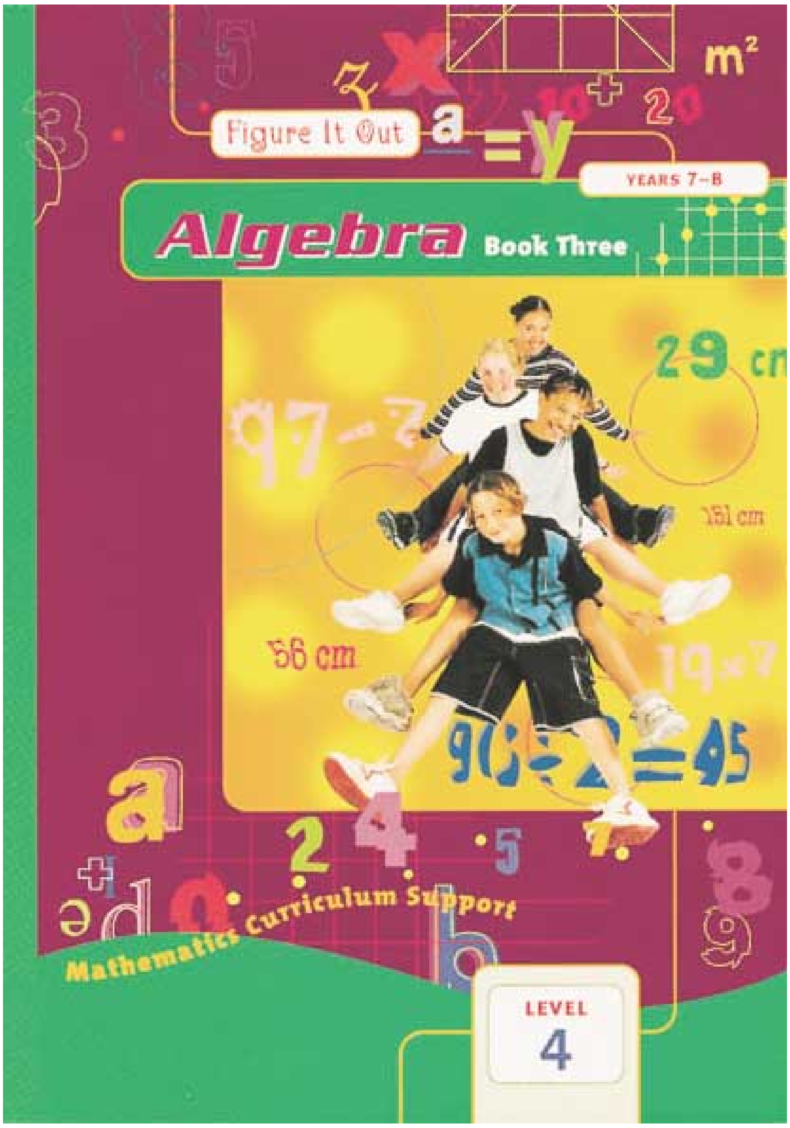
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+ distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around,* (level 4–4+, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers

Algebra : Book Three

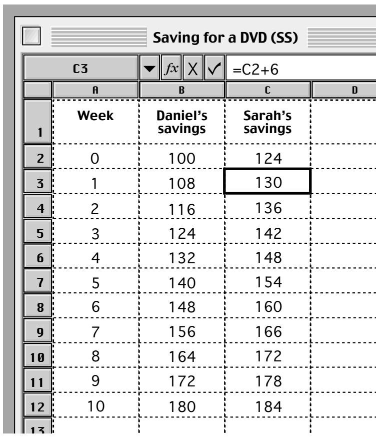
Page 1 Save Some, Spend Some

ACTIVITY

1. a. The formula is =B3+8. It adds 8 to the value 108 in cell B3, so the value in cell B4 becomes 116.

   b. The formula is =C2+10–4 or just =C2+6. It adds 6 to the value 124 in cell C2, so the value in cell C3 becomes 130.

2. a.



Saving for a DVD (SS)

C3 =C2+6

	A	B	C
1	Week	Daniel's savings	Sarah's savings
2	0	100	124
3	1	108	130
4	2	116	136
5	3	124	142
6	4	132	148
7	5	140	154
8	6	148	160
9	7	156	166
10	8	164	172
11	9	172	178
12	10	180	184

   b. After 10 weeks, Daniel has saved $180 and Sarah has saved $184.

   c. Daniel: after 18 weeks. (He has $236 after 17 weeks and $244 after 18 weeks.)
   Sarah: after 20 weeks. (She has $238 after 19 weeks and $244 after 20 weeks.)

3. a. =C9+10

   b. She will be able to buy the DVD player after 15 weeks. (She will have $236 after 14 weeks and $246 after 15 weeks.)

4. a. The new formulae are: =B5+11 in cell B6, =B6+11 in cell B7, =B7+11 in cell B8, and =B8+11 in cell B9. (The different figure in cell B6 shows that Daniel increases his savings after week 3.)

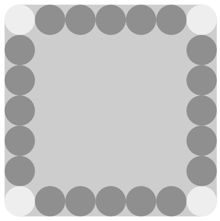
   b. After 14 weeks. (He will have $234 after 13 weeks and $245 after 14 weeks.)

Pages 2–3 Table Mats

ACTIVITY

1. a. 4 x 5 + 4 = 24

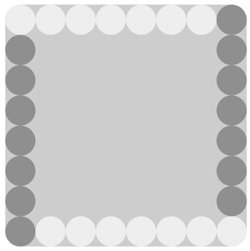
   b.



   There are 5 blue circles on each side, so there are 4 sets of 5 blue circles and a yellow circle in each corner.

   c. A table mat with 100 circles on each side will have 4 x 98 blue circles and 4 yellow corner circles. So it will have a total of 4 x 98 + 4 = 396 circles.

2. a.



   There are 2 sets of 7 orange circles and 2 sets of 7 pink circles. So there are 4 sets of 7 circles or 4 x 7 = 28 circles.

   b. 4 x 199 = 796 circles

3.

Number of circles on side	Total number of circles	
	Hine's short cut	Larissa's short cut
4	4 x 2 + 4 = 12	4 x 3 = 12
10	4 x 8 + 4 = 36	4 x 9 = 36
15	4 x 13 + 4 = 56	4 x 14 = 56
90	4 x 88 + 4 = 356	4 x 89 = 356
63	4 x 61 + 4 = 248	4 x 62 = 248
136	4 x 134 + 4 = 540	4 x 135 = 540
500	4 x 498 + 4 = 1 996	4 x 499 = 1 996

4. a. Answers may vary. A rule is: 2 times the blue set of circles plus 2 times the yellow set of circles. Using this rule, the short cut for the design with 4 circles on each side is 2 x 4 + 2 x 2 = 12 circles. For the design with 5 circles on each side, the short cut is 2 x 5 + 2 x 3 = 16 circles.

b.

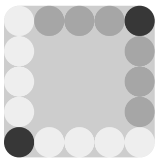
Number of circles on side	Total number of circles
4	12 (2 x 4 + 2 x 2)
5	16 (2 x 5 + 2 x 3)
8	28 (2 x 8 + 2 x 6)
27	104 (2 x 27 + 2 x 25)
156	620 (2 x 156 + 2 x 154)

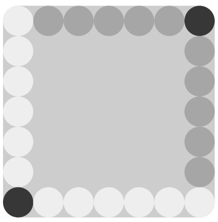
5. a. 4 x 6 – 4. In each short cut, the number of circles on any side is first multiplied by 4 (the number of sides). But the 4 corner circles will then have been included twice, so 4 circles must be subtracted.

b.

Number of circles on side	Total number of circles
6	4 x 6 – 4 = 20
9	4 x 9 – 4 = 32
14	4 x 14 – 4 = 52
59	4 x 59 – 4 = 232
131	4 x 131 – 4 = 520

6. a. Answers will vary. One design is shown below.


5 circles on each side


7 circles on each side

b. A rule for the design above is: 2 times the number of colour a counters in one side plus 2 times the number of colour b counters in one side, plus 2. For the design above with 5 circles on each side, the short cut is (2 x 4) + (2 x 3) + 2 = 16. For the design with 7 circles on each side, the short cut is (2 x 6) + (2 x 5) + 2 = 24.

c. For the design above:
(2 x 99) + (2 x 98) + 2 = 396 circles

Pages 4–5 Stacking Up

ACTIVITY

1. a. 17 cm. The stack of 4 containers measures 41 cm, and the stack of 2 containers measures 25 cm. So the 2 extra containers add 16 cm (41 – 25), which is 8 cm for each new container. So 1 container measures 25 – 8 = 17 cm.

b. 89 cm

c. A rule for the height of a stack with any number of containers is: the height of the first container, 17 cm, plus 8 cm for each of the other containers in the stack.

2. a. 12 cm. Each hat below the top hat adds 6 cm to the height of the stack. The hat at the top of the 3-hat stack is 24 – 2 x 6 = 12 cm.

b. The height of a stack with 20 hats would be 19 x 6 + 12 = 126 cm. A rule for the height of a stack with any number of hats is: the height of the top hat, 12 cm, plus 6 cm for each of the other hats in the stack.

3. a. 57 cm. The extra 5 CDs need 10 cm, so each additional CD in a rack occupies a height of 10 ÷ 5 = 2 cm. The height of a rack with 10 CDs is 27 cm. The 10 CDs need 20 cm, so an extra 7 cm is used for the frame of the rack. The height of a rack with 25 CDs must therefore be 25 x 2 + 7 = 57 cm.

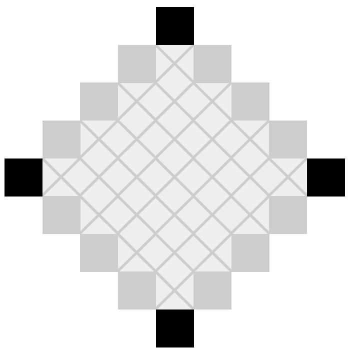
b. 45. (97 – 7 = 90. 90 ÷ 2 = 45. So 7 + 45 x 2 = 97.)

4. Answers will vary depending on the measurements.

Pages 6–7 Design Day

ACTIVITY

1. a.



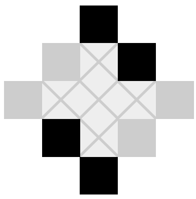
b. For the third design, George paints 4 sets of 3 green squares and 4 blue squares.
His rule is: 4 times the number in each set of green squares (the design number) plus 4 blue squares. This is the same as 4 x 3 + 4 = 16 painted squares. So the fifth design will have 4 x 5 green squares + 4 blue squares, which is 4 x 5 + 4 painted squares.

c. 4 x 20 + 4 = 84 painted squares

d.

Design	Number of painted squares
1st	4 x 1 + 4 = 8
2nd	4 x 2 + 4 = 12
3rd	4 x 3 + 4 = 16
5th	4 x 5 + 4 = 24
37th	4 x 37 + 4 = 152
100th	4 x 100 + 4 = 404

2. a.



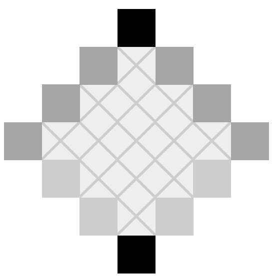
b. In Kelly's first design, she paints 2 sets of 2 blue squares and 2 sets of 2 green squares.
Her rule is: 4 times the number in a set of either green or blue squares (the design number plus 1). So her fifth design will have 4 sets of either green or blue squares to paint, which is 4 x 6 = 24 squares.

c. 52. (4 x 13 = 52)

d.

Design	Number of painted squares
7th	32 (4 x 8)
8th	36 (4 x 9)
15th	64 (4 x 16)
47th	192 (4 x 48)
126th	508 (4 x 127)
199th	800 (4 x 200)

3. a.



b. Answers may vary. One rule is: 2 times the number in a set of orange squares (1 more than the design number) plus 2 times the number in a set of green squares (the design number), plus 2 blue squares. So the hundredth design will have 2 sets of 101 orange squares, 2 sets of 100 blue squares, and 2 blue squares, which is 2 x 101 + 2 x 100 + 2 = 404 painted squares.

c.

Design	Number of painted squares
5th	2 x 6 + 2 x 5 + 2 = 24
9th	2 x 10 + 2 x 9 + 2 = 40
20th	2 x 21 + 2 x 20 + 2 = 84
37th	2 x 38 + 2 x 37 + 2 = 152
89th	2 x 90 + 2 x 89 + 2 = 360

Page 8 Kidding Around

ACTIVITY

1. a. i. 1 goat

ii. 2 goats would be dosed before Gussy.

b.

Number of goats in front of Gussy	8	9	12	16	18	25	33	42	46	100
Number of goats dosed before Gussy	3	3	4	6	6	9	11	14	16	34

c. If the number of goats in front of Gussy is a multiple of 3, you divide that number by 3 to find the number of goats dosed before Gussy. If the number of goats in front of Gussy is not a multiple of 3, you round that number to the next multiple of 3 and divide by 3. For example, for 10, 11, or 12 goats in front of Gussy at the start, Gussy would have to wait for 12 ÷ 3 = 4 goats to be dosed.

2. a.

Number of goats in front of Gussy	8	9	12	16	18	25	33	42	46	100
Number of goats dosed before Gussy	2	2	3	4	4	5	7	9	10	20

b. If the number of goats in front of Gussy is a multiple of 5, you divide that number by 5 to find the number of goats dosed before Gussy. If the number of goats in front of Gussy is not a multiple of 5, you round that number to the next multiple of 5 and divide by 5.

Page 9 Stepping Stones

ACTIVITY

1. Long stride, short step (LS)
2. Matiu can cross the stream in 5 different ways: SSSS; LL; SSL; SLS; LSS
3. a.

Number of stepping stones	1	2	3	4	5	6
Different ways to cross	2	3	5	8	13	21

b. Each number in the second row of the table is the sum of the two preceding numbers. (The next three numbers in the table would be 34, 55, and 89. They are part of the Fibonacci sequence mentioned in question **4b.**)

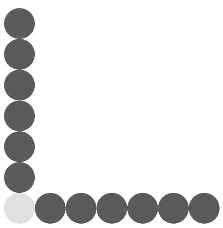
4. a. 144. (55 + 89, the two preceding numbers)

b. Leonardo Fibonacci (1170–1240) was an Italian mathematician. His major work was the book *Liber Abacci*, published in 1202. He is best remembered for the solution to a problem related to rabbit reproduction. His solution led to the Fibonacci number sequence, 1, 1, 2, 3, 5, 8 …, where each number in the sequence is the sum of the two preceding numbers.

Pages 10–11 Letter Designs

ACTIVITY

1. a. The L design below has 2 x 6 + 1 = 13 counters.



b. There are 6 blue counters on each side, so there are 2 x 6 blue counters. There is 1 orange counter in the corner of the L, so there are 2 x 6 + 1 counters altogether.

c.

Number of counters on each side	Total number of counters
4	2 x 3 + 1 = 7
5	2 x 4 + 1 = 9
7	2 x 6 + 1 = 13
23	2 x 22 + 1 = 45
87	2 x 86 + 1 = 173

2. a. A short cut for the first design is 4 + 3 = 7 counters, and a short cut for the second design is 5 + 4 = 9. A rule is: the number of orange counters plus that number minus 1. The L design with 4 counters on each side has 4 orange counters on one side and 3 blue on the other, and the design with 5 counters on each side has 5 orange counters and 4 blue counters. The number of blue counters is always 1 less than the number of orange counters. Or, alternatively, the number of orange counters is always 1 more than the number of blue counters.

b.

Number of counters on each side	Total number of counters
9	9 + 8 = 17
10	10 + 9 = 19
15	15 + 14 = 29
76	76 + 75 = 151
127	127 + 126 = 253

3. a. The L design has 6 counters on each side. This suggests that the design has 2 x 6 = 12 counters. But the corner counter is included in each side and has been counted twice, so 1 must be subtracted, giving the total number of counters as 2 x 6 – 1 = 11.

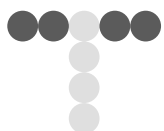
b. 2 x 256 – 1 = 511 counters

4. a. A short cut for the first design is 2 + 3 x 2 = 8. The T design with 4 counters in the stem has 2 green counters and 3 sets of 2 blue counters. (The stem includes the middle counter in the top bar.) The T design with 5 counters in the stem has 2 green counters and 3 sets of 3 blue counters. A rule is: 2 green counters plus 3 times the number of blue counters in the stem.

b. Using the rule shown above:

Number of counters in stem	Total number of counters
4	8 (2 + 3 x 2)
7	17 (2 + 3 x 5)
11	29 (2 + 3 x 9)
32	92 (2 + 3 x 30)
71	209 (2 + 3 x 69)
478	1 430 (2 + 3 x 476)

5. a.–b. Answers will vary. Another T design is:

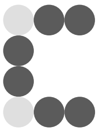
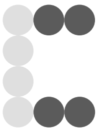
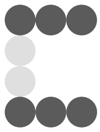


This T design with 4 counters in the stem has 4 green counters and 2 sets of 2 blue counters, which is 4 + 2 x 2 = 8 counters. The same T design with 5 counters in the stem has 5 green and 2 sets of 3 blue counters, which is 5 + 2 x 3 = 11 counters. So a rule is: the number of green counters in the stem plus 2 times the stem minus 2.

c. 746 counters. The rule given in **a** would give 250 + 2 x 248 = 746.

6. Answers will vary. One rule for the design in one colour is: the left side plus 2 times the left side minus 2. So, for the first design, this is 4 + 2 x 2 = 8. For 100 in the stem, this is 100 + 2 x 98 = 296. Another rule for this design is: 3 times the left side minus 4. For 100 in the stem, this is 3 x 100 – 4 = 296.

Other rules could be based on designs such as:

i

ii

iii


For i, a rule is: 3 times the number of blue counters in the stem plus 2. So for 100 counters on the left side, this is 3 x 98 + 2 = 296.

For ii, a rule is: the number of counters on the left side plus 2 times that number minus 2. So for 100 counters on the left side, this is 100 + 2 x 98 = 296. (This is the same rule as the one for the first one-colour rule above.)

For iii, a rule is: the number of counters in the stem minus 2, plus 2 times the number of counters in the stem minus 1. So for 100 counters on the left side, this is 100 – 2 + 2 x (100 – 1) = 98 + 2 x 99
= 296.

Pages 12–13 Problem Smorgasbord

ACTIVITY

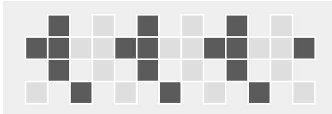
1. 13. (There are 16 boys.)
2. 9 m
3. Grandad. (He is 66 years old.)
4. Huani earns $18, and Kiri earns $6.
5. 5
6. 435

Pages 14–15 Frieze

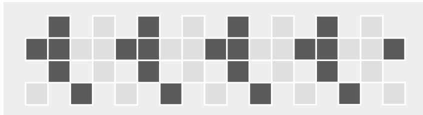
ACTIVITY

1. a. 11 x 5 + 2 = 57 squares

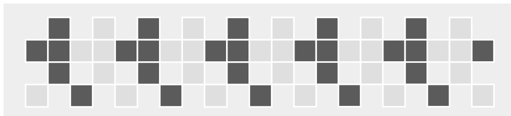
b.



c. Design for 8 x 5 + 2



Design for 10 x 5 + 2



Short cut for number of squares	Total number of squares	Total number of blue squares	Total number of yellow squares
4 x 5 + 2	22	2 x 5 + 1 = 11	2 x 5 + 1 = 11
6 x 5 + 2	32	3 x 5 + 1 = 16	3 x 5 + 1 = 16
8 x 5 + 2	42	4 x 5 + 1 = 21	4 x 5 + 1 = 21
10 x 5 + 2	52	5 x 5 + 1 = 26	5 x 5 + 1 = 26

d. 2 501 blue squares (500 x 5 + 1)
2 501 yellow squares (500 x 5 + 1)

e. If the first set of 5 squares is blue, there is 1 more set of blue squares than there are yellow squares. There are 2 additional yellow squares: 1 at the beginning and 1 at the end of the design. So there are 501 x 5 = 2 505 blue squares and 500 x 5 + 2 = 2 502 yellow squares. (Reverse these totals if the first set of 5 squares is yellow.)

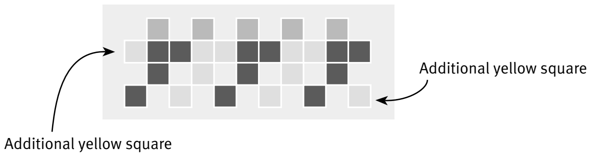
2. a.

Number of pink squares	Total number of blue and yellow squares
4	3 x 4 + 1 = 13
7	6 x 4 + 1 = 25
12	11 x 4 + 1 = 45
37	36 x 4 + 1 = 145
243	242 x 4 + 1 = 969

b.

Number of pink squares	Total number of blue and yellow squares
2	1 x 4 + 1 = 5
8	29
19	73
25	97
76	301

3. A rule is: 2 plus the number of pink squares times 4. A short cut for the frieze design shown is 2 + 5 x 4 = 22 blue or yellow squares. (Each pink square is linked to 3 + 1 = 4 blue or yellow squares. There is 1 additional yellow square at the beginning of the design and 1 additional yellow square at the end of the design.



Note that when there is an odd number of pink squares, the 2 additional squares are the same colour.)

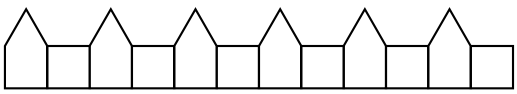
The completed table is:

Number of pink squares	Total number of blue and yellow squares
5	2 + 5 x 4 = 22
4	2 + 4 x 4 = 18
10	2 + 10 x 4 = 42
15	2 + 15 x 4 = 62
256	2 + 256 x 4 = 1 026
20	82
54	218

Pages 16–17 Stick Houses

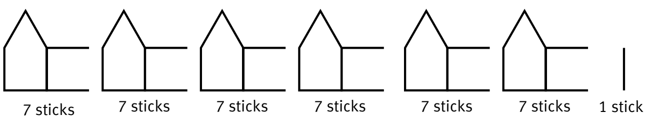
ACTIVITY

1. a.



A block of 6 houses has 6 x 7 + 1 = 43 sticks.

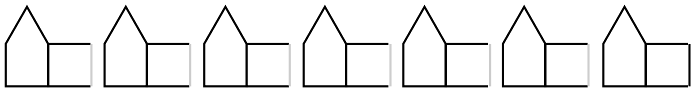
b. There are 6 sets of 7 sticks and an additional stick to complete the garage wall at the end of the block.



Number of houses	Number of sticks
4	4 x 7 + 1 = 29
5	5 x 7 + 1 = 36
10	10 x 7 + 1 = 71
35	35 x 7 + 1 = 246
83	83 x 7 + 1 = 582
156	156 x 7 + 1 = 1 093

Number of houses	Number of sticks
2	15
6	43
11	78
20	141
120	841
200	1 401

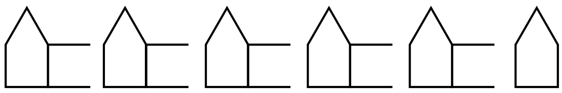
2. a. A block of 7 houses would have 7 sets of 8 sticks or 7 x 8 sticks altogether. But the 6 yellow sticks used to complete the garage walls for 6 of the 7 houses will not be needed when the houses are joined. So we subtract 6 sticks. Therefore, 7 x 8 – 6 = 50 sticks are needed.



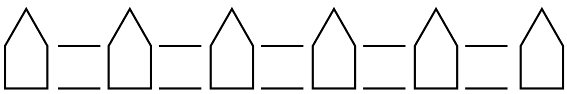
b.

Number of houses	Number of sticks
4	4 x 8 – 3 = 29
7	7 x 8 – 6 = 50
12	12 x 8 – 11 = 85
21	21 x 8 – 20 = 148
55	55 x 8 – 54 = 386
100	100 x 8 – 99 = 701

3. a. Answers will vary. A short cut for this design for a block of 6 houses is 5 x 7 + 5 = 40.



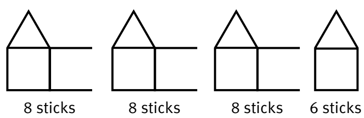
There are 5 houses using 7 sticks each and 5 sticks for the last house. Another short cut for this design is 6 x 5 + 5 x 2 = 40. There are 6 houses using 5 sticks each and 2 sticks for each of the 5 garages.



b. Answers will vary. Based on the short cuts in **a**:

Number of houses	Number of sticks	
	First rule	Second rule
4	3 x 7 + 5 = 26	4 x 5 + 3 x 2 = 26
6	5 x 7 + 5 = 40	6 x 5 + 5 x 2 = 40
10	9 x 7 + 5 = 68	10 x 5 + 9 x 2 = 68
20	19 x 7 + 5 = 138	20 x 5 + 19 x 2 = 138
87	86 x 7 + 5 = 607	87 x 5 + 86 x 2 = 607
196	195 x 7 + 5 = 1 370	196 x 5 + 195 x 2 = 1 370

4. a. Answers will vary. A short cut based on the design below is 3 x 8 + 6 = 30. There are 3 houses using 8 sticks each and 6 sticks for the last house.



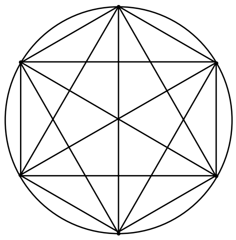
b. 99 x 8 + 6 = 798

Page 18 Web Circles

ACTIVITY

1. **a.** 10. (There are 4 red + 3 blue + 2 green + 1 brown = 10 straight lines.)

   b. 5 + 4 + 3 + 2 + 1 = 15 straight lines



   c. 190. (19 + 18 + 17 + 16 + 15 + 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 190 straight lines)

2. **a.** Answers will vary. One way is to use Ian's approach and then rearrange the numbers from 19 to 1 like this:
   (19 + 1) + (18 + 2) + ... + (12 + 8) + (11 + 9) + 10.
   The value for the sum in each of the 9 pairs of brackets is 20, or 10 + 10. There are 19 sets of 10 altogether, so the short cut is 19 x 10 = 190 straight lines.

   A simpler approach is to consider the number of points in the web circle and the number of straight lines going to each point. In the 20-point web circle, there are 19 straight lines for each point (1 to every other point), so there is a total of 20 x 19 = 380 straight lines. However, this method counts each line twice (each line gets counted at both the points that it joins), so the actual number is 380 ÷ 2 = 190. The short cut for this is 20 x 19 ÷ 2 = 190.

   b. 4 950. (Using the short cuts above, this is 99 x 50, or 100 x 99 ÷ 2.)

   c. Based on the short cuts in **b**, a rule for x points would be $(x - 1) \times \frac{1}{2}x$, or $\frac{x \times (x - 1)}{2}$.

Page 19 Marooned

ACTIVITY

1. **a.** 25 trips. (Using the rule given for question **3**, the short cut is 4 x 6 + 1 = 25.)

   b. 49 trips would be needed (4 x 12 + 1).

   c. 81 trips would be needed (4 x 20 + 1).

2. 4 fewer trips are required because each extra adult generates 4 trips.

3. A rule for any number of adults and 2 children is: 4 times the number of adults, plus 1. It takes 4 trips to get 1 adult across, plus a final trip for the 2 children.

4. 9 adults (4 x 9 + 1 = 37), which in reverse is (37 – 1) ÷ 4 = 9.

INVESTIGATION

Answers will vary. 2 extra trips are needed for each child after the first 2 children (to get them to the shore at the end). For any number of adults and 3 children, the rule is: 4 times the number of adults, plus 3. The rule for any number of adults and any number of children is: 4 times the number of adults, plus twice the number of children, minus 3.

Pages 20–21 Patterns, Rules, and Spreadsheets

ACTIVITY ONE

a. $63

b. $8

c. The savings that the formulae work out will be the same in both Savings columns:

Week	Savings ($)
0	63
1	71
2	79
3	87
4	95
5	103
6	111
7	119

d. $479

e. The calculation 8 x 75 + 63 = 663 indicates the savings after 75 weeks. That is, $8 per week for 75 weeks, plus the $63 Hannah started with.

f. **i.** 8 x 83 + 63 = $727

   ii. 8 x 147 + 63 = $1,239

g. Answers may vary, but the second method is more efficient because it calculates directly rather than using Fill Down. For example, for 52 weeks with method 1, Hannah first needs to have filled down to 51 weeks, but with the second method, she just needs to calculate 8 x the week number + 63.

ACTIVITY TWO

1. **a.** 501. (The value in cell A2 is 1, so the formula =504–3*A2 in cell B2 produces 504 – 3 x 1 = 501. The starting number is therefore 501.)

   b. 501, 498, 495, 492, 489, 486, 483, 480. (The –3 in the formula indicates that Tom is skip-counting backwards in 3s.)

   c. **i.** 414. (504 – 3 x 30)

   ii. 285. (504 – 3 x 73)

   iii. 69. (504 – 3 x 145)

2. **a.** 17. (The value in cell A2 is 1, so the formula =5+12*A2 in cell B2 produces 5 + 12 x 1 = 17.)

   b. 17, 29, 41, 53, 65, 77, 89, 101. (The +12 in =5+12*A2 indicates that Tom is skip-counting forwards in 12s.)

   c. **i.** 1 193. (5 + 12 x 99)

   ii. 3 077. (5 + 12 x 256)

   iii. 10 553. (5 + 12 x 879)

ACTIVITY THREE

1. **a.**

Number Sequence 23, 30, 37, _ (SS)

B2 =16+7*A2

	A	B	C
1	Position of number	Number	
2	1	23	
3	2	30	
4	3	37	
5	4	44	
6	5	51	
7	6	58	
8	7	65	
9	8	72	

The spreadsheet above shows the first 8 numbers and the formula =16+7*A2 to calculate the first number, 23, in cell B2.
(The formulae used may vary, but the sequence of numbers will be the same.)
Successive numbers increase by 7.

The formula =16+7*A3 goes in cell B3 to calculate the 2nd number, the formula =16+7*A4 goes in cell B4 for the 3rd number, and so on.

   b. **i.** 275. (16 + 7 x 37)

   ii. 1 325. (16 + 7 x 187)

   iii. 8 969. (16 + 7 x 1 279)

2. **a.**

Sequence 10001, 9992, 9983, (SS)

B2 =10010-9*A2

	A	B	C
1	Position of number	Number	
2	1	10001	
3	2	9992	
4	3	9983	
5	4	9974	
6	5	9965	
7	6	9956	
8	7	9947	
9	8	9938	

Formulae may vary. The spreadsheet above shows the first 8 numbers and the formula =10010–9*A2 to calculate the first number, 10 001, in cell B2. Successive numbers decrease by 9.

The formula =10010–9*A3 goes in cell B3 to calculate the 2nd number, the formula =10010–9*A4 goes in cell B4 for the 3rd number, and so on.

   b. **i.** 9 506. (10 010 – 9 x 56)

   ii. 9 263. (10 010 – 9 x 83)

   iii. 1 019. (10 010 – 9 x 999)

ACTIVITY FOUR

a.

Gardening Services (SS)

C2 =25+13*A2

	A	B	C
1	Number of hours	Mika's charge ($)	Hine's charge ($)
2	0	13	25
3	1	30	38
4	2	47	51
5	3	64	64
6	4	81	77
7	5	98	90
8	6	115	103
9	7	132	116

(The formula in cell C2 is =25+13*A2. The value in cell A2 is 0, so the formula calculates the value in cell C2 as 25. This is Hine's travelling charge of $25.)

b. The charges for 3 hrs are the same, $64. For any time less than 3 hrs, Mika's charge is less than Hine's. For any time greater than 3 hrs, Hine's charge is less than Mika's.

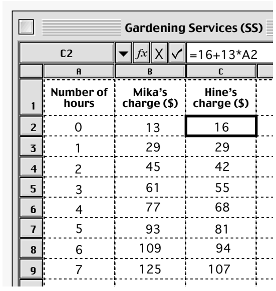
c.

Gardening Services (SS)

B2 =13+16*A2

	A	B	C
1	Number of hours	Mika's charge ($)	Hine's charge ($)
2	0	13	25
3	1	29	38
4	2	45	51
5	3	61	64
6	4	77	77
7	5	93	90
8	6	109	103
9	7	125	116

The formula in cell B2 changes to =13+16*A2. Charges for 4 hrs are now the same, $77. For any time less than 4 hrs, Mika's charge is less than Hine's. For any time greater than 4 hrs, Hine's charge is less than Mika's.

d. 

Gardening Services (SS)

C2 =16+13*A2

	A	B	C
1	Number of hours	Mika's charge ($)	Hine's charge ($)
2	0	13	16
3	1	29	29
4	2	45	42
5	3	61	55
6	4	77	68
7	5	93	81
8	6	109	94
9	7	125	107

The formula in cell C2 changes to =16+13*A2. Now the charge for both gardeners' services for 1 hr is the same, $29. For more than 1 hr, Hine's charge is less than Mika's.

Page 22 Car Journeys

ACTIVITY

1. Margot's family left Tauranga at 9.00 a.m. Their journey took 7 hrs, 30 min. Jenny's family left Tauranga at 10.15 a.m. Their journey took approximately 6 hrs.

2. a. i. The drink stop was at 10.30 a.m. Lunch and a swim was from 11.45 to 1.30 p.m. The petrol stop was at 3.00 p.m.

ii. The drink stop took 30 min. Lunch and a swim took 1 hr 45 min. The petrol stop took 15 min.

b. The slowest part of the journey for Margot's family occurred between 3.15 and 4.30 p.m. The graph between these times is less steep than for any other part of the graph except for during stops. The less steep the graph, the lower the speed. (They covered 50 km in 1 hr and 15 min, and their speed during this period was 40 km/h.)

3. Jenny's car passed Margot's car at approximately 12.30 p.m. and 3.45 p.m. (Margot's car passed Jenny's car at approximately 2.40 p.m.)

4. a. 100 km/h. (They travelled 100 km in 1 hr.)

b. Approximately 67 km/h. (They travelled 100 km in $1\frac{1}{2}$ hrs.)

5. Answers will vary.

Page 23 Surfboard Sums

ACTIVITY

1. a. $700

b. The value of the surfboard is continuously decreasing.

2. a.

Time from new (months)	Value of surfboard ($)
0	700
3	500
6	450
9	400
12	350
15	325
18	300
21	275
24	250

b. i. During the first 3 months

ii. In the 3 month periods between 12 and 24 months

c. During the first 3 months, the value decreases by $200 in 3 months or approximately $67 per month. Between 12 and 24 months, the value decreases by $25 in each 3 month period or approximately $8 per month.

3. Probably about $225. The gradual flattening of the graph suggests the value of the surfboard will not decrease by an amount greater than that for the previous 3 months, that is, $25. So, if its value at 24 months is $250, then by 27 months, its value is likely to be at least $225.

4. 30 months is the latest time that Vinny could sell his surfboard for $200 or more. If the surfboard's value at 27 months is $225, it is unlikely to fall in value by more than $25 in the next 3 months.

Page 24 Holiday Pay

ACTIVITY

1. a.

Day	Offer 1 Daily pay ($)	Offer 1 Cumulative total ($)	Offer 2 Daily pay ($)	Offer 2 Cumulative total ($)
1	30	30	5	5
2	30	60	10	15
3	30	90	15	30
4	30	120	20	50
5	30	150	25	75
6	30	180	30	105
7	30	210	35	140
8	30	240	40	180
9	30	270	45	225
10	30	300	50	275
11	30	330	55	330
12	30	360	60	390
13	30	390	65	455
14	30	420	70	525
15	30	450	75	600

b. See completed graph for question **3a** below.

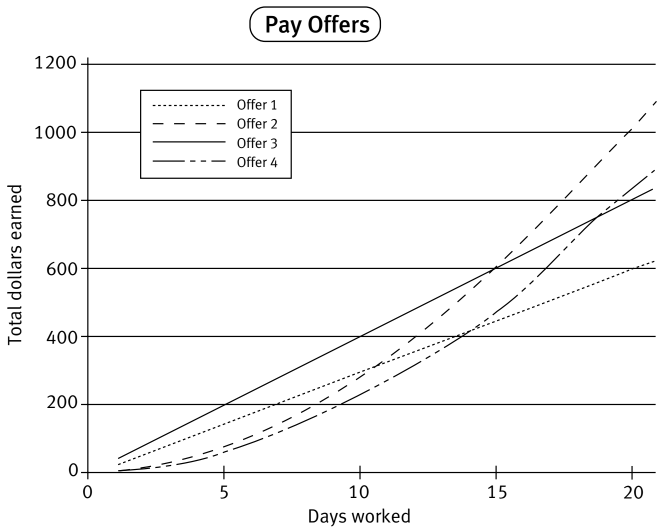
c. For 10 days, offer 1 is best. (Jane could make $300 with offer 1 and $275 with offer 2.)

d. For 11 days, Jane can make $330 with either offer.

2. For 3 weeks (15 days), offer 2 is best. Jane could make $600 with offer 2 and $450 with offer 1.

3. a.

Day	Offer 3 Daily pay ($)	Offer 3 Cumulative total ($)	Offer 4 Daily pay ($)	Offer 4 Cumulative total ($)
1	40	40	4	4
2	40	80	8	12
3	40	120	12	24
4	40	160	16	40
5	40	200	20	60
6	40	240	24	84
7	40	280	28	112
8	40	320	32	144
9	40	360	36	180
10	40	400	40	220
11	40	440	44	264
12	40	480	48	312
13	40	520	52	364
14	40	560	56	420
15	40	600	60	480



(The graph above has been extended to 20 days so that it can be used in **3b**.)

Reports will vary, but the following points should be noted:

A comparison of all four offers shows that offers 2 and 3 would both give Jane the same pay for 15 days, and both are more than the other offers. After 15 days, offer 1 would be $450, offers 2 and 3 would be $600, and offer 4 would be $480.

b.

Day	Offer 1 Daily pay ($)	Offer 1 Cumulative total ($)	Offer 2 Daily pay ($)	Offer 2 Cumulative total ($)
15	30	450	75	600
16	30	480	80	680
17	30	510	85	765
18	30	540	90	855
19	30	570	95	950
20	30	600	100	1 050

Day	Offer 3 Daily pay ($)	Offer 3 Cumulative total ($)	Offer 4 Daily pay ($)	Offer 4 Cumulative total ($)
15	40	600	60	480
16	40	640	64	544
17	40	680	68	612
18	40	720	72	684
19	40	760	76	760
20	40	800	80	840

Jane is unlikely to be offered the choice of offer 2 for 4 weeks, as she would earn $450 in the fourth week.

It would take 19 days for offers 3 and 4 to be equal ($760), but in day 20, offer 4 would give $840 compared with $800 for offer 3. However, by day 18, Jane could make $855 from offer 2, which exceeds the best that she could make in 20 days from offer 4 ($840). By day 20, offer 2 would give $1,050 and offer 1 would give only $600.

Teachers' Notes

Overview — Algebra: Book Three (level 4)

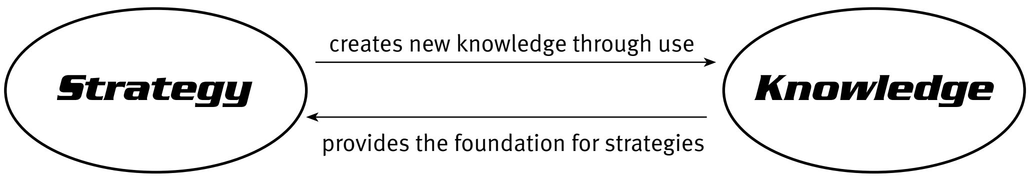
Introduction to Algebra

Teaching and learning algebra has always posed difficulties for teachers and their students. Even today, there is no consensus about when it should be introduced and exactly what should be included in an introduction to algebra. Historically, algebra has formed an important part of the secondary curriculum. However, its inclusion as a strand in the national curriculum statement, *Mathematics in the New Zealand Curriculum,* has meant that teachers at all levels have been grappling with what should be taught at these levels. Internationally, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum.

One view is that algebra is an extension of arithmetic. Another view is that it is a completion of arithmetic. Some argue that algebra begins when a set of symbols is chosen to stand for an object or situation. Others argue that all basic operations are algebraic in nature: for example, the underlying structure of a part-whole mental strategy for adding 47 to 36 is algebraic because it may involve seeing 47 + 36 as 47 + 33 + 3, giving 50 + 30 + 3 and then 83, and such mental action constitutes algebraic thinking, in spite of the absence of algebraic-looking symbols.

The Figure It Out series aims to reflect the trends in modern mathematics education. So this series promotes the notion of algebraic thinking in which students attend to the underlying structure and relationships in a range of mathematical activities. While the student material includes only limited use of algebraic symbols, the teachers' notes show how mathematical ideas formulated in words by learners can be transformed into symbolic form. Teachers are encouraged to introduce the use of symbols in cases where they themselves feel comfortable and where they think that their students are likely to benefit. It is intended that with this book (Book Three), increasing numbers of students will begin to work with algebraic symbols.

The basis for the material in the students' books is consistent with the basis of the Number Framework, which highlights the connections between strategies students use to explore new situations and the knowledge they acquire.



To help students develop sensible strategies or short cuts for working with new mathematical situations, the activities encourage them to create their own visual and pictorial images to represent mathematical ideas and relationships. These strategies can then be applied to a range of similar, as well as new, mathematical situations.

There are four *Algebra* books in this series for year 7–8 students:
Link (Book One)
Level 4 (Book Two)
Level 4 (Book Three)
Level 4+ (Book Four)

Page 1

Save Some, Spend Some

Achievement Objectives

- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

In this activity, students interpret, make, and solve problems using spreadsheets.

Note that there may be slight variations in how spreadsheets make the calculations in selected cells. Some require direct use of Fill Down in the Calculate menu while others automatically complete the calculations as soon as the appropriate cells are selected or allow the use of Copy and Paste commands. Even if students have had experiences with spreadsheets in other books in the Figure It Out series, you will probably need to work with them as they carry out these tasks.

In questions **3** and **4**, the students see the effect of making increased savings. Sarah makes changes to her savings pattern after 7 weeks when she stops purchasing her magazine at a cost of $4 a week. So the formula changes from =C8+6 in cell C9 for the savings after 7 weeks to =C9+10 in cell C10 for the savings after 8 weeks. The effect of this change is significant. The new savings plans proposed in questions **3** and **4** for both Daniel and Sarah are shown below. The first spreadsheet shows the formulae that calculate the values shown in the second spreadsheet.

Saving for a DVD (SS)

C3 =C2+6

	A	B	C	D
1	Week	Daniel's savings	Sarah's savings	
2	0	100	124	
3	=A2+1	=B2+8	=C2+6	
4	=A3+1	=B3+8	=C3+6	
5	=A4+1	=B4+8	=C4+6	
6	=A5+1	=B5+11	=C5+6	
7	=A6+1	=B6+11	=C6+6	
8	=A7+1	=B7+11	=C7+6	
9	=A8+1	=B8+11	=C8+6	
10	=A9+1	=B9+11	=C9+10	
11	=A10+1	=B10+11	=C10+10	
12	=A11+1	=B11+11	=C11+10	
13	=A12+1	=B12+11	=C12+10	
14	=A13+1	=B13+11	=C13+10	
15	=A14+1	=B14+11	=C14+10	
16	=A15+1	=B15+11	=C15+10	
17	=A16+1		=C16+10	

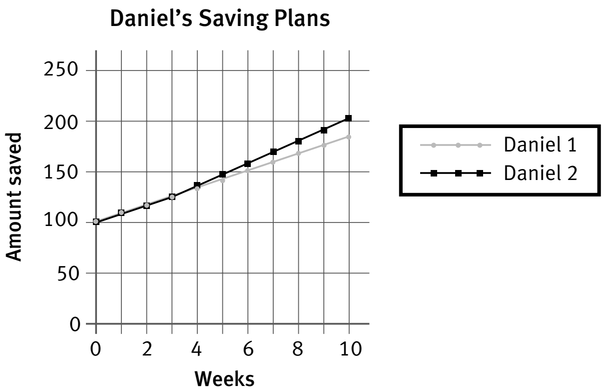
→

Saving for a DVD (SS)

C3 =C2+6

	A	B	C	D
1	Week	Daniel's savings	Sarah's savings	
2	0	100	124	
3	1	108	130	
4	2	116	136	
5	3	124	142	
6	4	135	148	
7	5	146	154	
8	6	157	160	
9	7	168	166	
10	8	179	176	
11	9	190	186	
12	10	201	196	
13	11	212	206	
14	12	223	216	
15	13	234	226	
16	14	245	236	
17	15		246	

As an extension exercise, the students may wish to graph the differing savings plans in order to highlight what is going on. For example, the two scenarios presented for Daniel's savings can be represented as shown:



Sarah's savings plans could then be shown in another colour on the same graph or on a separate graph.

Pages 2–3 Table Mats

Achievement Objectives

- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students find short cuts and rules to work out the number of circles on the borders of different table mats. The designs referred to in the following notes are shown in the student book or in the Answers.

In question **1**, the short cuts for the number of circles on the border of each table mat designed by Hine lead to a rule that can be applied generally. The rule is: any table mat has 4 (blue) sets of 2 fewer circles than the number of circles on a side, plus 4 corner circles (yellow). This rule can be expressed algebraically: a table mat with x circles on a side has $4 \times (x - 2) + 4$ circles or $4(x - 2) + 4$ altogether.

In question **2**, Larissa's table mat designs lead to a different rule: any table mat has 4 sets of 1 fewer circles than the number of circles on a side. So a table mat with x circles on a side has $4 \times (x - 1)$ or $4(x - 1)$ circles altogether.

In question **4**, another rule for Hine's new table mat design, in addition to the one given in the Answers, is: any table mat has 2 sets of the number of blue circles on a side, plus 2 sets of 2 circles fewer than the number of blue circles on a side. So any of Hine's new table mat designs with x circles on a side has $2 \times x + 2 \times (x - 2)$ or $2x + 2(x - 2)$ circles altogether.

In question **5**, the short cut for Larissa's new table mat design with 5 circles on each side is given as $4 \times 5 - 4$. While there are 5 circles on each side, each corner circle is included twice in this count. So we subtract 4 corner circles to correct this double counting.

These short cuts lead to a rule that can be expressed in words and also algebraically: a table mat with x circles on each side has $4 \times x - 4$ circles or $4x - 4$ altogether.

While the rules arising from the short cuts above are different, they all produce the same results for particular table mats.

Number of circles on side	Number of circles using rule			
	$4(x - 2) + 4$	$4(x - 1)$	$2x + 2(x - 2)$	$4x - 4$
3	4 x (3 – 2) + 4 = 8	4 x (3 – 1) = 8	2 x 3 + 2 x (3 – 2) = 8	4 x 3 – 4 = 8
10	4 x (10 – 2) + 4 = 36	4 x (10 – 1) = 36	2 x 10 + 2 (10 – 2) = 36	4 x 10 – 4 = 36
27	4 x (27 – 2) + 4 = 104	4 x (27 – 1) = 104	2 x 27 + 2 x (27 – 2) = 104	4 x 27 – 4 = 104
98	4 x (98 – 2) + 4 = 388	4 x (98 – 1) = 388	2 x 98 + 2 x (98 – 2) = 388	4 x 98 – 4 = 388

Most students will not be ready to use these rules involving symbolic algebra. It will be enough that they see how the different short cuts arise for particular table mats, that is, that they can relate the short cuts to the different table mat designs and can use them to predict the number of circles for table mats of any size.

More able students might like to attempt to show the algebraic equivalence of the different rules. The first three rules all reduce to the fourth rule, $4x - 4$. The equivalences are shown below:

Rule 1: $4(x - 2) + 4 = 4x - 8 + 4$
$= 4x - 4$

Rule 2: $4(x - 1) = 4x - 4$

Rule 3: $2x + 2(x - 2) = 2x + 2x - 4$
$= 4x - 4$

Pages 4–5

Stacking Up

Achievement Objectives

- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, the students must first work out the height that each additional container, hat, CD, chair, or desk contributes to the height of the respective stack. Explanations for each object's height are outlined in the Answers. The table below shows short cuts for working out the height of stacks with different numbers of containers. Students might make a table like this one to help them figure out how the rule works for stacks with any number of containers.

Number of containers in stack	Short cut to work out height of stack	Height of stack (cm)
2	29 + 1 x 9	38
4	29 + 3 x 9	56
5	29 + 4 x 9	65
17	29 + 16 x 9	173
89	29 + 88 x 9	821

A rule for the height of a stack with any number of containers is the height of the first container, 29 centimetres, plus 9 centimetres for each of the other containers in the stack. So for a stack with x containers, the height, y, is $y = 29 + (x - 1) \times 9$. This can be expressed as $y = 29 + 9(x - 1)$ or, more simply, as $y = 29 + 9x - 9$ or $20 + 9x$. This is also $y = 9x + 20$. Students who grasp this symbolic algebra may like to use this rule to check the values in the table above. A table using $y = 9x + 20$ is shown here.

x	$y = 9x + 20$
2	9 x 2 + 20 = 38
4	9 x 4 + 20 = 56
5	9 x 5 + 20 = 65
17	9 x 17 + 20 = 173
89	9 x 89 + 20 = 821

The table below shows short cuts for working out the height of stacks with different numbers of hats in question **2**. As with the containers in question **1**, students may find that a table like the one below will help them figure out how the rule works for stacks with any number of hats.

Number of hats in stack	Short cut to work out height of stack	Height of stack (cm)
4	18 + 3 x 7	39
10	18 + 9 x 7	81
12	18 + 11 x 7	95
25	18 + 24 x 7	186
63	18 + 62 x 7	452
127	18 + 126 x 7	900

A rule for the height of a stack with any number of hats is the height of the top hat, 18 centimetres, plus 7 centimetres for each of the other hats in the stack. This can be expressed symbolically as $y = 18 + (x - 1) \times 7$ or $y = 18 + 7(x - 1)$. This can be simplified to $y = 18 + 7x - 7$ or $y = 7x + 11$. The following table shows different algebraic expressions for the height, y, of stacks with x hats.

x	$y = 7x + 11$
4	7 x 4 + 11 = 39
10	7 x 10 + 11 = 81
12	7 x 12 + 11 = 95
25	7 x 25 + 11 = 186
63	7 x 63 + 11 = 452
127	7 x 127 + 11 = 900

Students completing such a table of values successfully will see that symbolic algebra provides a simple way to figure out the height of any stack of hats.

In question **3**, a short cut for the height of 25 CDs is given in the Answers as 25 x 2 + 7 = 57 centimetres. The height, y, of a rack with x CDs is then $y = x \times 2 + 7$ or $y = 2x + 7$.

Students may approach question **3b** in several ways. The most straightforward is to reason as follows: The rack is 97 centimetres tall, but we know that the frame takes up 7 centimetres of this, so the CDs take up 90 centimetres. Each CD requires 2 centimetres, so there must be 90 ÷ 2 = 45 CDs.

Students who are comfortable with algebraic manipulation might be interested to see how we can adapt the rule found above ($y = 2x + 7$) to solve question **3b**. In this question, we are told that $y = 97$ and are asked to find x, the number of CDs. Algebraically, we are told that $97 = 2x + 7$. We can then reason as follows to find x: $90 = 2x$ (subtracting 7 from each side), so $45 = x$ (dividing both sides by 2), which is the answer we found above.

In question **4**, the students will need to make their own measurements. Students who have shown skill in working with algebraic symbols should be encouraged to devise and then test their own algebraic expressions for the height of stacks with any number of chairs or desks.

Pages 6–7 Design Day

Achievement Objectives

- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

In this activity, students identify, use, and explain rules based on different patterns of colour for the number of squares on the edge of designs that are based on patiki patterns.

In question **1**, George's designs have changing numbers of green squares. Short cuts and rules for these and the others in this activity are given in the Answers.

Students who successfully complete the table in question **1d** might see if they can write an algebraic expression for the number of painted squares in any of George's designs. For example, the xth design will have 4 sets of x green squares and 4 blue squares. So the number of painted squares, y, in the xth design is $y = 4 \times x + 4$ or $y = 4x + 4$. The thousandth design will therefore have $4 \times 1\,000 + 4 = 4\,004$ painted squares.

In question **2**, Kelly uses a different pattern for her painted squares. Students using algebra to write a rule for Kelly's designs might write $y = 2 \times (x + 1) + 2 \times (x + 1)$ for the number of painted squares in the xth design. This can be written more simply as $y = 2(x + 1) + 2(x + 1)$, which can be simplified further to $y = 4(x + 1)$ or even $y = 4x + 4$. Notice that this last algebraic rule is the same as the algebraic rule for George's designs. This is to be expected, as the number of squares being painted is the same for both designs.

In question **3**, George changes his pattern from 2 colours to 3. An algebraic rule for George's new designs using 3 colours is as follows: If y is the number of painted squares in the xth design, then $y = 2 \times (x + 1) + 2 \times x + 2$. This can be written more simply as $y = 2(x + 1) + 2x + 2$, which can be simplified further to $y = 2x + 2 + 2x + 2$, which is really just $y = 4x + 4$. Again, this last algebraic rule is the same as the algebraic rule for George's first designs as well as for Kelly's designs.

Page 8 Kidding Around

Achievement Objectives

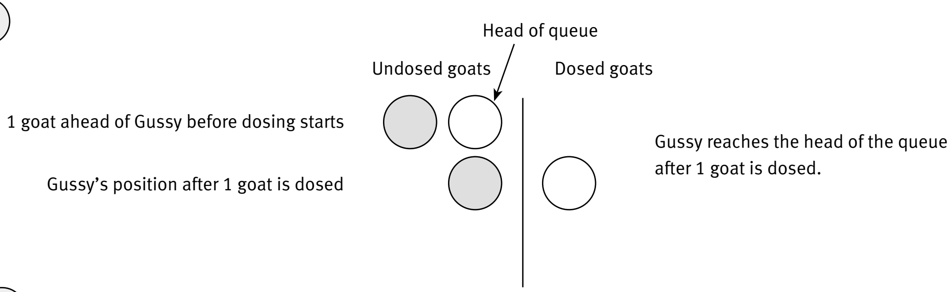
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

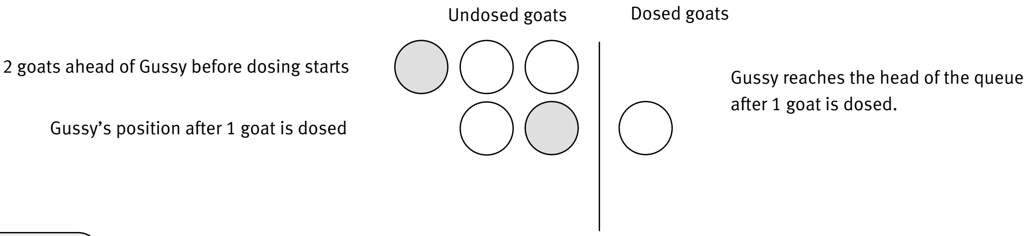
In this activity, students work to identify patterns based on groupings of a particular size.

The students will probably need to systematically explore this problem using counters to represent goats. The counter for Gussy should be a different colour from the counters for the other goats. The students should check what happens when there is 1 goat ahead of Gussy, then 2, then 3, and so on, up to at least 13 goats ahead of Gussy. In the diagrams below, Gussy is the shaded counter.

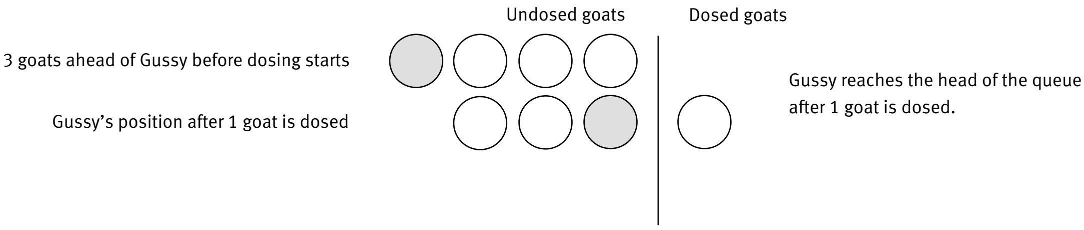
1 goat ahead of Gussy



2 goats ahead of Gussy

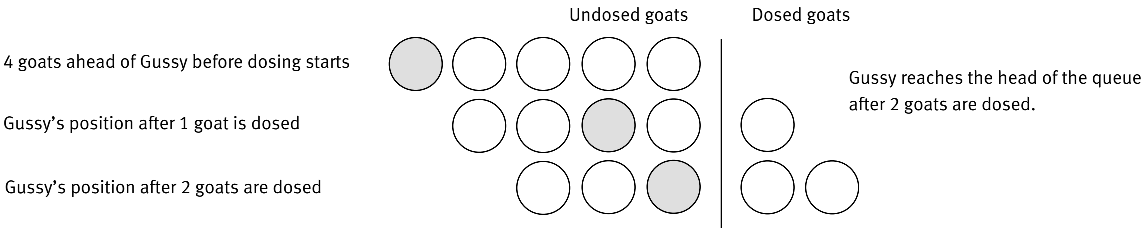


3 goats ahead of Gussy



Notice that Gussy reaches the head of the queue after just 1 goat is dosed regardless of whether there are 1, 2, or 3 goats ahead of her. When there are 4 goats ahead of her, 2 goats will be dosed before Gussy reaches the head of the queue.

4 goats ahead of Gussy



In fact, this is the outcome whether there are 4, 5, or 6 undosed goats ahead of her.

The following table shows what happens when there are 1, 2, 3, … undosed goats in front of Gussy:

Number of goats in front of Gussy	Number of goats dosed before Gussy
1	1
2	1
3	1
4	2
5	2
6	2
7	3
8	3
9	3
10	4
11	4
12	4
13	5
14	5
15	5
16	6
17	6
18	6
19	7
20	7
21	7
22	8
23	8
24	8

As noted in the Answers, when the number of goats in front of Gussy is a multiple of 3 (shaded rows in the table), the number of goats dosed before Gussy is the number of undosed goats in front of Gussy divided by 3. So, for example, when there are 18 undosed goats ahead of Gussy, the number of goats dosed before Gussy is $18 \div 3 = 6$.

When the number of undosed goats in front of Gussy is not a multiple of 3, round that number up to the next multiple of 3 and divide the rounded number by 3. So, for example, when there are 16 undosed goats ahead of Gussy, first round 16 up to 18. Then the number of goats dosed before Gussy is $18 \div 3 = 6$. When there are 100 undosed goats ahead of Gussy, round 100 to 102 because 100 is not a multiple of 3 and 102 is the first multiple of 3 after 100. Then the number of goats dosed before Gussy is $102 \div 3 = 34$.

The following table shows what happens when Gussy sneaks past 4 goats ahead of her instead of 2, each time a goat is dosed:

Number of goats in front of Gussy	Number of goats dosed before Gussy
1	1
2	1
3	1
4	1
5	1
6	2
7	2
8	2
9	2
10	2
11	3
12	3
13	3
14	3
15	3
16	4
17	4
18	4
19	4
20	4

Here, the number of goats dosed before Gussy changes immediately after each multiple of 5 rather than after each multiple of 3 as earlier.

The students may be puzzled that we look at multiples of 3 and 5 when the number of goats that Gussy is sneaking past is 2 and 4 respectively. The reason for doing this is clear once we realise that whenever Gussy sneaks past 2 (or 4) goats, another goat is being dosed, so that really we need to consider the goats in groups of 3 (or 5).

Students who manage to reason this outcome may like to see if they can generalise the results when Gussy sneaks past 5, 6, … 100 undosed goats ahead of her. In fact, when Gussy sneaks past any number, x, of undosed goats ahead of her, the number of goats dosed before Gussy changes immediately after each multiple of $x + 1$ undosed goats ahead of her.

Page 9 Stepping Stones

Achievement Objectives

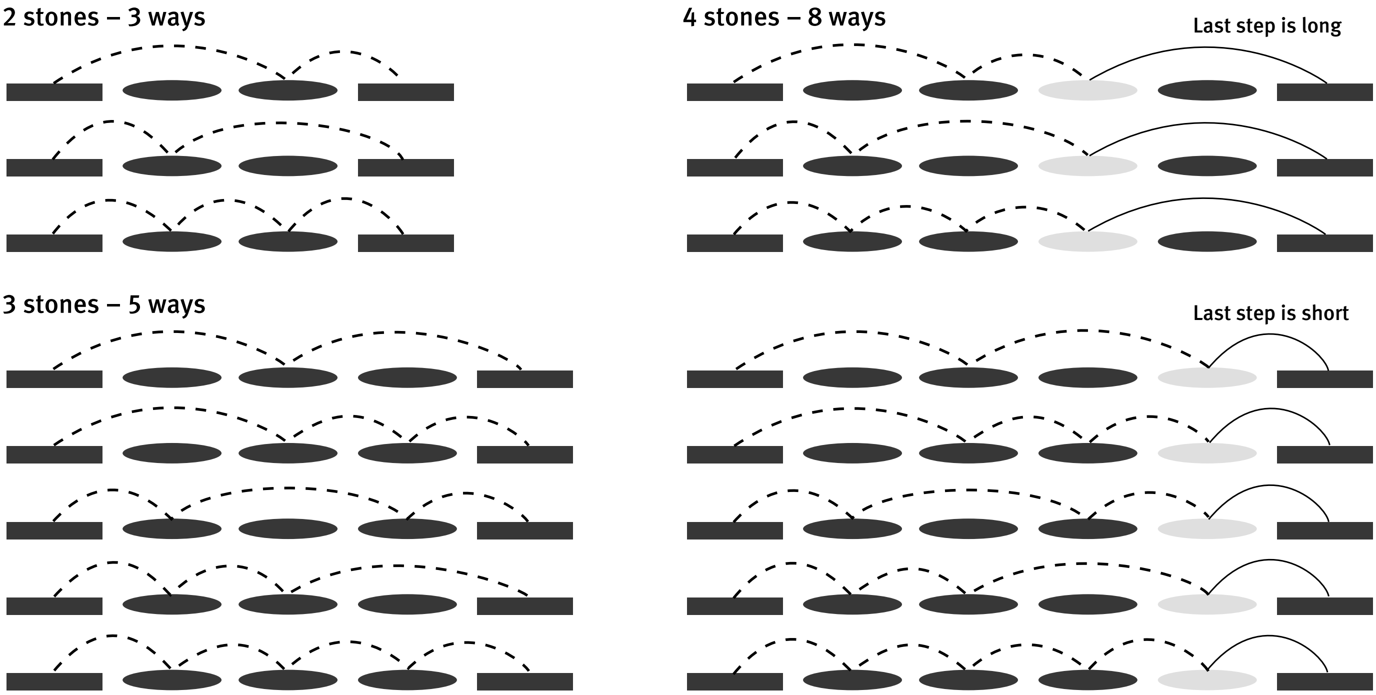
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

In this activity, students must work systematically to investigate the different ways that the stream can be crossed when there are 0, 1, 2, 3, 4, … stepping stones. The number sequence, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, … that emerges is known as the Fibonacci sequence, where each number is the sum of the two preceding numbers. There is no simple way of applying this as a rule for a particular stepping stone, but the students should be able to spot the pattern.

When the students have found the pattern (question **4**) that gives the number of ways a stream with n stepping stones may be crossed, encourage them to think about why the pattern works.

The 4-stepping-stone case is considered in the diagram below (the same argument can be applied to any other number of stones).



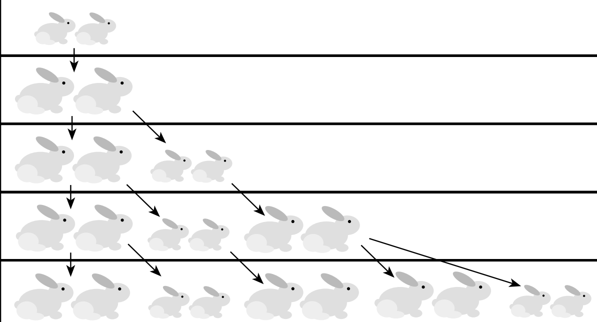
In order to get to the far side, the last step must be either a long step from stone 3 or a short step from stone 4 (these are the only options). There are 3 ways to get to stone 3, so there are 3 ways to get to the far side via a long step from stone 3. There are 5 ways to get to stone 4, so there are 5 ways to get to the far side via a short step from stone 4. Because these are all the ways to get to the far side, there is a total of 3 + 5 = 8 ways to cross a stream with 4 stones.

Note that the number of ways (3) to get to the third stone is the same as the number of ways to cross a stream with 2 stones. Similarly, the number of ways (5) to get to the fourth stone is the same as the number of ways to cross a stream with 3 stones.

Leonardo Fibonacci’s number sequence appeared as a solution to a problem in his book *Liber Abacci*, which was based on the arithmetic and algebra that Fibonacci accumulated during his many travels in Europe and North Africa.

The problem and its solution are as follows.
A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair that from the second month on becomes productive?

Students who are familiar with rabbits may query the productivity at 2 months, the lack of any deaths, and the licence taken in assuming that female rabbits always give birth to a single male/female pair. However, the gestation period (the time carried in its mother’s womb) for rabbits is accurately given as 30 days (1 month), and the problem is still valid in terms of the mathematics involved. A solution to this original Fibonacci problem is shown in the following table.

Months	Rabbit population growth	Number of pairs of rabbits
0		1
1		1
2		2
3		3
4		5

The number of pairs of rabbits after any month is the sum of the pairs of rabbits at the end of the 2 preceding months. So, after 5 months, there are 3 + 5 = 8 pairs of rabbits, after 6 months there are 5 + 8 = 13 pairs of rabbits, after 12 months, there are 89 + 144 = 233 pairs of rabbits, and so on.

For more on the rabbit problem and much more on Fibonacci numbers, see:
www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
Another interesting site devoted to Fibonacci is:
www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Fibonacci.html

Pages 10–11 Letter Designs

Achievement Objectives

- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students base their short cuts and rules on the pattern of coloured counters in the designs.

In question **1**, students check Leakhana's prediction by making the L design with counters. The design should have 6 counters of the same colour in each side and 1 counter of a different colour for the corner. This arrangement will then match Leakhana's prediction of 2 x 6 + 1 = 13 for the number of counters in the L design.

As the students work out the short cuts for the total number of counters for different L designs in the table in question **1c**, they generalise a rule for any L design. It is always challenging for students to write these rules in their own words. One way to write the rule is: find 1 fewer than the number of counters in a side, double the value of this number, then add 1. The students might also try to use algebraic symbols. For example, if there were x counters in each side of an L design, the total number of counters, y, in the design is given by $y = 2 \times (x - 1) + 1$ or $y = 2(x - 1) + 1$. Students who manage this should always check that the rule works by trying different values for x. So, when $x = 87$ (the number of counters in each side), the total number of counters in the design is 2 x (87 – 1) + 1 = 173.

In question **2**, the students may find it helpful if they make an L design using the colour pattern shown in the examples. It is more likely that they will then see that an L design with, say, 68 counters in each side will have 68 + 67 counters altogether. This process of generalising is central to the understanding of introductory algebra. So, if there were x counters in each side of an L design, the total number of counters, y, in the design is $y = x + (x - 1)$.

A third way to generalise the L design is developed in question **3**. There are 6 counters in each of the 2 sides of the L. While this may suggest that there are 2 x 6 = 12 counters altogether in the design, the corner counter is counted twice. So we subtract a counter to give 2 x 6 – 1 = 11 counters.

This way of visualising the arrangement of counters in these L designs leads to the algebraic expression $y = 2 \times x - 1$ or $y = 2x - 1$. Here, y is the total number of counters in a design with x counters on each side of the L.

The L design algebraic rules from questions **1–3** are shown in this table:

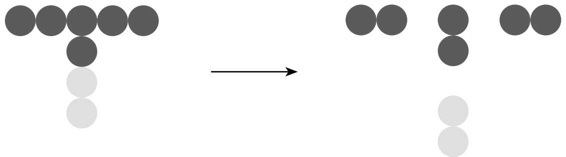
Question	Rule
1	$y = 2(x - 1) + 1$
2	$y = x + (x - 1)$
3	$y = 2x - 1$

The rules are, in fact, equivalent.
The rules from questions **1** and **2** simplify to $y = 2x - 1$ as follows:

$$\begin{aligned} y &= 2(x - 1) + 1 \\ &= (2 \times x - 2 \times 1) + 1 \\ &= 2x - 2 + 1 \\ &= 2x - 1 \end{aligned}$$

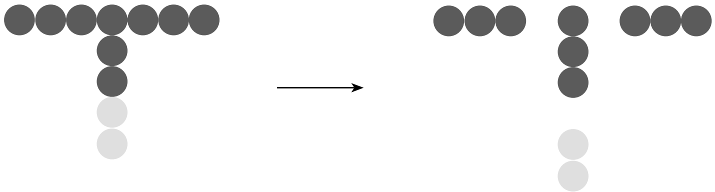
$$\begin{aligned} y &= x + (x - 1) \\ &= x + x - 1 \\ &= 2x - 1 \end{aligned}$$

In question **4**, each T design has 2 green counters and 3 sets of blue counters. For example, the T design with 4 counters in the stem can be visualised in the following way:



We see this as 3 sets of 2 blue counters plus 2 green counters or 3 x 2 + 2 counters altogether.

Similarly, the T design with 5 counters in the stem can be visualised as 3 sets of 3 blue counters and 2 green counters or 3 x 3 + 2 counters altogether.



Visualising this way leads to the generalisation $y = 3 \times (x - 2) + 2$, where x is the number of counters in the stem of the T and y is the total number of counters. This is usually written as $y = 3(x - 2) + 2$ and is the essence of the rule that the students use to complete the table for question **4b**. This algebraic rule simplifies to $y = 3x - 4$ as follows:

$$
\begin{aligned}
y &= 3(x - 2) + 2 \\
&= (3 \times x - 3 \times 2) + 2 \\
&= 3x - 6 + 2 \\
&= 3x - 4
\end{aligned}
$$

Students who can manage this algebra can confirm their work by completing the calculations shown in this table:

x	$3(x - 2) + 2$	$3x - 4$
4	3 x (4 – 2) + 2 = 8	3 x 4 – 4 = 8
7	3 x (7 – 2) + 2 = 17	3 x 7 – 4 = 17
11	3 x (11 – 2) + 2 = 29	3 x 11 – 4 = 29
32	3 x (32 – 2) + 2 = 92	3 x 32 – 4 = 92
71	3 x (71 – 2) + 2 = 209	3 x 71 – 4 = 209
478	3 x (478 – 2) + 2 = 1 430	3 x 478 – 4 = 1 430

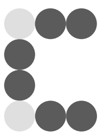
They should see that the two expressions are equivalent and that both give the same answer for any particular values of x.

In question **5**, the students see if they can find and explain other rules for T designs. An example of a different T design, with a short cut of 4 + 2 x 2 = 8 counters, is shown in the Answers. For this example, a T design with x counters in the stem has x green counters and 2 sets of $x - 2$ blue counters. If there are y counters altogether, then $y = x + 2(x - 2)$ or $y = 3x - 4$, as in question **4** (see above) and question **6** (see below).

In question **6**, the students see if they can find their own rules for Cathy's C designs, using designs made in one or more colours. Some rules and short cuts for 100 counters on the left side are given in the Answers. Those who have shown that they can manage working with symbolic algebra might try to figure out ways to represent their designs algebraically. The designs in the Answers are all shown with 4 counters on the left side. Three possible solutions for these are shown below.

Solution One

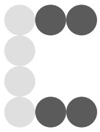
This C design has 3 x 2 (blue) + 2 green counters.



An algebraic rule for this design is $y = 3(x - 2) + 2$, where x is the number of counters on the left side and y is the total number of counters used.

Solution Two

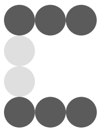
This C design has 4 green + 2 x 2 blue counters.



An algebraic rule for this design is $y = x + 2(x - 2)$.

Solution Three

This C design has 2 green + 2 x 3 blue counters.



An algebraic rule for this design is $y = (x - 2) + 2(x - 1)$.

The three different algebraic rules for the designs above all simplify to $y = 3x - 4$:

$$\begin{aligned} y &= 3(x - 2) + 2 \\ &= 3x - 6 + 2 \\ &= 3x - 4 \end{aligned} \qquad \begin{aligned} y &= x + 2(x - 2) \\ &= x + 2x - 4 \\ &= 3x - 4 \end{aligned} \qquad \begin{aligned} y &= (x - 2) + 2(x - 1) \\ &= x - 2 + 2x - 2 \\ &= 3x - 4 \end{aligned}$$

Pages 12–13 Problem Smorgasbord

Achievement Objectives

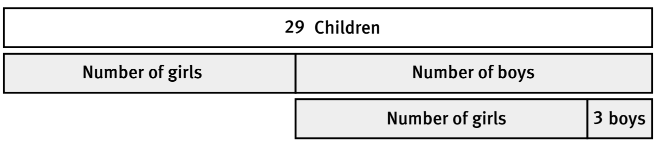
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

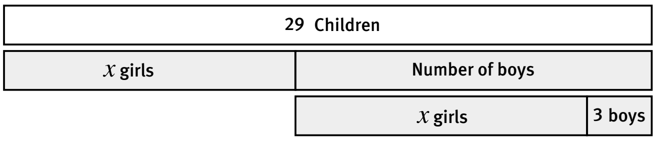
In this activity, students must use a variety of problem-solving strategies.

In question **1**, the students might use a trial-and-improvement strategy to work out that there are 13 girls and 16 boys. (10 girls would mean 19 boys, 12 girls would mean 17 boys, and so on.)

An alternative approach is to visualise the problem like this:

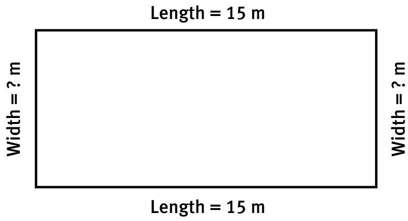


The diagram or model shows that the number of boys equals the number of girls plus 3. This means that twice the number of girls would give 26 children. So there are 13 girls. This model is only a small step away from using algebra for equations.



When there are x girls, there are $x + 3$ boys. So we can write an equation for the problem: $x + (x + 3) = 29$. If $2x + 3 = 29$, $2x = 26$, so $x = 13$. This means that there are 13 girls and therefore 16 boys.

In question **2**, students will find it helpful to draw a diagram of the rectangular pool and then label the lengths.

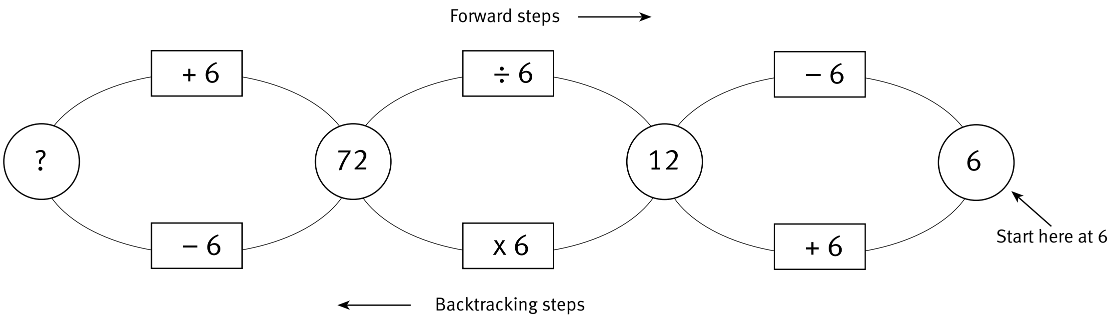


The perimeter (the distance around the edge), of the pool is 48 metres. So, width + width + 15 m + 15 m = 48 m. This means that 2 x width = 18 metres, so the width is 9 metres. Once again, some students could use an algebraic approach. In this case, an equation for the problem is $48 = 2 \times 15 + 2 \times x$ (where x is the width) or $48 = 30 + 2x$. This means that $18 = 2x$ (subtracting 30 from both sides) and that $x = 9$ (dividing both sides by 2).

In question **3**, if the students check the outcome when a 30-year-old is talking, this is what happens:

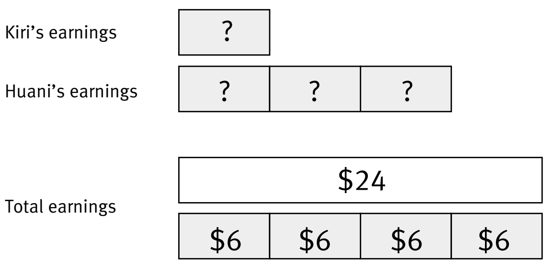
Starting age	30
Add 6	36
Divide by 6	6
Subtract 6	0

The outcome here is 0, so it is likely that the person who is talking is considerably older than 30, that is, Grandad. The students might use a trial-and-improvement strategy to work out the age of the person talking and confirm that it is Grandad. They might also try a working backwards strategy. This involves reversing the procedure, starting with 6, then following the sequence of instructions: add 6, multiply by 6, subtract 6. This backtracking diagram or flow chart shows what happens.



So the age of the person talking is 72 – 6 = 66, and the person is Grandad.

In question **4**, students might try the visual approach suggested by Huani.



This can lead to an algebraic approach involving equations: If Kiri earns x, Huani earns $3x$. Together, $x + 3x = 24$ or $4x = 24$, giving $x = 6$. So Kiri earns $6, and Huani earns $18.

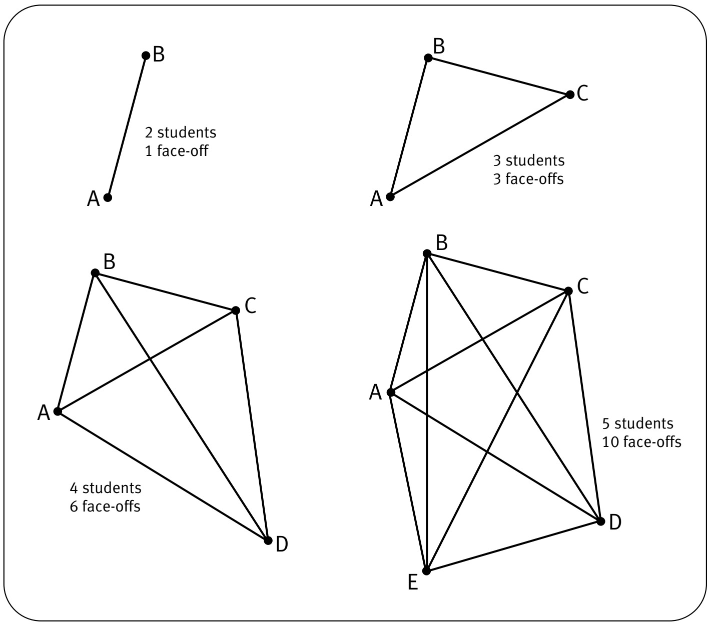
A visual model can also be used for question **5**:

24 chocolates at start	
15 chocolates shared	9 left

Each person gets 3 of the chocolates that were shared, so there were 5 people altogether (David and 4 of his friends) who each had 3 chocolates.

In question **6**, the students should explore how many face-offs there are with just 2, then 3, then 4 students, and so on. They might record their results in a table and see if they can work out a rule that will help them predict the number of face-offs for 30 students.

Their exploratory work might involve diagrams like these. Each line represents a face-off or question, so 3 lines mean that 3 questions are needed.



Some students may quickly reason that when, for example, there are 5 students, each student faces off with the other 4 students, making a total of 20 face-offs. But a face-off from A to B is the same as a face-off from B to A, so there are in fact one-half of 20 or 10 face-offs. They might then reason that with 30 students, each student faces off with the other 29 students, giving what appears to be 30 x 29 = 870 face-offs. But as indicated earlier, a face-off from A to B is the same as a face-off from B to A. So, altogether, there are 870 ÷ 2 = 435 face-offs or questions needed for 30 students.

It is unlikely that many students will accomplish the reasoning above. An alternative strategy is to make a table of their results from their initial exploratory work.

Number of students	Number of questions needed	Pattern
2	1	(2 x 1) ÷ 2 = 1
3	3	(3 x 2) ÷ 2 = 3
4	6	(4 x 3) ÷ 2 = 6
5	10	(5 x 4) ÷ 2 = 10
6	15	(6 x 5) ÷ 2 = 15
7	21	(7 x 6) ÷ 2 = 21
8	28	(8 x 7) ÷ 2 = 28
9	36	(9 x 8) ÷ 2 = 36
10	45	(10 x 9) ÷ 2 = 45

This rule can also be expressed algebraically. For x students, y questions are needed. The pattern in the table above suggests that $y = \frac{x(x-1)}{2}$.

However, while this rule may seem obvious once you have seen the pattern for the number of questions needed, it can nevertheless be challenging for many students.

The students may also notice a pattern in the table above with regard to the way the number of questions increases. For example, if there are 4 students, we need 6 questions, but if we add another student, we will need 4 more (that is, 10) questions. This makes sense because the fifth student will need to face off with each of the other 4 students, which gives a total of 4 + 6 = 10 questions. So the number of questions needed can be found as the sum of the two numbers in the row above.

The following table provides another way to record face-offs for a quick-draw tables tournament. Here, there are 8 students involved in face-offs. Students must find a short cut way to count face-offs A v B, A v C, and so on.

v	A	B	C	D	E	F	G	H
A								
B	B v A							
C	C v A	C v B						
D	D v A	D v B	D v C					
E	E v A	E v B	E v C	E v D				
F	F v A	F v B	F v C	F v D	F v E			
G	G v A	G v B	G v C	G v D	G v E	G v F		
H	H v A	H v B	H v C	H v D	H v E	H v F	H v G	

For these 8 students, there are 8 rows of 8 squares in the table to be filled in. But each row has a shaded space, representing A v A, B v B, and so on, which makes no sense. So, there are actually 8 rows of 7 spaces to be filled. But half of these are duplicated (see the empty spaces above the shaded spaces), so in fact, there are really just $\frac{1}{2}$ of 8 rows of 7 spaces to be filled. This is $\frac{1}{2}$ of 8 x 7, which is 56 ÷ 2 = 28 spaces, which is the formula we found before. Each of these spaces represents a face-off or question, so when there are 8 students, 28 questions are needed. Similarly, when there are 10 students, there are $\frac{1}{2}$ of 10 x 9 or 90 ÷ 2 = 45 questions needed. And when there are 30 students, $\frac{1}{2}$ of 30 x 29 = 435 questions are needed.

Note that the mathematics in this question is identical to the mathematics in the Web Circles activity on page **18**.

Pages 14–15 Frieze

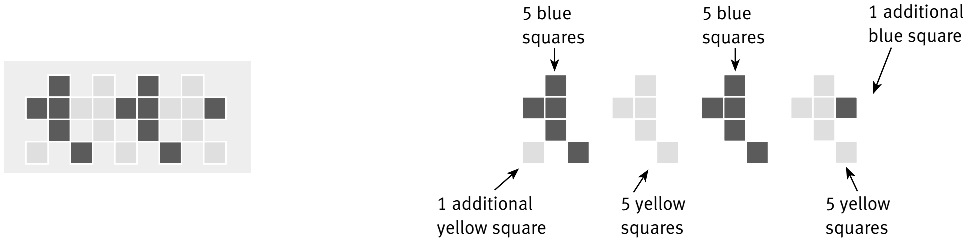
Achievement Objectives

- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students base their short cuts and rules on the patterns of coloured squares in the frieze designs.

In question **1**, the frieze design made by painting a total of 4 x 5 + 2 squares either blue or yellow includes alternate sets each of 5 blue or yellow squares.

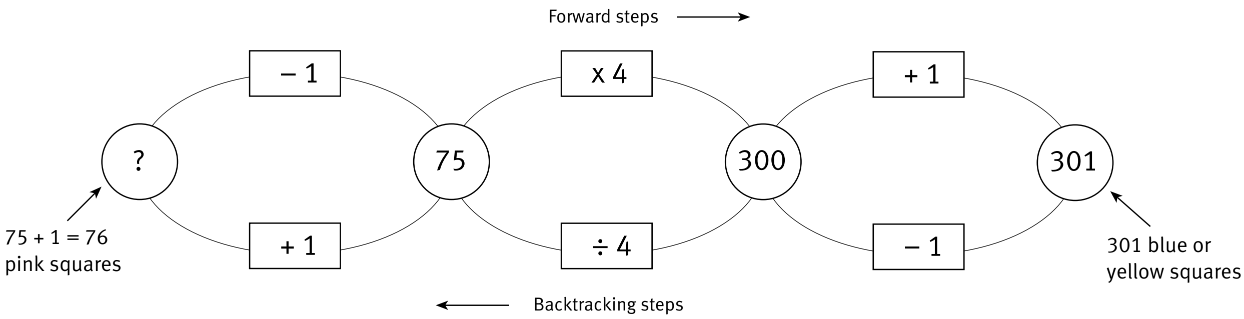


The design in question **1a** has 6 sets of 5 blue squares and 5 sets of 5 yellow squares. There is also 1 additional yellow square at the beginning and 1 at the end of the design. So, altogether, there are 11 x 5 + 2 squares painted blue or yellow. Note that when there is an even number of sets of 5 squares, the square at the beginning is coloured differently from the square at the end, and when there is an odd number of sets of 5 squares, the 2 squares are coloured the same. The students will need to be aware of this difference between designs with even and odd numbers of sets of 5 squares to complete questions **1d** and **1e**. Students who have difficulty with question **1e** might benefit from considering a design with a smaller odd-numbered short cut, for example, 7 x 5 + 2. As an alternative to drawing such a pattern, they could cover part of a larger pattern with paper or card.

In question **2**, there is always 1 fewer set of 4 squares (composites of blue or yellow squares) than there are pink squares. There is also 1 additional square at the end of the design. In the first design shown for this question, there are 4 pink squares and 3 sets of 4 squares that are blue or yellow. There is also a yellow square at the end. So for 4 pink squares, there are 3 x 4 + 1 = 13 yellow or blue squares.

In the second design, there are 7 pink squares and 6 x 4 + 1 = 25 blue or yellow squares. So, a design with 100 pink squares has 99 x 4 + 1 = 397 blue or yellow squares altogether. And a design with x pink squares has $(x - 1) \times 4 + 1$ or $4(x - 1) + 1$ blue or yellow squares altogether.

In question **2b**, the students must reverse their thinking to find the number of pink squares in designs with given total numbers of yellow or blue squares. They should try this for themselves and explain their thinking. The following backtracking flow chart shows the thinking involved to find the number of pink squares when there are 301 yellow or blue squares.



This set of three operations beginning with 301 can be written arithmetically as (301 − 1) ÷ 4 + 1 = 76.

In the frieze design in question **3**, for each of the 5 pink squares, there are 4 blue or yellow squares. There are also 2 additional yellow squares.

Note that when there is an odd number of pink squares, the 2 additional squares are the same colour. When the number of pink squares is even, the 2 additional squares are different colours. So, when there are 100 pink squares, there are 100 x 4 + 2 blue or yellow squares altogether. The number of pink squares is an even number, so there are 50 x 4 + 1 blue squares and the same number of yellow squares. When there are 101 pink squares, there are 101 x 4 + 2 blue and yellow squares altogether. There is an odd number of pink squares, so there are 51 x 4 blue squares and 50 x 4 + 2 yellow squares when the first set of 4 squares is blue. On the other hand, when the first set of 4 squares is yellow, there are 51 x 4 yellow squares and 50 x 4 + 2 blue squares.

A frieze design with x pink squares has x x 4 + 2 or $4x$ + 2 blue or yellow squares altogether.

Pages 16–17 Stick Houses

Achievement Objectives

- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

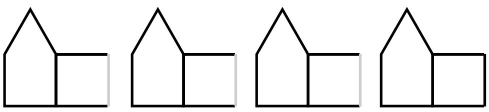
ACTIVITY

In this activity, students base their short cuts and rules on the arrangements of sticks used in designs for blocks of houses.

In question **1**, each house and its adjoining garage needs 7 sticks (5 for the house and 2 for the garage). An additional stick is needed for the end wall of the last garage. So, a block of 5 houses needs 5 x 7 + 1 = 36 sticks, and a block of 100 houses needs 100 x 7 + 1 = 701 sticks. A block of x houses therefore needs y sticks where $y = x$ x 7 + 1 sticks. This is usually written as $y = 7x + 1$. A block with 1 000 houses therefore needs 7 x 1 000 + 1 = 7 001 sticks.

In the second table in question **1c**, the students need to reverse their thinking in order to find the number of houses for a given number of sticks. So, for example, if the number of sticks used is 78, we subtract 1 (the stick used for the final garage) and then divide by 7 to find the number of houses (11). This can be expressed as (78 – 1) ÷ 7 = 11.

In question **2**, Rebecca first makes each house with its garage separate by including an extra stick for the vertical wall of all but the last garage.



A block with 4 houses therefore needs 4 x 8 sticks. But when the houses and garages are joined, the extra 3 sticks are no longer needed. So there are 4 x 8 – 3 sticks altogether. A block with 100 houses therefore needs 100 x 8 – 99 sticks, and a block with x houses needs y sticks where $y = x$ x 8 – $(x – 1)$. This is the same as $y = 8x – (x – 1)$.

Although the two algebraic rules for blocks of houses shown in questions **1** and **2** are different, they each produce identical values for y (the number of sticks) for particular values for x. Students who can manage the algebra above might check their rules by making the calculations in a table such as the one on the next page.

x	$y = 7x + 1$	$y = 8x - (x - 1)$
1	7 x 1 + 1 = 8	8 x 1 − (1− 1) = 8
2	7 x 2 + 1 = 15	8 x 2 − (2 − 1) = 15
3	7 x 3 + 1 = 22	8 x 3 − (3 − 1) = 22
4	7 x 4 + 1 = 29	8 x 4 − (4 − 1) = 29
5	7 x 5 + 1 = 36	8 x 5 − (5 − 1) = 36
87	7 x 87 + 1 = 610	8 x 87 − (87 − 1) = 610

While the equivalence between the two algebraic expressions may seem obvious, it is helpful to confirm it in this way since the simplification of $8x - (x - 1)$ is often carried out incorrectly.

Correct

$$8x - (x - 1) = 8x - x + 1$$
$$= 7x + 1$$

Incorrect

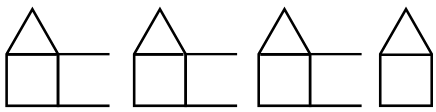
$$8x - (x - 1) = 8x - x - 1$$
$$= 7x - 1$$

error

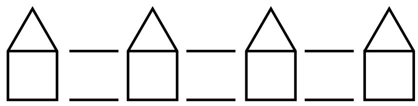
Students who are puzzled by this will find it helpful if they investigate what has been done to make the first simplification incorrect and the second correct. They should convince themselves by checking the expressions for particular values of x.

In questions **3** and **4**, the students devise their own short cuts and rules for the number of sticks in the designs. In question **3**, they will be able to confirm their rule by completing the table for question **3b**. The number of sticks for particular houses using any rule must be the same.

Two possible arrangements for the design in question **4** and their rules are shown below. The number of houses is x, and the number of sticks is y.



$y = 8(x - 1) + 6$



$y = 6x + 2(x - 1)$

These rules simplify as follows:

$$8(x - 1) + 6 = 8x - 8 + 6$$
$$= 8x - 2$$

$$6x + 2(x - 1) = 6x + 2x - 2$$
$$= 8x - 2$$

So the simplest way to calculate the number of sticks for, say, 100 houses is 8 x 100 − 2 = 798.

Page 18 Web Circles

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

In this activity, the students find a rule to predict the number of straight lines in any web circle. It is soon clear that simply counting the lines will not be possible. An alternative strategy is needed. Ian decides to start with a web circle with just 5 points. He is able to easily count the 10 lines he has drawn and should see clearly that 4 lines radiate from each of the points, so there are 5 x 4 = 20 straight lines. But this counts each line twice (each line gets counted at both points where it meets the circle), so there are in fact 20 ÷ 2 = 10 straight lines.

This method can be used for web circles with any number of points. If there are x points, there are always (x – 1) straight lines to that point, and therefore a total of $\frac{1}{2} \times x \times (x - 1)$ straight lines. This can be written as $\frac{1}{2}x(x - 1)$ or $\frac{x(x-1)}{2}$.

If the students have worked through Problem Smorgasboard on page **13** of the student book, they may realise that the last problem on that page is identical to this one in terms of the mathematics involved.

On page 13, we have people; here we have points. In each case, we have to work out the number of connections that can be made between members of the group. If there are 30 people, every one of them can connect with all but themselves, that is, with 29 people. But 30 x 29 counts each connection twice, so there are $\frac{1}{2}$ x 30 x 29 = 435 distinct connections. In this activity, there are 20 points, each able to connect with every point but itself, that is, 19 points. $\frac{1}{2}$ x 20 x 19 = 190.

Page 19 Marooned

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)

ACTIVITY

In this activity, the students should use counters to represent the adults and children involved in the transfer by dinghy between the launch and shore. Give them the problem and let them spend time in groups to see if they can find a way forward. A systematic approach to the problem is outlined in the table on the next page, with the circles (❍) representing children and the squares (■) representing adults.

Trip	People on launch	People in dinghy	People on shore
0	■■■■■■○○		
1	■■■■■■	○○›	
2	■■■■■■	‹○	○
3	■■■■■○	■›	○
4	■■■■■○	‹○	■
5	■■■■■	○○›	■
6	■■■■■	‹○	○■
7	■■■■○	■›	○■
8	■■■■○	‹○	■■
9	■■■■	○○›	■■
10	■■■■	‹○	○■■
11	■■■○	■›	○■■
12	■■■○	‹○	■■■
13	■■■	○○›	■■■
14	■■■	‹○	○■■■
15	■■○	■›	○■■■
16	■■○	‹○	■■■■
17	■■	○○›	■■■■
18	■■	‹○	○■■■■
19	■○	■›	○■■■■
20	■○	‹○	■■■■■
21	■	○○›	■■■■■
22	■	‹○	○■■■■■
23	○	■›	○■■■■■
24	○	‹○	■■■■■■
25		○○›	■■■■■■
26			○○■■■■■■

This can be summarised in a table, showing that each additional adult adds another 4 trips:

Adults	1	2	3	4	5	6	7	8	9	10	11	12
Trips	5	9	13	17	21	25	29	33	37	41	45	49

So a rule is: the number of trips is equal to the number of adults multiplied by 4, plus 1. If there are x adults and 2 children, the number of trips, y, can be expressed as $y = 4 \times x + 1$ or $y = 4x + 1$. So, for 20 adults, there are 4 x 20 + 1 = 81 dinghy trips, and for 100 adults, there are 4 x 100 + 1 = 401 dinghy trips. (The students will probably realise that in this scenario, the children do a lot of rowing!)

In question **4**, there are 37 trips altogether. Students who know their multiplication facts for 4 will quickly see that 4 x 9 + 1 = 37. So 9 adults are involved.

INVESTIGATION

Many students, including able students, will find the investigation challenging. The tables below show what happens when there is 1 adult and the number of children increases from 2 to 3.

Number of trips	People on launch	People in dinghy	People on shore
	■○○		
1	■	○○ →	
2	■	← ○	○
3	○	■ →	○
4	○	← ○	■
5		○○ →	■

One extra child generates 2 extra dinghy trips (see the 2 shaded rows):

Number of trips	People on launch	People in dinghy	People on shore
	■○○○		
1	■○	○○ →	
2	■○	← ○	○
3	■	○○ →	○
4	■	← ○	○○
5	○	■ →	○○
6	○	← ○	○■
7		○○ →	○■
			○○○■

The following table shows the number of trips generated by different numbers of children. (We are not concerned here with the number of adults involved.)

Children	2	3	4	5	6
Trips	1	3	5	7	9

Note how the number of trips increases by 2 for each additional child after 2 children. There must always be at least 2 children. The rule linking the number of trips with the number of children is:
the number of trips = 2 x the number of children – 3. This rule is based on the pattern shown in the table above.

Pages 20–21 Patterns, Rules, and Spreadsheets

Achievement Objectives

- find a rule to describe any member of a number sequence and express it in words (Algebra, level 4)
- use a rule to make predictions (Algebra, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITIES ONE TO FOUR

In this activity, students interpret, make, and then solve problems using spreadsheets. It builds on the earlier spreadsheet activity on page **1** of the student book.

In question **1**, the students see and use two ways to generate the same number sequences using computer spreadsheets. Both methods model what happens to Hannah's initial savings of $63 if she saves a further $8 each week.

When method 1 is used, the formula in any cell calculates a value that is 8 more than in the cell directly above it. So the formula =B4+8 in cell B5 simply adds 8 to the value in cell B4, the formula =B5+8 in cell B6 adds 8 to the value just calculated for cell B5, and so on.

When method 2 is used, students see that the value calculated for any cell is independent of the value in the cell directly above it (in contrast to the first method). The second method works by multiplying the number of weeks by 8 and adding 63. So, for example, the formula =8*A9+63 in cell C9 multiplies the value in A9 by 8 and then adds 63. The value in A9 is 6 (6 weeks), so the value in C9 is 8 x 6 + 63 = 111.
So after 5 weeks, Hannah will have saved $103.

Different spreadsheet applications have different ways of working. In some applications, the Fill Down command in the Calculate menu is used to enter formulae. Other applications operate automatically. Formulae are entered by "pulling" the first cell down or across as required.

Note with **Activity One** that when the first method is used to calculate savings after 52 weeks, the savings after each week from week 1 to week 52 must be calculated. While this doesn't take long on a spreadsheet, it is actually quicker and easier to read as a formula if the second method is used. For example, Hannah's savings after 52 weeks are calculated from 8 x 52 + 63, that is, $8 for each of 52 weeks plus the $63 in the account at the start. Here a single calculation is made instead of the 52 calculations with method 1.

In **Activity Two**, question **1**, the parts –3*A2, –3*A3, –3*A4, and so on of the formulae in successive cells in column B indicate that successive values in the sequence decrease by 3. So Tom is skip-counting backwards in 3s from 501. The 145th number in Tom's sequence will be 504 – 3 x 145 = 69. And in question **2**, the parts +12*A2, +12*A3, +12*A4, and so on of the formulae in column B indicate that successive values in the sequence increase by 12. So Marnie is skip-counting forwards in 12s from 17. The 879th number in Marnie's sequence will be 5 + 12 x 879 = 10 553. Note that, as with the second method in **Activity One**, the formulae used in both questions in **Activity Two** require only a single calculation to work out the value in any cell.

In **Activity Four**, the students make spreadsheets to compare charges for gardening services offered by Mika and Hine. The spreadsheets in the Answers show the effects of firstly making a change to Mika's hourly rate and then a change to Hine's fixed travelling charge.

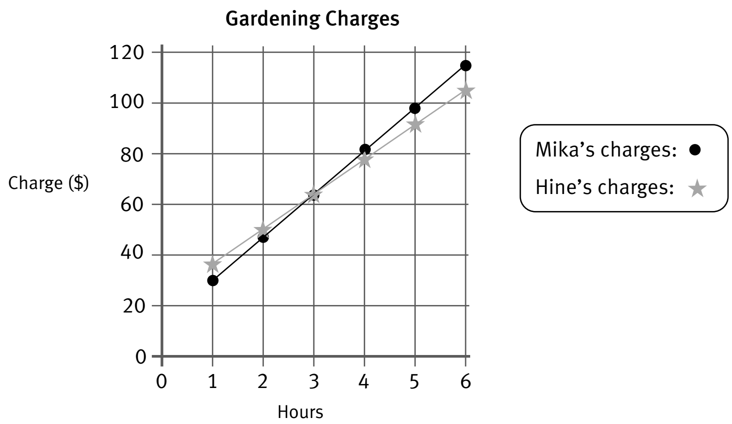
For more than 1 hour's work, Hine's charges are less than Mika's. Working for 0 hours is really meaningless, so Hine's new charges will always be cheaper or, in one case only, the same as Mika's. Students who reason successfully with the tasks above might see if they can find a way that Mika could respond by altering his charges to better compete with Hine. This challenge will be more interesting if conditions are placed upon it, for example: "How can Mika alter his travel charge or hourly rate so that he is cheaper than Hine for up to 3 hours' work, and charges the same as Hine for 4 hours' work?"

The formulae in the spreadsheets above have a direct relationship with algebra and algebraic thinking. For example, the formula used in cell B2 for Mika's original charging plan is =13+17*A2. For cell B3, the formula is =13+17*A3, and for cell B4, the formula is =13+17*A4, and so on. This set of formulae can be replaced by a single algebraic formula, $y = 13 + 17 \times x$, or more simply, $y = 17x + 13$. Here, y stands for the total charge for x hours' gardening.

Equations of the form $y = 17x + 13$ and $y = 13x + 25$ (the two equations corresponding to Mika and Hine's initial charging schemes) are called linear equations because they describe straight lines.

As an extension exercise, the students could graph these lines to highlight the difference in Mika and Hine's charges. By plotting both lines on the same grid, the students will easily see the contrast in their charges.

For instance, a graph showing a comparison of their first charging schemes is:



This displays the information given in the spreadsheet very clearly. For example, we can see from the graph that the lines cross when Mika and Hine charge the same amount for the same period worked (3 hours for $64), and that Mika is cheaper for under 3 hours, while Hine is cheaper for over 3 hours.

Page 22 Car Journeys

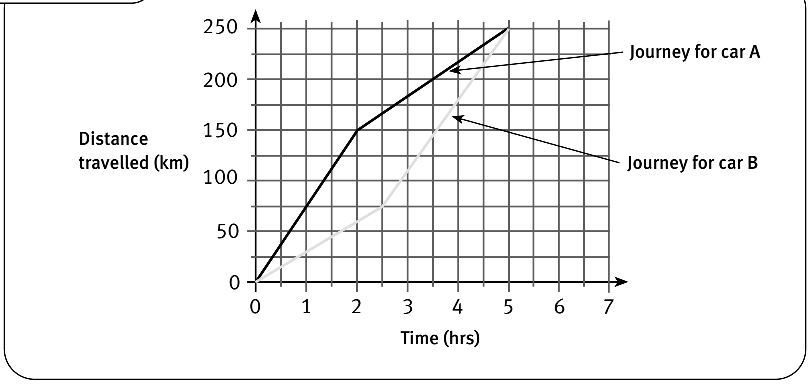
Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

In this activity, students first interpret distance–time graphs that illustrate two car journeys from Tauranga to Whangarei on the same day. While distance–time graphs may seem clear and simple to interpret, many students see quite different messages depicted in the graphs. For example, it is quite common for students to interpret these graphs as “going up a hill” rather than the way the distance travelled changes with time. Students may therefore benefit from working through tasks such as the one below.

Car Journeys



Each car represented in this graph takes 5 hours to complete 250 kilometres. So the average speed, that is, the total distance travelled divided by the time taken, for each car is 250 km ÷ 5 hrs = 50 km/h. Note, however, that car A completes the first 150 kilometres in 2 hours. The average speed for this part of the journey is 75 kilometres per hour. The next 100 kilometres take 3 hours for car A. So the average speed for the second part of the journey for car A is $33\cdot\dot{3}$ kilometres per hour. Car B travels the first 75 kilometres in 2.5 hours at an average speed of 75 km ÷ 2.5 hrs = 30 km/h and the remaining 175 kilometres also in 2.5 hours. The average speed for the second part of the journey for car B is therefore 175 km ÷ 2.5 hrs = 70 km/h. Students might see if they can use the information above to tell, from the graph, which part of each journey is the fastest and which is the slowest. The rule that should emerge is the steeper the line, the greater the speed. This makes sense, as it means that more distance is being covered for a given amount of time. A line close to vertical represents very high speed (a great distance in very little time), and a horizontal line represents zero speed (time passes, but no distance is travelled).

In question **1**, the students must see that the starting time for Margot’s family was 9.00 a.m. and the starting time for Jenny’s family was 10.15 a.m. Margot’s family completed the 400 kilometre journey at 4.30 p.m., that is, after 7 hours, 30 minutes (including stops), while Jenny’s family completed the journey at about 4.10 p.m, that is, after about 6 hours (including stops).

In question **2**, the horizontal parts of the graphs represent stops because no distance is covered for the particular time interval. In question **3**, the students locate the points where the graphs intersect. The times and distances for these points indicate when and where the passing occurred.

In question **4**, the students must calculate the speed travelled over a particular section of the journey. They do this by dividing the distance travelled in that section by the time it took. So, for example, in question **4b**, Margot’s family travelled 100 kilometres (175 km – 75 km) between 10 a.m. and 11.30 a.m., which gives a speed of 100 kilometres per 1.5 hours, or approximately 67 kilometres per hour.

In question **5**, the students use what they have learned from the earlier questions to draw graphs related to their own experiences, either real or imaginary. Students completing this question successfully will have demonstrated a clear understanding of how, for example, distance can change with time for particular everyday situations.

Surfboard Sums

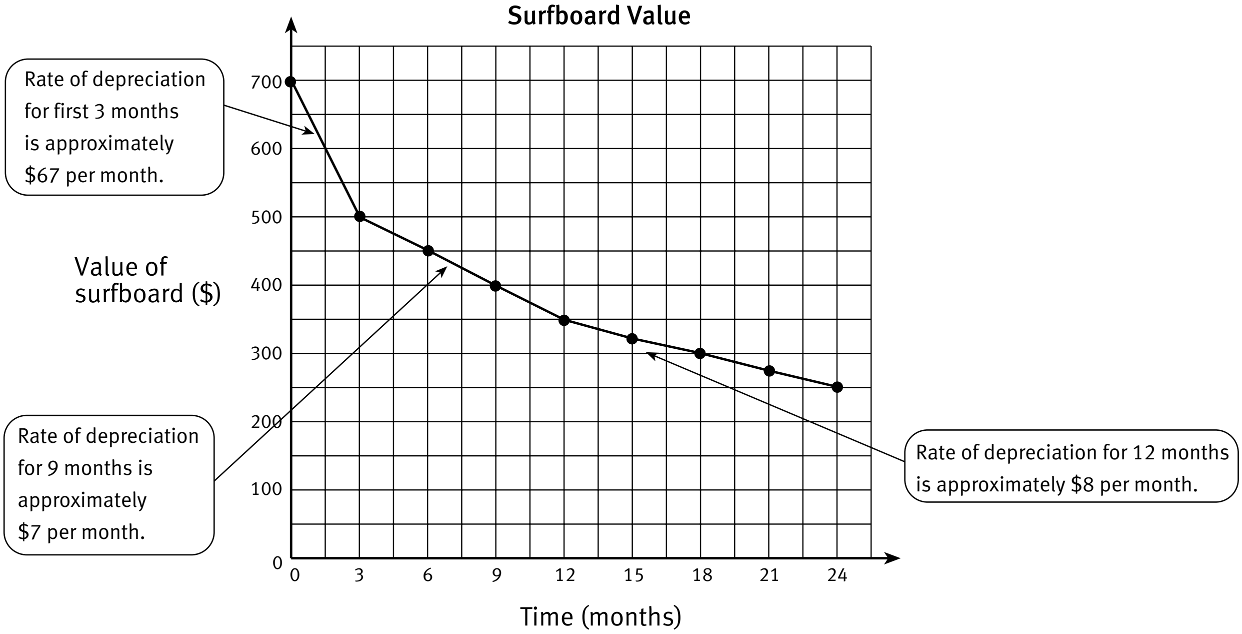
Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- interpret information and results in context (Mathematical Processes, developing logic and reasoning, level 4)

ACTIVITY

In this activity, students interpret the graph showing how the value of material goods such as a surfboard usually decreases over time.

There are three distinct periods that need to be considered. During the first 3 month period, the surfboard loses value more rapidly than at any other time. In fact, it loses $200 in value, which is almost 30% of its price when new. Its rate of loss of value (rate of depreciation) is about $67 per month (found from $\$200 \div 3 = 66.66\dot{6}$). For the next 9 months, the surfboard loses value steadily from $500 to $350. The line for this part of the graph is much less steep than the line for the first 3 months, and the rate of depreciation over the 9 months is almost $17 per month (found from $\$150 \div 9 = 16.66\dot{6}$). For the third period, that is, for the second 12 months that Vinny owns the surfboard, there is a loss of $100 as a result of a steady rate of depreciation that is a little more than $8 per month (found from $\$100 \div 12 = 8.33\dot{3}$).



In questions **3** and **4**, the students use the graph to make sensible predictions, or extrapolations, for periods beyond the data shown on the graph. They are also asked to justify their predictions. While it is safe to simply extend the graph by assuming that the rate of depreciation remains the same for the period beyond 24 months, a more in-depth analysis is possible by looking at the shape of the graph over the first 24 month period.

The gradual flattening of the graph as a consequence of the rate of depreciation reducing as time goes on suggests that the value of the surfboard will not decrease by an amount greater than for the previous 3 months, that is, $25. So, if its value at 24 months is $250, then by 27 months, its value is not likely to be less than $225. For the same reason, if the value of the surfboard at 27 months is $225, it is likely to fall in value by not more than $25 in the next 3 months. So Vinny is likely to get at least $200 for the surfboard by selling it at 30 months.

It is unlikely that many students will make the argument above without some initial prompting. One way to help might be to first extend the straight line representing the part of the graph between 12 and 24 months to the point representing the value of the surfboard at 27 months and the point when the value of the surfboard is $200. The students might then decide whether it is more likely that these points will actually lie on the line, above the line, or even below the line. It is argued above that it is more likely that these points will be on or above this line. The students should be encouraged to explain their reasoning for their decisions.

Page 24 Holiday Pay

Achievement Objectives

- use words and symbols to describe and generalise patterns (Mathematical Processes, developing logic and reasoning, level 4)
- sketch and interpret graphs on whole number grids which represent simple everyday situations (Algebra, level 4)
- record information in ways that are helpful for drawing conclusions and making generalisations (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, the students first construct a table and then a graph to compare two offers of pay for selling CDs. It is expected that each day Jane will sell $600 worth of CDs. So, if Jane were to accept offer 1, each day she would get 10% of the difference between $300 and $600, that is, 10% of $300, which is $30. Offer 2 is very different. For sales of $600 on day 1, she would get 1% of the difference between $100 and $600, that is 1% of $500, or $5. On day 2, she would get 2% of $500, or $10, on day 3, she would get 3% of $500, or $15, and so on.

A table for students to complete by entering the results of their calculations is begun for them. Some students may instead want to use a computer spreadsheet for this purpose. The first spreadsheet below shows formulae for calculating the values shown in the second spreadsheet.

Pay Offers 1&2 (SS)

F17 =F16+E17

	A	B	C	D	E	F
1		Offer 1			Offer 2	
2	Day	Daily pay ($)	Cumulative total ($)		Daily pay ($)	Cumulative total ($)
3	1	=0.1*300	=B3		=(A3/100)*500	=E3
4	=A3+1	=0.1*300	=C3+B4		=(A4/100)*500	=F3+E4
5	=A4+1	=0.1*300	=C4+B5		=(A5/100)*500	=F4+E5
6	=A5+1	=0.1*300	=C5+B6		=(A6/100)*500	=F5+E6
7	=A6+1	=0.1*300	=C6+B7		=(A7/100)*500	=F6+E7
8	=A7+1	=0.1*300	=C7+B8		=(A8/100)*500	=F7+E8
9	=A8+1	=0.1*300	=C8+B9		=(A9/100)*500	=F8+E9
10	=A9+1	=0.1*300	=C9+B10		=(A10/100)*500	=F9+E10
11	=A10+1	=0.1*300	=C10+B11		=(A11/100)*500	=F10+E11
12	=A11+1	=0.1*300	=C11+B12		=(A12/100)*500	=F11+E12
13	=A12+1	=0.1*300	=C12+B13		=(A13/100)*500	=F12+E13
14	=A13+1	=0.1*300	=C13+B14		=(A14/100)*500	=F13+E14
15	=A14+1	=0.1*300	=C14+B15		=(A15/100)*500	=F14+E15
16	=A15+1	=0.1*300	=C15+B16		=(A16/100)*500	=F15+E16
17	=A16+1	=0.1*300	=C16+B17		=(A17/100)*500	=F16+E17
18						

→

Pay Offers 1&2 (SS)

F17 =F16+E17

	A	B	C	D	E	F
1		Offer 1			Offer 2	
2	Day	Daily pay ($)	Cumulative total ($)		Daily pay ($)	Cumulative total ($)
3	1	30	30		5	5
4	2	30	60		10	15
5	3	30	90		15	30
6	4	30	120		20	50
7	5	30	150		25	75
8	6	30	180		30	105
9	7	30	210		35	140
10	8	30	240		40	180
11	9	30	270		45	225
12	10	30	300		50	275
13	11	30	330		55	330
14	12	30	360		60	390
15	13	30	390		65	455
16	14	30	420		70	525
17	15	30	450		75	600
18						

Notice that by day 11, the pay for offer 2 has caught up with the pay for offer 1. The total pay for each offer is then $330. After day 11, the total pay for offer 2 quickly outstrips the pay for offer 1.

In question **3**, the students investigate the effects of two further offers. For all offers, it is assumed that each day Jane would sell CDs worth a total of $600. If she were to accept offer 3, then each day she would get 10% of the difference between $200 and $600, that is, 10% of $400, which is $40. For offer 4, on day 1, she would get 1% of the difference between $200 and $600, that is 1% of $400, or $4, on day 2, she would get 2% of $400, or $8, on day 3, she would get 3% of $400, or $12, and so on.

The next spreadsheet shows the pay for all four offers for up to 4 weeks or 20 working days.

Pay Offers 1–4 (SS)

F22 =F21+E22

	Offer 1		Offer 2		Offer 3		Offer 4	
Day	Daily pay ($)	Cumulative total ($)	Daily pay ($)	Cumulative total ($)	Daily pay ($)	Cumulative total ($)	Daily pay ($)	Cumulative total ($)
1	30	30	5	5	40	40	4	4
2	30	60	10	15	40	80	8	12
3	30	90	15	30	40	120	12	24
4	30	120	20	50	40	160	16	40
5	30	150	25	75	40	200	20	60
6	30	180	30	105	40	240	24	84
7	30	210	35	140	40	280	28	112
8	30	240	40	180	40	320	32	144
9	30	270	45	225	40	360	36	180
10	30	300	50	275	40	400	40	220
11	30	330	55	330	40	440	44	264
12	30	360	60	390	40	480	48	312
13	30	390	65	455	40	520	52	364
14	30	420	70	525	40	560	56	420
15	30	450	75	600	40	600	60	480
16	30	480	80	680	40	640	64	544
17	30	510	85	765	40	680	68	612
18	30	540	90	855	40	720	72	684
19	30	570	95	950	40	760	76	760
20	30	600	100	1050	40	800	80	840

The pay for each of the offers up to 15 days and also up to 20 days is highlighted. Notice that for 15 days, the pay is the same for offers 2 and 3 and that by day 19, the pay is the same under offers 3 and 4. However, by day 20, offer 2 is by far the best offer. It provides $1,050 for working 20 days and is $210 more than for offer 4, the next best offer.

While offers 2 and 4 provide more than the other two offers as time goes on, in the long term, they are unrealistic. The students may want to check this out. They will find, for example, that by day 250, that is, after 50 five-day weeks or about 1 year, the total pay will be as follows:

Offer	Total pay for 250 days ($)	Total value of CDs sold ($)
1	7 500	150 000
2	156 875	150 000
3	10 000	150 000
4	125 500	150 000

Note that the total value of sales of $600 worth of CDs per day for 250 days is $150 000. Clearly, offer 2 would have the business operating at a loss. The same is likely for offer 4.

It may be argued, however, that offers 1 and 3 are also unrealistic because, in the long run, they may be much too low to attract employees to sell CDs. The students might find it interesting to use a spreadsheet to explore a range of more realistic offers.

The students will find that the graphs for offers 1 and 3 are straight lines but the other graphs gradually curve upwards since the values for total pay increase exponentially. These curved graphs show that the total pay increases at a gradually increasing rate after each day. This is one reason that pay offers such as these are unsustainable in the long term.

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