interesting connection with another area of mathematics, that is, the relationship between the dimensions of a rectangle and its area.

The two addends that a number is split into can be visualised as representing the length and width of a rectangle. By multiplying the addends together, we are finding the area of that rectangle. Show the students diagrams of the rectangles provided below to help them see how the area of a rectangle changes as the length and width change.

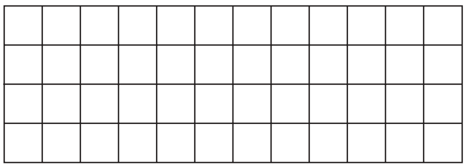
Some rectangles where the length + width equals 16 units are:



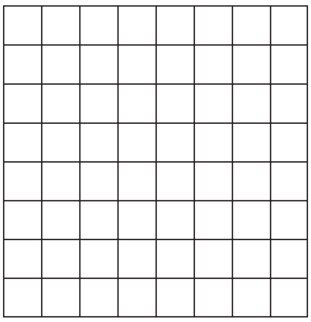
Length = 15 units; width = 1 unit; area = 15 square units.



Length = 13 units; width = 3 units; area = 39 square units.



Length = 12 units; width = 4 units; area = 48 square units.



Length = 8 units; width = 8 units; area = 64 square units.

Question **2d** gives an odd number, so the whole numbers that result in the greatest product have a difference of 1. Thirteen is split into 7 and 6 to give the greatest product, which is 42.

As an extension to this question, have the students split 13 into two equal addends using fractions. This would be 6.5 + 6.5. The product of these two numbers is 42.25.

From this, the students should conclude that a square rectangle always has a greater area than an oblong rectangle that has been made by splitting a number.

ACTIVITY TWO

This activity extends the previous investigation by looking at three-addend splits. This can be visualised as comparing the dimensions of a rectangular prism with its volume. Again, the students should find that the prism with sides that are equal in length (that is, a cube) yields the greatest volume.

Question **3** in this activity is the real goal of this investigation. Explain this to the students and show them how the investigation is structured to lead them to this question. Some students may wish to extend this still further to see if this pattern applies with four or more splits.

Students who have grasped the idea could be challenged to investigate multiplication using a calculator. "How can the keys [2], [3], [4], [5], and [x] be pressed once each time to make the largest product?" The factors that do this are 52 x 43 because these digits are the nearest to making the dimensions of a square.

Pages 4–5 Aiming High

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

GAME

Photocopy the game board copymaster provided at the back of this booklet to give each student a recording sheet for the game.

This game is a great way to strengthen place value understanding and to devise some problem-solving strategies involving addition.

Explain the rules to the students through a short demonstration. Emphasise the rule that does not allow them to change the place of the number once it has been entered as well as the rule that it is important to enter a number on every turn. These are vital rules to encourage them to develop a strategy other than trial and error.

After the students have played a few games, increase the pace at which the dice is thrown. The increase in frequency of the throws may force them to change their strategies because they are no longer being given sufficient time to do an exact addition as they go.

Discuss possible strategies, such as starting high by using the first few throws to a score of about 900 and then concentrating on placing the remaining throws in the tens and ones to get the last 100 points. Alternatively, they could try starting low for the first six throws in the ones column and then use the last four throws to try to get near the target.

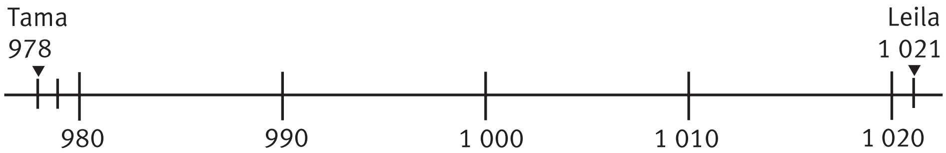
Have the students list the strategies they use and keep a tally of the winning ones to see which one gives the best results.

ACTIVITY TWO

Question **1** in this activity poses a problem that involves finding the score nearest to the target number by looking at differences. Have the students discuss strategies that are easy and efficient for them to use in finding the differences.

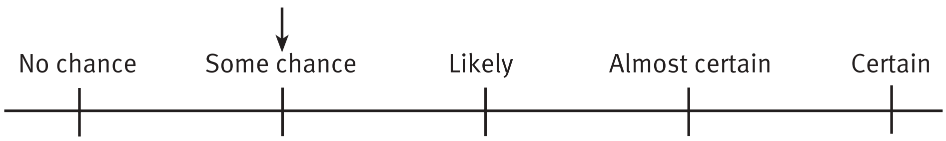
They may choose to do a counting-on strategy for Tama's score to build it up to 1 000. For example, 978 plus 2 makes 980 and plus 20 makes 1 000. So Tama is 22 away from 1 000. For Leila, they may just subtract 1 000 from 1 021 to find the difference of 21.

The use of a partial number line could help the students to see why Leila is closer to 1 000 than Tama. For example:



In question **2**, you may need to encourage the students to make a list of all the possible results for Mere. This would show that Mere could end with a total ranging from 987 to 992 if she put the dice score in the ones place. With 987 (a 1), she would lose, and with 988 (a 2), she would tie. But if she throws a 1 or a 2, she could put them into the tens place and make a total of 996 or 1 006.

Question **3b** encourages the students to discuss the chances of making exactly 500. They could use a continuum with four or five descriptors or a number line to 100 and place the exact score of 500 somewhere along the line and discuss their reasons for the placement. For example:



"I placed it here because you could get exactly 500, but you would be very lucky to throw the numbers you would need at the end."

The students could then keep a tally of 100 games played by the class and see how many times a score of 500 was achieved.

Extend the game by using two dice for each throw, with the students placing each of the two digits in any of the ones, tens, or hundreds places for every turn. This will encourage further strategies, such as rounding, to keep a running total.

Other ways to extend the challenge level include adding a thousands column and changing the target to 10 000 or adding a tenths column and keeping the target at 1 000.

Page 6 Number Scavenge

Achievement Objectives

- explain the meaning of the digits in any whole number (Number, level 3)
- explain the meaning of the digits in decimal numbers with up to 3 decimal places (Number, level 3)
- solve practical problems which involve whole numbers and decimals and which require a choice of one or more arithmetic operations (Number, level 3)
- express quantities as fractions or percentages of a whole (Number, level 4)

ACTIVITY

This activity is designed to give students an enjoyable investigation that will help them recognise numbers in everyday use by researching in newspapers and magazines.

Ensure that the students understand each of the number types needed. This is an opportunity to emphasise that a percentage is a fraction "out of a hundred".

It may be interesting to tally the percentages found to see the most common ones used in the newspapers and magazines.

Although there are three types of averages (median, mean, and mode), the usual one found in newspapers is the mean or average, such as the average temperatures for the month in the weather section or average scores in the sports section. For example, the weather page often lists data such as "mean annual rainfall" or "average sun for October". These may also be good examples of the use of rounding.

The students are asked to find a decimal that is not money because common usage reads a quantity such as $3.57 as "three dollars fifty-seven" instead of "three point five seven dollars", which would be the correct way of describing a decimal number. Dollars and cents are also often written without a decimal point in newspapers (for example, 50 cents, not 0.50 cents). An entry in the business pages may be 7.25% where the unit is one percent. An entry such as $2.85 million would be a good example of a decimal where the unit is one million dollars.

Finding a common fraction may be difficult, but sometimes car dealers advertise repayments such as $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ to show the amount that is charged each year.

Metric measures are used in the property, sports, food, car, and weather sections of newspapers. It may be an interesting challenge to find as many different metric unit references as possible from the same newspaper.

Words such as "about", "almost", or "approximately" would indicate that an estimate and/or a rounding has been made in a report.

Temperatures are listed in the weather section of a newspaper.

If the students cannot find a winning score of approximately 20 percent, have them find a sports team result and estimate the winning margin as a percentage.

Page 7

Hit the Target

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

GAME

This game is an ideal context to introduce the correct use of brackets to control the order of operations when forming equations that combine addition or subtraction with multiplication or division. The game also encourages flexible thinking with numbers and operations while applying number facts.

Demonstrate the game with two students while the others observe. Deal the cards face up and ask the students to come up with ways of getting close to the target number. Show them how to record each suggestion. Make a point of finding a way to make an equation using brackets. For example, if the cards were worth 3, 4, 1, and 5, the target of 10 could be made with $3 \times (4 + 1) - 5 = 10$.

Note that the rules say all four cards must be used. If some students need an easier version of the game, you could allow them to use just some of the card values.

Make the game more difficult by allowing a decimal point to be placed in front of a card value. This will encourage the students to start working with fractional amounts and to include the square root symbol as an operation to be used. For example, if the cards were 9, 5, 4, and 5, the target of 10 could be made with $\sqrt{9} + 5 + 0.5 \times 4 = 10$.

Control the time taken to play the game by adjusting the range of target numbers to suit the time available. Alternatively, reduce the set of cards given to each group to about 18 cards.

Page 8

Pathways

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity is a great way of encouraging the students to solve problems by combining number operations.

Before the students attempt to answer the questions, encourage them to explore the problem to get a feeling for what is happening and show them how they could record their attempts. For example, follow the path along the top route. One way to record this route would be:

x 5   – 70   x 3   ÷ 6   + 15

20 → 100 → 30 → 90 → 15 → = 30

Use problem-solving groups of four students and have them discuss starting strategies for question **1**. "Trial and error" will be a common strategy, so make sure that this develops into "trial and learn" (from the result). It will also be useful to emphasise the need to be systematic in choosing and recording which pathways have been followed because it is easy to get confused or waste time repeating a rule that didn't work.

A good strategy for this type of problem is simply to try out and keep a record of different pathways and their answers and then see which part of question **1** matches the path followed.

The students may also get a feeling for the pathways by looking at the last two steps along each pathway. They may notice that two of the possible three ways use division, which will reduce the aggregated totals, while the other way (+ 45 + 15) will increase the total at the end.

Page 9 Flying Feet

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

ACTIVITY

This activity allows students to explore the way the factors of a number relate to a context.

Encourage the students to form a mental picture of each of the children described on the page as they cross the 48 paving stones. You could encourage them to make a diagram of the first 12 stones and then act out or draw the way each child crosses those stones. This would give them a good feel for the problem.

Emphasise the fact that each stride takes the same amount of time, regardless of how long the stride is. This is vital in understanding the problem.

Their answers to question **1a** will give you an indication of whether or not the students have grasped the idea, while question **1b** leads them to connect the factors of 48 with the strides taken. Make sure that this connection is understood. A simple chart form of recording may help to show this:

Person	Length of stride	x	Number of strides	= 48 stones
Mira	1	x	48	= 48
Api	2	x	24	= 48
Matiu	3	x	16	= 48
Eru	4	x	12	= 48

In questions **2** and **3**, the students may find it helpful to draw a diagram showing the stones numbered from 1 to 48. Check that the students have numbered the stone closest to the pool as number 48.

Extend the question by introducing a fifth person who has a stride that uses a factor of 48 not yet mentioned. Ask the students what this stride might be. (It would be either 6 or 8 because 6 x 8 = 48.) This question would confirm which students have made the connection between the context and the factors of 48 because it completes the set of factors of 48.

As a further extension, have the students make up a similar problem using all the factors of 60.

Page 10 Space Marauders

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- explain the meaning of the digits in 2- or 3-digit whole numbers (Number, level 2)

ACTIVITY

This activity will help the students to find efficient ways to add using the place value as well as the face value of the digits involved. It will also help them learn how to record equations using pronumerals and algebraic conventions.

After the students have read and understood the scoring system, work through the example of Kimberley's first game to ensure that they have grasped the reasoning behind her recording system. This is a good opportunity to point out the algebraic convention for recording multiplication: "Kimberley's 8S means 8 times S." Explain that in cases such as this, we do not usually record the multiplication sign as an x because it becomes confusing when other letters are being used to represent unknown or variable numbers (because unlike + and –, x is a letter of the alphabet as well as a symbol).

The other aspect that they should explore and discuss is the way that both the positive and negative quantities that belong in the H (hits) or tens column can be recorded and calculated. In Kimberley's first game, the hits on the space station are recorded as + (2 x –20), which is then recorded as –40 after the calculation has been done and the brackets have been removed.

Page 11 Double and Halve

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

The strategy of doubling and halving can be used to show the connection between division and multiplication by a basic fraction, such as a half or a quarter.

ACTIVITY ONE

In this activity, the strategy of halving also shows the connection between the common fraction of one-half and its decimal equivalent of 0.5 because this will be the result of halving any whole number until a number with a decimal point is reached.

The interest in this activity is in the number of steps it takes to get to the number with a decimal point. Some students may assume that larger numbers will require more steps, so be sure to compare the number of steps it takes when we start from 2 000 (five steps) with the number of steps it takes from 2 500 (three steps).

Encourage the students to explore the number of steps it takes to reach a number with a decimal point for numbers that end in the same quantity, for example, 7 500, 6 500, 3 500, and 500. This way, they will see that the number of steps depends on the first significant figure (that is, the first non-zero figure, in this case, the 5). All the other significant figures have no influence.

The students could then investigate putting any odd number in the ones place, then in the tens place, and then in the hundreds place. They will find that it is the place the odd number is in that decides how many steps it takes to reach a decimal fraction.

An investigation into even numbers is quite a different matter because for any even number, you first have to halve the number until you produce a number where the first significant figure is odd and then you proceed with the steps to a decimal fraction.

By comparing the number of steps for 1 600 and 50, we can see the effect:

Steps for 1 600		Steps for 50	
800		25	
400		12.5	two steps
200			
100			
50			
25			
12.5	seven steps		

ACTIVITY TWO

Here the process is reversed so that the students can see the effect of doubling. Note that, in the first question, Jarod will not reach exactly 1 000 000 by doubling from 1 000, but it will take him 10 steps to go past that number.

A good context that many students will relate to is the way the size of the memory on computers usually grows in doubles. A chart such as the one below may be helpful. As an extension, the megabyte increases could be shown as powers of 2.

1	megabyte	=	2^1
2	megabytes	=	2^2
4	megabytes	=	2^3
8	megabytes	=	2^4
16	megabytes	=	2^5
32	megabytes	=	2^6
64	megabytes	=	2^7
128	megabytes	=	2^8
256	megabytes	=	2^9
512	megabytes	=	2^{10}

Page 12

Find the Number

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity challenges the students to use clues based on the properties of number to solve the problems. The number range selected in the activity encourages the students to develop a deeper knowledge of numbers between 50 and 200.

You may need to explain the number properties of odd, even, and square numbers and the meaning of the associated vocabulary (digit, multiple) before the students start working on a set of clues. You could divide the students into groups, with each group working on a different set of clues.

An alternative approach, which would heighten the activity as a problem-solving situation and also provide you with feedback about the entry knowledge of the students, would be to give out a set of clues to each group without giving them the accompanying board of numbers. Then tell them to use all the clues to find an unknown number and give them some process instructions, such as:

- One of the group must read out all the clues slowly twice.
- The group discusses the clues to identify those they understand and those they are not sure about.
- They write out the clues they are not sure about on the whiteboard.
- You or students from another group explain the meaning of the clues that they don't understand.
- The group attempts to solve the problem.
- The group discusses strategies that might work, such as listing or highlighting numbers that are eliminated or remain after each clue until the solution is found.

It is best to have no more than four students in each group so that all the students can be involved in the discussion.

These clues can also be used in a more co-operative problem-solving situation if the group's set of clues is copied onto paper and each clue is cut out separately. Give each student responsibility for one of the clues. They can read it out to the others, but they cannot hand it over. When a solution is proposed, they must check that it meets the requirements of their clue. If everyone in the group agrees, then this is the solution.

Page 13 Ways to 43

Achievement Objectives

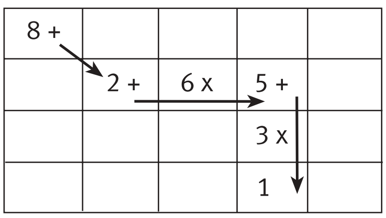
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)

ACTIVITY

The cumulative calculations required in this activity will give the students plenty of opportunities to practise using the order of operations conventions.

One of the challenges facing the students is how to record their pathway of calculations. Discuss this with them. Some students may choose to list each set of calculations, such as 8 + 2 = 10, 6 x 5 = 30, and 3 x 1 = 3, and then the pathway, in this case, 10 + 30 + 3 = 43.

Others may use a template of a 5 by 4 set of squares and write in the operations and numbers of the squares that are on the path they have chosen. The path for 8 + 2 + 6 x 5 + 3 x 1 would look like this:



After the students have explored various ways of recording their pathway of calculations, make sure that you show them how to record all the steps taken in one equation only rather than in a series of equations. Help them to see the connection between this equation and the way they record their chosen path. This will provide the opportunity for you to work with the students on the conventions of the order of operations that should be used in complex equations. The students can then practise writing complex equations for other pathways.

Use special challenges to add interest to the activity. For example, be the first to find:

- the shortest route
- a long route (for example, 8 + 7 + 4 + 1 + 9 + 2 + 5 + 6 + 2 + 3 – 6 + 9 – 1 – 3 – 0 – 4 – 1 + 2 = 43)
- the most routes
- a route to a different prime number (for example, 8 + 3 + 6 + 9 + 8 + 1 + 2 = 37)
- an equation that uses division (for example, 8 ÷ 2 x 6 + 9 + 8 + 2 = 43)
- an equation that uses all four operations (for example, 8 x 7 ÷ 2 + 9 + 8 – 2 = 43).

Page 14 Writing 1 000

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- demonstrate knowledge of the conventions for order of operations (Number, level 4)

ACTIVITY

This activity will develop fluency with combinations of numbers that make 1 000. This is a useful way to help the students understand the bridge between 3-digit and 4-digit numbers.

Question **2** encourages the students to use a compensation strategy to make a series of addition equations that are equivalent to 1 000. In your discussion with them, ensure that each student can clearly see the connection between their equations.

Question **3** links addition of equal groups to an equation using multiplication to encourage more efficient ways of thinking about 1 000. This is then extended in question **4** into equations involving combinations of operations.

In question **4b**, the students should discuss and share the ways they used compensation to help them derive new equations as they expanded their original equation.

Challenge the students to come up with expressions that have a pattern in them, such as:
40 x 20 + 20 x 10
8 x 50 + 4 x 100 + 1 x 200
60 x 30 – 40 x 20
50 x 25 – 20 x 10 – 10 x 5

Up the Ladder

Achievement Objectives

- read any 3-digit whole number (Number, level 2)
- explain the meaning of the digits in 2- or 3-digit whole numbers (Number, level 2)

GAME

This ladder game provides a fun challenge that builds understanding of the place value of 3-digit numbers as well as practice in ordering those numbers.

The key learning opportunity comes from discussion of the strategies used to place numbers so that a player gives themselves the best chance of winning. This involves putting numbers close together on the ladder if they are close in size. It also involves placing smaller numbers nearer the bottom of the ladder and larger numbers nearer the top.

The importance of the digit in the hundreds place should be discussed after the students have played the game a few times.

The students should be encouraged to track the numbers that have been used so that they think about the possibilities left to them. For example, if all three nines have been used, they will not be able to make a number in the nine hundreds until all the cards have been used and the pack is reshuffled.

This type of game is ideal to use as an independent activity in a learning centre after it has been used for instructional purposes.

ACTIVITY

Have the students discuss the aspects that will influence Vanna's and Logan's chances. These aspects are the digits already chosen for this round and the number of winning digits still available for selection.

If the students analyse the numbers already on the ladder, they should realise that Vanna and Logan have gone through the whole pile of cards once, and on the second time through, they have six cards left for the tenth round. (In practice, the students will probably find that, in their own games, they will need to go through the cards more than twice to fill their 10 rungs, especially at the start and until they get used to using strategies for ordering their numbers.) After Vanna and Logan have drawn two cards each, the two cards left are a 0 and a 5. Vanna has a 50 percent chance of winning on this round because she can win if she draws the 5 (to make 593 or 539). Logan cannot win with either card because he needs a 2 or a 3 (to make a number between 407 and 218).

VARIATIONS AND EXTENSIONS

This game can be extended to 4-digit numbers or decimal numbers if a decimal point is placed on each rung. The range could be reduced by changing the top and bottom numbers on the ladder. The students could explore the effect on the game if the number of sets of digit cards used is changed.

Playzone Discount

Achievement Objectives

- make sensible estimates and check the reasonableness of answers (Number, level 2)
- find a given fraction or percentage of a quantity (Number, level 4)

GAME

This game has two mathematical features: the calculation of a percentage discount and the use of a winning strategy to get four in a row on a 5 by 5 array.

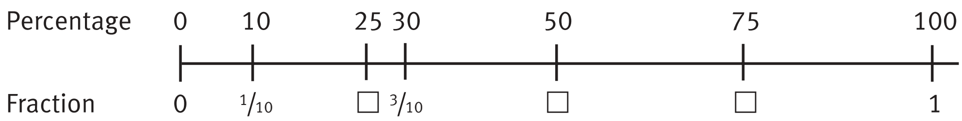
If the students are familiar with finding percentages, they can play the game as a percentages maintenance activity and focus on working out a winning strategy. The player who goes first could choose the centre square and try to get a line of three in a row with both ends unblocked. Once the player has this, victory is assured. The player who goes second should use their turn to defend immediately by trying to block the first player until the first player makes a mistake. If both players do not make any mistakes, either the player going first will win or the game will end in a draw.

If the strategy for winning is the learning outcome you want the students to focus on, you may let them do their discount calculation on a calculator.

If the main purpose of the game is to introduce percentage calculations to the students, they should not use a calculator because the intention is to develop mental strategies for doing these.

Make sure that the students understand that percentage means “out of 100”. One interesting way to do this is to explain the origins of the percentage symbol, %. The / represents “out of” or division and the two small zeros on either side of the / represent the zeros in 100. So the symbol represents “out of 100”.

The students could make a chart or a double number line showing the equivalent values of the percentages as fractions.



Have the students develop and discuss strategies for estimating and calculating a percentage amount, starting with the 50 percent coupon, then the 25 percent, and then the 10 percent. Do these first because they are equivalent to unit fractions, and so they allow the students to get used to dividing the amount by the denominator of the unit fraction before they move on to looking at the more complex calculations involved with the improper fractions.

With 75 percent ($^{3}/_{4}$) and 30 percent ($^{3}/_{10}$), the students need to see these as “three lots of a quarter” and “three lots of one-tenth” respectively to develop a suitable strategy for calculating the percentage amounts.

The students could make a chart showing the discount amounts for each of the Playzone games.

Playzone game	Discount				
	10%	25%	30%	50%	75%
Exotic Raiders $40	$4	$10	$12	$20	$30
Blitz $80	$8	$20	$24	$40	$60
Metal Strike $130	$13	$33*	$39	$65	$98*
Team Pro $200	$20	$50	$60	$100	$150
Lightning $150	$15	$38*	$45	$75	$113*
Sound Barrier $100	$10	$25	$30	$50	$75

* rounded

The students could then use this chart as they try to win the game. However, you could challenge them to use mental strategies to work out the percentages instead of using the chart.

If estimation is the main purpose of the game, the students should definitely not make the chart above. It would be better to explore strategies for estimating. Begin by asking the students, "Which Playzone game would be the easiest to look at in terms of estimating the discounts?"

The Sound Barrier Playzone game is probably the easiest as its price rounds to $100 and therefore the percentage gives a one-to-one indication of the discount. The discount for this game can then be used as a guide for other games. Team Pro will be twice as much. Lightning will be half as much again as Sound Barrier. Exotic Raiders will be just under half as much as Sound Barrier.

After the students have played the game a few times, discuss the best strategies for winning. The students could compare the 5 by 5 board strategies with the common game of noughts and crosses, which uses a 3 by 3 board or grid.

Page 17 Different Approaches

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

The purpose of this activity is to encourage the students to improve their mental strategies so that they can mentally solve addition and subtraction problems quickly. The contest between the calculator users and the brain users is not about showing that one of these approaches is better than the other. The calculator approach is there as a benchmark to show the efficiency of a good mental strategy. There is not one "correct" strategy; a good one is one that finds the answer quickly.

As part of the discussion in question **1a**, ask the students what knowledge or strategy the "brain" people might need to use to beat the "calculator" people.

You might need to point out that the calculator people would have been expected to use key presses only, as indicated by the question, and would not have combined their own mental strategies with the calculator to speed up their response. For example, in problem **1v**, they would have been expected to press [9] [+] [9] [+] [9] [+] [9] [+] [9] = , not [5] [x] [9] [=].

In problem **1b**, the students should be encouraged to adjust the numbers or the operation in ways that will give them accurate solutions quickly.

Strategies such as rounding or adjusting would suit question **1i**, where 49 can be thought of as the "tidy" number 50 and then adjusted by taking 2 from the answer. Question **1ii** can be worked out quickly if the students recognise that the digits are expanded to show their place value, so the quick solution is simply expressing it as the compact numeral 934. Question **1iv** encourages the use of compensation to see that 704 – 299 is the same as 705 – 300. The students could then also round 705 to 700 as long as they remember to insert the 5 into the answer. Question **1v** will be easy for those students who understand that a repeated addition sequence is equivalent to a multiplication situation.

Question **2** asks the students to apply their understanding of mental strategies to create questions that are suited to mental reasoning versus calculator usage. To make questions suited to mental thinking, they should be looking to select:

- numbers that can be easily rounded to a tidy number (a number with a zero on the end)
- numbers that can be joined to make a ten or a hundred
- equations that look hard but are easy when a simple adjustment is made to them
- equations that involve doubles or near doubles
- equations that involve addition of repeated groups.

Make sure that the questions that the students create are not just basic facts because these are likely to only involve recall thinking as opposed to strategy thinking.

For the questions designed to suit the calculator people, you may need to specify the number of digits to be used in each numeral so that the students don't just resort to very long numerals to try and make the questions difficult. Encourage the students to use the opposite to the ideas used to make questions designed for the brain people to win. Have the students list the strategies they see the brain people using to beat the calculator people on the questions designed for them to win. On the questions that favour the calculator use, the students should explain what it is about the questions that makes it hard for brain people to solve them quickly.

Page 18 It Pays to Win!

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity provides an interesting context for solving computational problems that involve 5- and 6-digit whole numbers.

The numbers involved have been deliberately chosen so that the students can use a range of strategies. Encourage them to try to work out each payment mentally and then check by writing out their methods or doing it on the calculator. If the calculator is used as the first strategy, it will help some students achieve success but the opportunity to solve the problems using the properties of the numbers involved will be lost. You should give guidance to individual students about which questions they should solve with a calculator, for example, question **2b**.

The students may attempt to solve the problems by counting forwards in hundreds or thousands, by adding, or by multiplying. Whatever strategies the students use should be valued positively. At the same time, the students should be encouraged to improve the speed and efficiency of their methods.

Before the students attempt the calculations involved in each question, make sure they show that they understand what is going on in the problem and how each performance payment works as an add-on to the basic salary. Encourage the students to record their thinking by carefully setting out their work and including their method of calculation for each step:

What Tama earned	**How I worked it out**
$82,000	Tama's basic salary
$17,000	17 times $1,000 for each win
$1,000	2 draws earns $500 plus $500
$4,600	46 kicks times $100
Total $104,600	I added all the amounts.

In question **2a**, the students could use a recording method that helps them compare the new payments with the old.

	What Tama earned last season	Tama's new rates applied to last season	Difference
Salary	$82,000	$82,000	$0
Wins	$17,000	$22,100	+$5,100
Draws	$1,000	$1,000	$0
Kicks	$4,600	$6,900	+$2,300
Total	$104,600	$112,000	+$7,400

Some students may choose to calculate the difference after working out the total income for each rate. Alternatively, they could set up a running total of the differences calculated at the end of each stage.

If calculators are used, the latter method will provide a great opportunity to show the students how to use the memory keys to record the running total. For example, after the $17,000 has been subtracted from the $22,100, enter the result of + $5,100 into the memory by pressing the M+ key. Use the same process to store the + $2,300 in the memory. The running total will come up when the MR key is used to recall the amount.

Page 19 Drink Stall

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

In this activity, the students are introduced to the concepts of ratio and proportion and how these are used to solve problems.

Explore Jenny's statement with the students to ensure that they know how the parts (ratio) are used to make up the whole. They need to see that when the cordial and water are mixed in a ratio of 1:4, the fraction of the whole that the cordial makes up is $^1/_5$ and the fraction of the whole that the water makes up is $^4/_5$.

Highlight the use of the ratio symbol (:) that means "to" in 1:4, which is thus read as "one to four".

Introduce the students to the use of a ratio table for considering the relationship between cordial amounts and the total drink produced in questions **1a** and **b**.

Cordial (litres)	1	2	3
Drink (litres)	5	10	15

The students could then take this idea and use it to show what each question is about before they solve it. For example, in question **2**, Toline's ratio table would look like this:

Sachets	4	8	?	16	?
Drink (litres)	5	10	15	20	25

You can use this table to discuss the connections between the different elements. Toline's table shows sachets in groups of 4 and drinks in groups of 5. It also shows how 4 sachets relate to 5 drinks in the same proportion as 8 sachets to 10 drinks.

Question **4** has an interesting connection between questions **a** and **b** that you could share with the students. That is, the number of litres of drink made with 15 lemons is the same as the number of cups of sugar used with 15 lemons. Discuss why this must always be true.

Question **6** is similar to question **4** in that it extends the ratio idea by involving three parts that combine to make the whole. Suggest to the students that they discuss what is happening in the problem to work out the ratios involved.

For question **6**, the girls' shares, Jenny:Mere:Toline, are shared in the ratio 2:1:1. This is said as "two to one to one". So Jenny gets 2 out of every 4 parts, while the others get 1 out of every 4 parts.
The ratio table would look like this:

Jenny's part	$2	$10	$20	$40
Mere's part	$1	$5	$10	$20
Toline's part	$1	$5	$10	$20
Total sales	$4	$20	$40	$80

Page 20

Grocery Grapplers

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity uses measurement contexts to enable students to understand what is happening in the problem. This will give them clues for deriving the equations they will need.

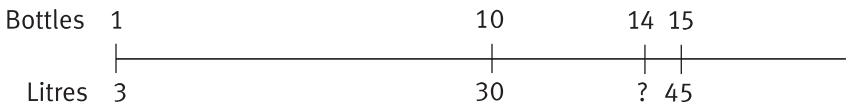
Ensure that the students understand the importance of the following three key stages of problem solving:
1. Understand the problem.
2. Solve the problem.
3. Evaluate the answer.

If you insist that each group explore each stage and report their findings before moving on to the next stage, the students will come to appreciate the value of each stage. This is best done using problem-solving groups of four students.

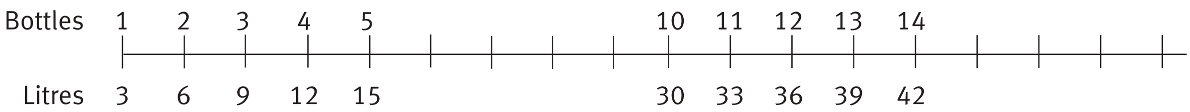
The students could use their own words, diagrams, or actions to explain to each other what they think is happening in each problem before they attempt to use numbers in equations or algorithms. Another idea is to have the students list the key words and facts, possibly using headings such as Given, Wanted, and Needed (what I have to do to get the wanted data).

Open or double number lines would help those students who find it difficult to come up with a suitable open number sentence to solve the problem.

Using an open number line to solve problem **1**:



Using a double number line to solve problem **1**:



The students can evaluate their answers by:
- checking that all the important information has been used;
- checking their calculations;
- checking that their answer makes sense.

Encourage the students to share two or three easy or efficient ways to solve the problems. Examples of alternative strategies are shown in the Answers. These alternative strategies show that the essence of problem solving in number is to deliberately manipulate the quantities that make the problem difficult in order to form easier steps that can be recombined successfully.

Question **6** requires a practical solution in that Nick and Ema need to buy more chocolate fish than they actually need. Six bags of fish gives only 72 fish, so they will need to buy an extra bag of 12 fish to get the 6 fish they need in order to have 78 chocolate fish altogether.

Page 21 **Birthday Treat**

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity uses a context that should appeal to students in this age group and should thus encourage them to develop their number problem-solving strategies further.

If the students make an error in question **1**, it will have a carry-over effect to the following questions. Have the students report their solutions for question **1** and resolve any difficulties before they continue with the next question. Take the opportunity to see if the students can find the multiplicative relationship between the $5.50 cost for the movie and the $55 total. This is a "10 times larger" relationship, and as there are 5 boys, the movies will take half of the $55, leaving half for food.

Another way of looking at this is to see that if there is $55 for 5 boys, each boy could have $11; that is, $5.50 for the movies and $5.50 for food.

After the students have answered question **4**, you might like to encourage them to develop some new types of burgers to add to the Burger Planet menu, along with different prices. They could use these new burgers to work out different options for the 5 boys.

Page 22 Shopping Around

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity is designed to encourage the students to develop rounding skills as they seek easy ways to solve money problems.

Ask the students to attempt to solve most of the problems mentally before they write out any working as this will encourage them to develop thinking strategies that are easy for them without relying on the working forms they have been taught.

Question **2** involves 25 cents over the 5 drink bottles if the base price is rounded to the nearest dollar, so you may choose to ask the students to adjust their rounded answer to be more accurate in this case. Similarly, with question **5**, if the students round to $6, they will need to subtract 6 x 50 cents = $3 from their rounded total.

Question **3** presents an opportunity for the students to use a variety of strategies, but at the same time, it sets a trap for those who do not read the question carefully. If the students simply cost out 10 CDs, they have missed the offer of a free 1 for every 4 they purchase. So they only have to purchase 8 CDs to get the 10 they want. Apart from the obvious calculation of 8 x $15.00, the students may choose to calculate this as (4 x $15) x 2 because of the structure of the question.

Question **4** doesn't require rounding, but it does involve calculating four subtotals. This will be too much for most students to do mentally. Give feedback to the students on how appropriately they are setting out the written workings they use to show the subtotals and the final total.

Extend this activity by organising a collection of advertising brochures from supermarket or appliance retailers. These brochures invariably use money amounts that encourage rounding for calculation purposes. The students can use these to pose similar questions for other students. They should be responsible for marking their questions because this will encourage them to set sensible tasks as well as maintain their own skill in calculating.

Page 23 Keep Your Shirt On

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

This activity encourages students to use rounding to simplify the process of calculating in multiplication and division contexts.

The error caused by rounding accumulates, so it would be a good idea to display the estimated answer and then the adjusted, accurate answer to help the students develop an awareness of this fact.

Question **1** recording may look like this:

How I estimated	How I adjusted the estimate
\$15.95 can be thought of as \$16.	This is 5c too much.
\$16 x 25 = \$16 x 100, then divide by 4 = \$1,600 ÷ 4 = \$400.	This is 5c x 25 too much (\$1.25). \$400 – \$1.25 = \$398.75 is therefore the accurate answer.

Question **2** is a great opportunity to ask the students to look for connections between question **1** and each part of question **2** to find easy ways of calculating the answers.

List the final equation for question **1** and then all the open equations for each part of question **2**:

1: 25 x \$15.95 = \$398.75
2a: 25 x \$19.95 = ?
2b: 75 x \$15.95 = ?
2c: 23 x \$15.95 = ?
2d: 25 x \$7.95 = ?

Ask the students to look for things that are the same and things that are different between the questions. Encourage them to use these observations to find easy ways of getting a solution. For example, question **2a** can be solved by adding \$4 x 25 to question **1** because the shirts are \$4.00 more but the same number (25) is being purchased. Question **2b** is three times the answer to question **1**. Question **2c** is two lots of \$15.95 less than question **1**. Question **2d** is 25 x \$8 less than question **1** (\$15.95 – \$8.00 = \$7.95).

Question **3b** asks for the approximate cost, so make sure that the students use rounded amounts in their calculations. The 50 shirts will cost approximately \$600, so they could find the cost of one shirt by dividing by 100 and then doubling. The equation for this is \$600 ÷ 100 x 2 = \$12. They could also divide by 10 and then by 5: \$600 ÷ 10 = \$60, then \$60 ÷ 5 = \$12. Alternatively, if they look at it simply as 600 ÷ 50, they may realise that they can divide both numbers by 10 and get 60 ÷ 5 = 12.

Encourage the students to report on two or three ways of thinking through the problems. Help them to form a recording model that accurately reflects the language they use.

Page 24 **Division Dilemmas**

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, levels 2–3)

ACTIVITY

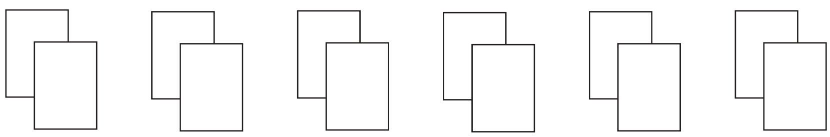
This activity models the use of different mental strategies in solving a division problem and then asks the students to apply these models to other questions. By doing this, the students will hopefully see that a division problem that cannot be solved in one step can be rearranged into a number of simpler steps based on the students' knowledge of number relationships. It is not intended that the students will become skilful at using every one of these methods; rather, the different options should be seen as models of how it is often possible to reconstruct a difficult problem in a variety of ways.

A picture of the thinking used in Ashley's method may take the form of an array where the 6-packs are lined up thus:

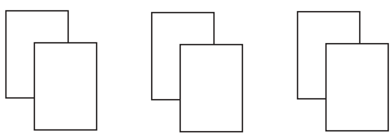
Packs	1 10	20	1 4
	O O O O O O O O O O	O O O O O O O O O O	O O O O
	O O O O O O O O O O	O O O O O O O O O O	O O O O
	O O O O O O O O O O	O O O O O O O O O O	O O O O
	O O O O O O O O O O	O O O O O O O O O O	O O O O
	O O O O O O O O O O	O O O O O O O O O O	O O O O
	O O O O O O O O O O	O O O O O O O O O O	O O O O
Balls	6 60	120	6 24

James's method might be explained by modelling and recording simple versions of equivalent equations. Ask the students: "If 12 ÷ 6 = 2 and 6 ÷ 3 = 2, how are these equations the same and how are they different?" The answer 2 is the same, so 12 ÷ 6 is equivalent to 6 ÷ 3. That is, the results are the same, even though the factors are different.

Use cards, blocks, or beans to show how 12 shared out among 6 people gives the same result as sharing 6 out among 3 people.



$12 \div 6 = 2$



$6 \div 3 = 2$

Have the students explore a number of division equations that have the same answer until they see the equivalence involved.

Zoe's method depends on knowing that 144 ÷ 6 is the same as 6 x ☐ = 144. It needs to be explained using the number properties that Zoe is thinking about. At this point, it could be helpful to explore the units digit in the six times table to see when it gives a result with 4 in the units column. This happens at 6 x 4 = 24 and at 6 x 9 = 54.

If the table is extended, it will be seen to occur next at 6 x 14 = 84, at 6 x 19 = 114, and then at 6 x 24 = 144 and 6 x 29 = 174. The key idea in Zoe's method is that something ending in 4 must be added to 120 to get 144, and 6 x 4 = 24 is the one that fits.

The students could then be organised into three groups. Each group can be assigned a method for solving question **2a**. After they have reported back on how to use Ashley's, James's, or Zoe's method, assign them another method to try on question **2b**. Continue this rotation for question **2c** so that all the students have tried all three methods of thinking discussed on this page. You could then ask them to choose the method they think would be easiest for each of the remaining questions.

Copymaster **Aiming High**

Throw	Hundreds	Tens	Ones
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Total			

Throw	Hundreds	Tens	Ones
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Total			

Throw	Hundreds	Tens	Ones
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Total			

Throw	Hundreds	Tens	Ones
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
Total			

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