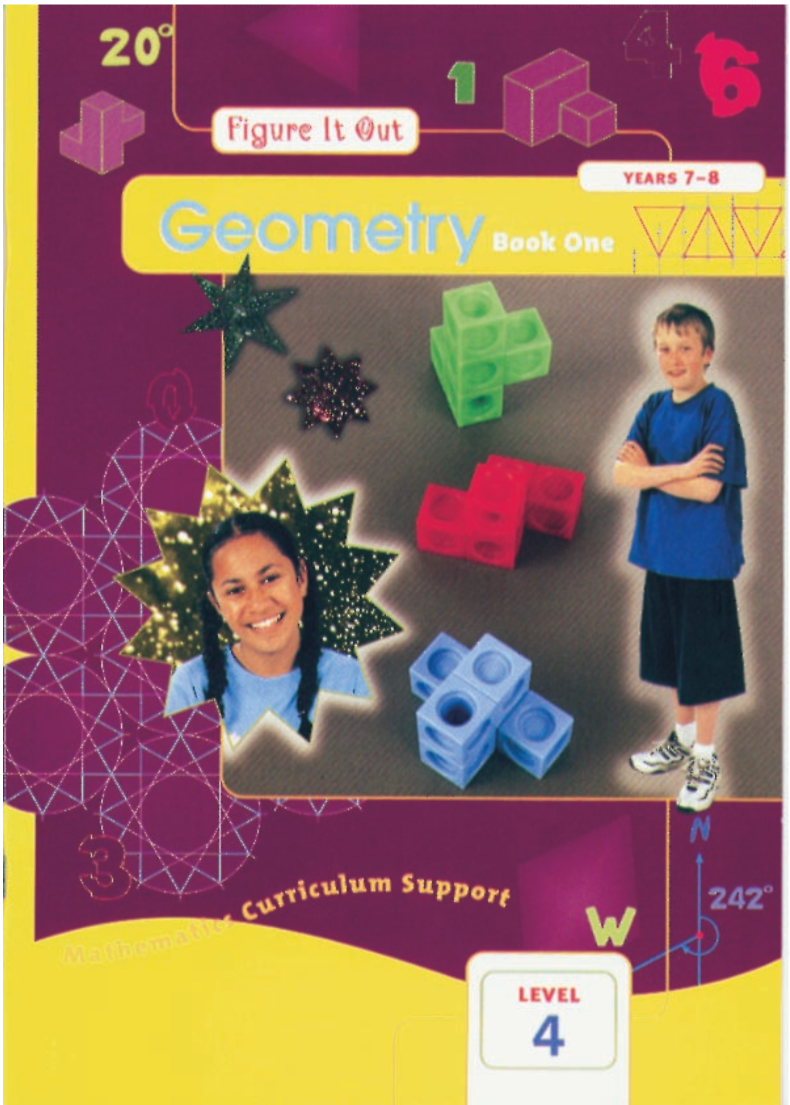
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

CONTENTS

The books for years 7–8 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand year 7–8 classrooms. The books have been developed and trialled by classroom teachers and mathematics educators and follow on from the successful series for levels 2–4 in primary schools.

Student books

The student books in the series are divided into three curriculum levels: levels 2–3 (linking material), level 4, and level 4+ (extension material). All the books are aimed at year 7–8 students in terms of context and presentation.

The following books are included in the series:
Number (two linking, three level 4, one level 4+, distributed in November 2002)
Number Sense (one linking, one level 4, distributed in April 2003)
Algebra (one linking, two level 4, one level 4+, distributed in August 2003)
Geometry (one level 4, one level 4+, distributed in term 1 2004)
Measurement (one level 4, one level 4+, distributed in term 1 2004)
Statistics (one level 4, one level 4+, distributed in term 1 2004)
Themes: *Disasters Strike!, Getting Around* (levels 4–4+, distributed in August 2003)

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and life experiences that are meaningful to students aged 11–13 years. The activities can be used as the focus for teacher-led lessons, as independent bookwork, or as the catalyst for problem solving in groups.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the student books includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, comment on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in your classroom

Where applicable, each page starts with a list of equipment that the students will need to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to a problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum*, page 7

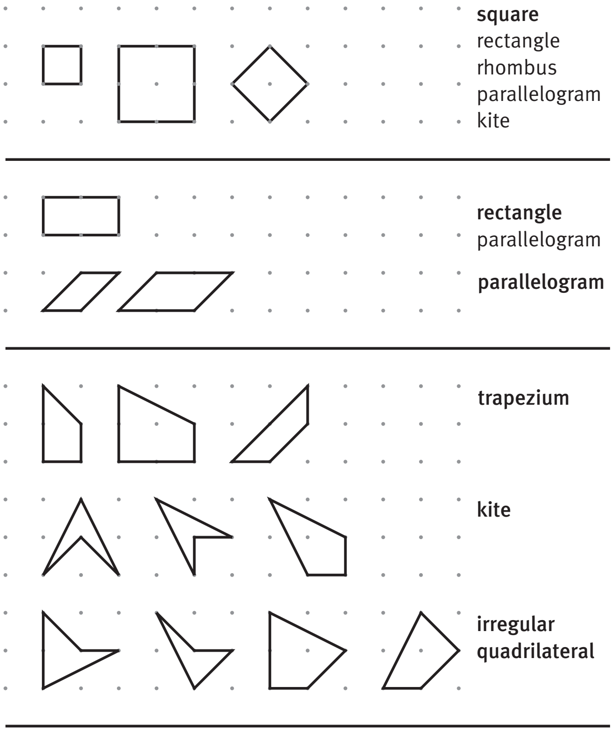
Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Answers Geometry: Book One

Page 1 Quad Queries

ACTIVITY

1.–2.a. The 16 different quadrilaterals and their labels are:



2. b. Some quadrilaterals have more than one label because they are subsets of other quadrilaterals. For example, a rectangle is also a parallelogram (because it has 2 pairs of equal, parallel sides). A square is a subset of 4 other quadrilaterals.

c. There are 6 different parallelograms and 3 different squares. (The 3 squares are also parallelograms.)

3. All its sides are the same length, and all its angles are equal. That makes it the only truly regular quadrilateral.

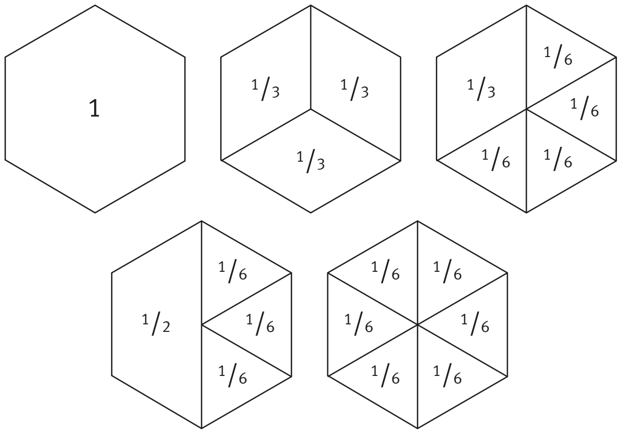
4. Descriptions will vary but should include the following information:

- square: a quadrilateral with all sides equal and all angles 90°
- rectangle: a quadrilateral with 2 pairs of equal, parallel sides and all angles 90°
- parallelogram: a quadrilateral with 2 pairs of equal, parallel sides
- rhombus: a quadrilateral with all sides equal
- kite: a quadrilateral with 2 pairs of equal, adjacent sides
- trapezium: a quadrilateral with (only) 1 pair of parallel sides
- irregular quadrilateral: a quadrilateral that does not fit any of the above definitions

Pages 2–3 Shaping Up

ACTIVITY ONE

1. Other ways are:



2. $1 = 1$

$\frac{1}{2} + \frac{1}{2} = 1$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

$\frac{1}{2} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

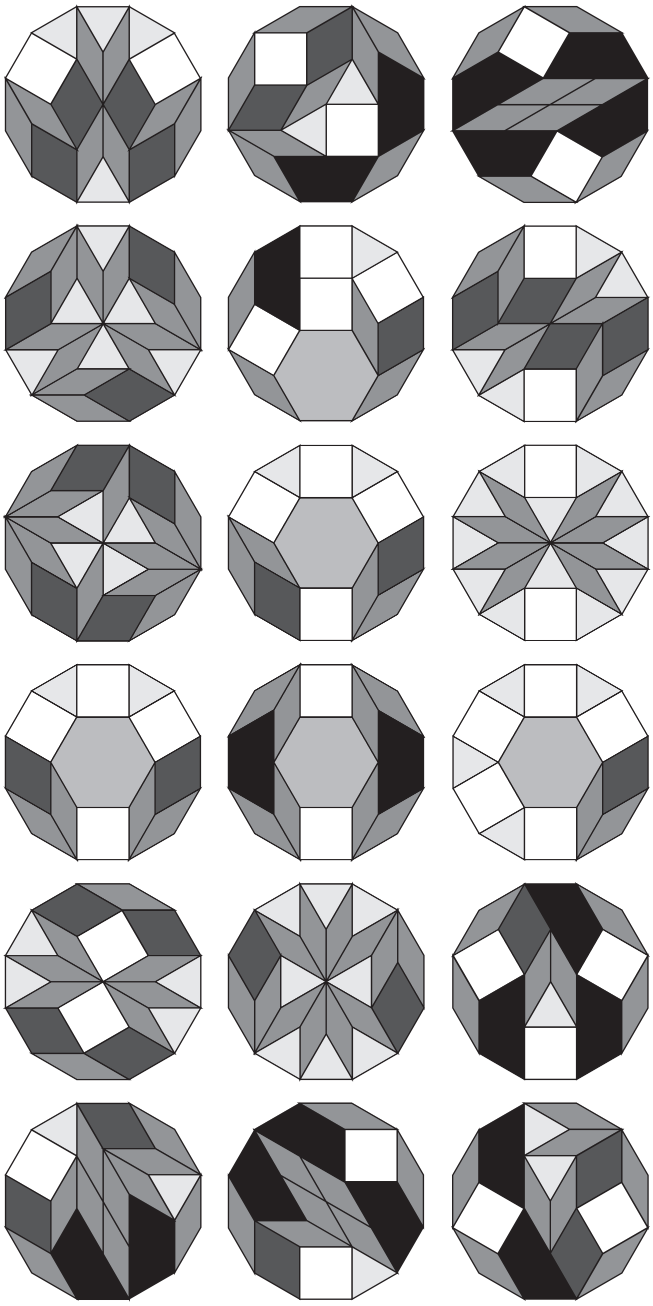
$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

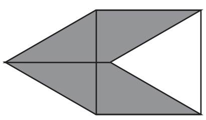
ACTIVITY TWO

1. There are many possible ways. Some of them are:

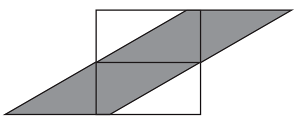


2. a. Area = 4 squares + 12 triangles + 4 30/150 rhombuses

   b. Methods may differ. As a possible explanation, you could put 2 rhombuses on top of a square. Then you can see that the "spare" bits could be cut off and used to exactly fill the gaps inside the square. So the area of 2 rhombuses must be the same as the area of 1 square.



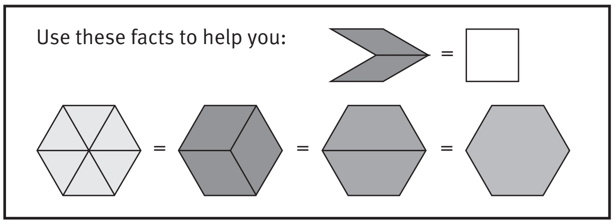
or



   c. Area = 6 squares + 12 triangles

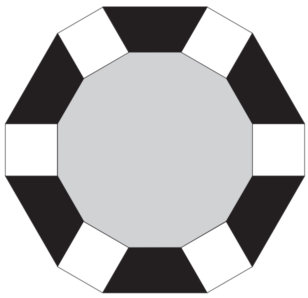
   d. Area = 6 squares + 2 hexagons

   e.–f. Practical tasks. Use the facts that you have discovered in the earlier parts of this question (summarised in the panel below) to show that the area of every dodecagon can be reduced to 6 squares and 2 hexagons.



3. a. Although it still has 12 sides, the sides are no longer all the same length. It is, in fact, a semi-regular dodecagon (semi-regular because there is a pattern to it).

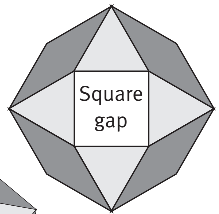
   b. Area = 12 squares + 5 hexagons. (The area of the dodecagon was 6 squares and 2 hexagons. To these, you have added 6 more squares and 6 trapeziums [the equivalent of 3 hexagons]).

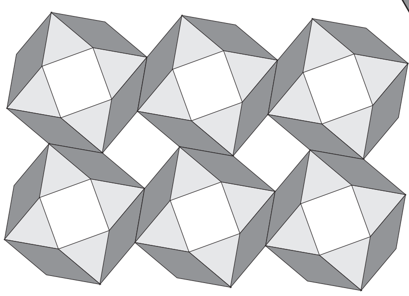


ACTIVITY THREE

1. a. No. To make a square shape you need right angles. With the exception of the square block and the 30/150 rhombus, the angles of all the blocks are 60° or 120°, which by themselves will not form 90°.

   You can, however, create a pattern that leaves a *gap* that is the size and shape of the square block. You can do this by making a semi-regular octagon from 4 triangles and 4 30/150 rhombus blocks. This octagon can be combined with others to form a tessellation featuring square gaps:

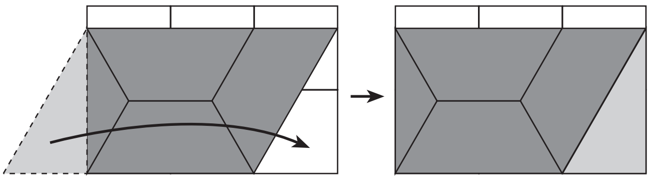




b. A regular octagon needs interior angles of 135° (not possible) and the decagon 144° (not possible). The angles of all the pattern blocks are multiples of 30°. 135° and 144° are not multiples of 30°.

2. 6 squares have a greater area than 2 hexagons. Here is one way of showing this:

2 hexagons have the same area as 4 trapeziums. Make a parallelogram from the 4 trapeziums and lay it on top of a 3-by-2 grid of squares, like this:

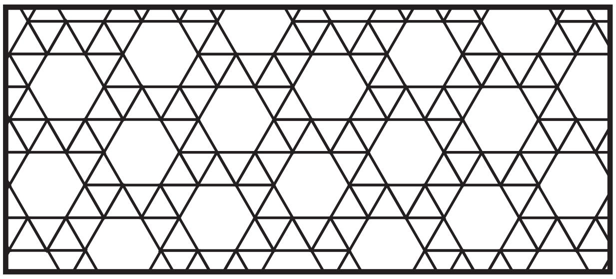


If the overhanging section of the trapezium on the left was cut off, it would exactly fit the gap on the right to make a rectangle. But this rectangle would be smaller than the rectangle formed by the 6 squares. (The difference is the narrow strip along the top.)

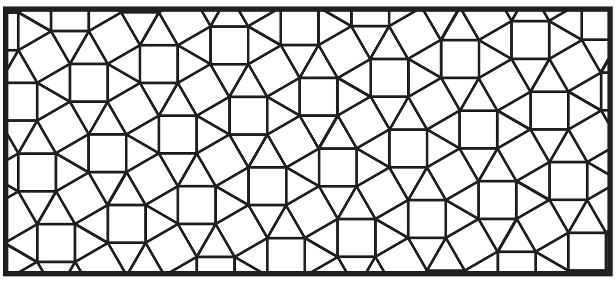
Pages 4–5 Pathway Patterns

ACTIVITY

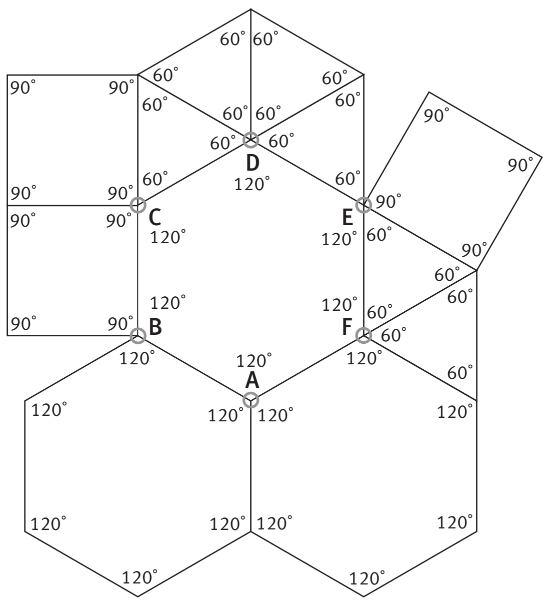
1. The only other pattern is:



2. The only other pattern is:



3. a. The angles are all 60°, 90°, or 120°:

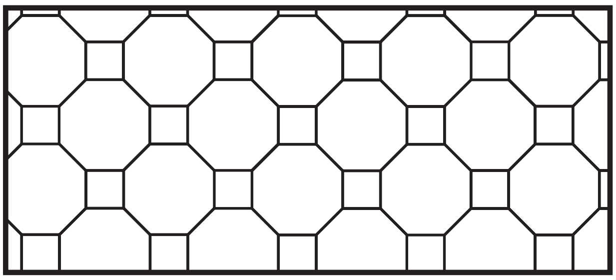


b. A = 360°, B = 330°, C = 360°, D = 360°, E = 330°, and F = 360°.

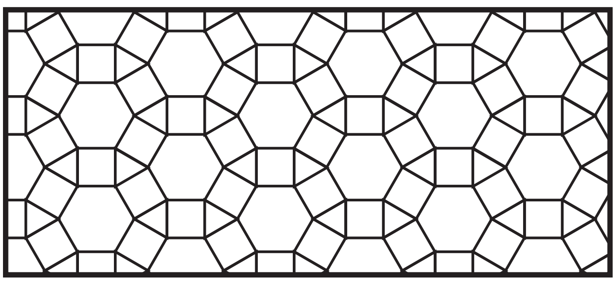
c. In both cases, the vertices of the tiles do not add up to 360°.

d. They will only tessellate if the angles of the vertices that meet at each point combine to give a sum of 360°.

4. a. The only possible pattern is:



b. The only possible pattern is:



Page 6 Hexagon Hunt

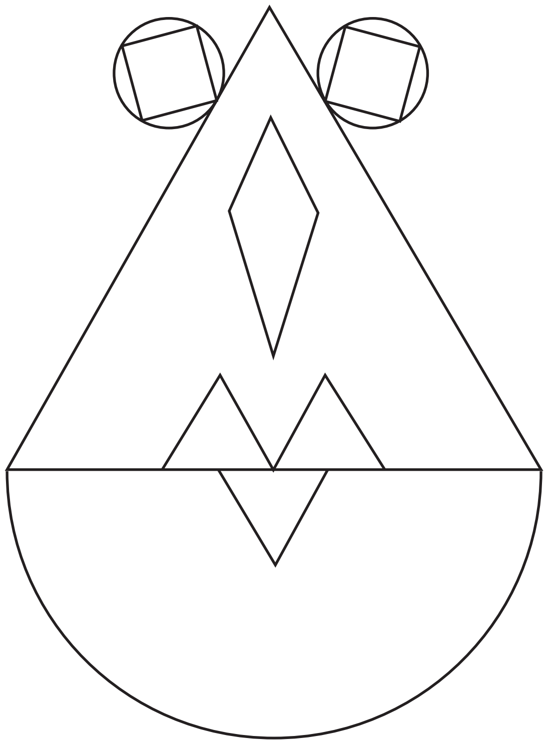
GAME

Practical activity (making the dice) and a game that involves using rotation and pattern blocks to make hexagons

Page 7 Alien Action

ACTIVITY

1. The face should look similar to this, but bigger:



2. Drawings should look similar but may vary in the details.
3. a.–c. Practical activities. Results will vary.

Page 8 Cute Cubes

ACTIVITY

Practical activity

Page 9 Fantastic Folding

ACTIVITY

Practical activity

Pages 10–11 A Different View

ACTIVITY ONE

1.

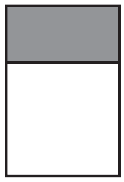
Object	Solid
a.	cylinder
b.	cone
c.	square-based pyramid
d.	half-cylinder
e.	sphere
f.	cube

2.

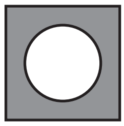
	View 1	View 2	View 3	View 4
a.	C	B	E	A
b.	C	A	E	D
c.	E	D	B	C
d.	D	A	C	E

ACTIVITY TWO

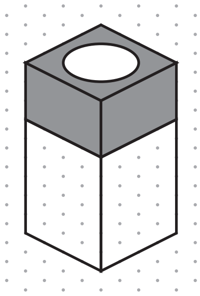
1. a.



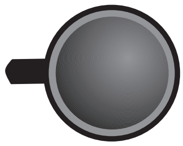
b.



c.



2. a.



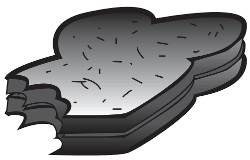
b.

c.

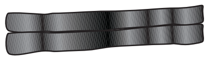
d.

e.

f. A possible view:



g. A possible view:



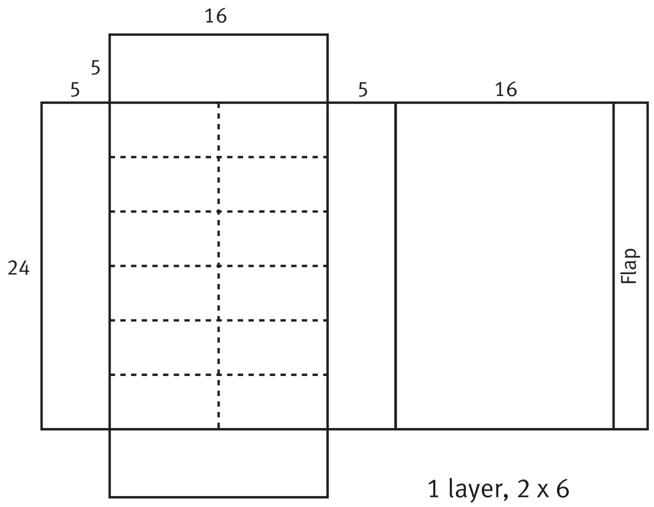
h.

3. Practical activity

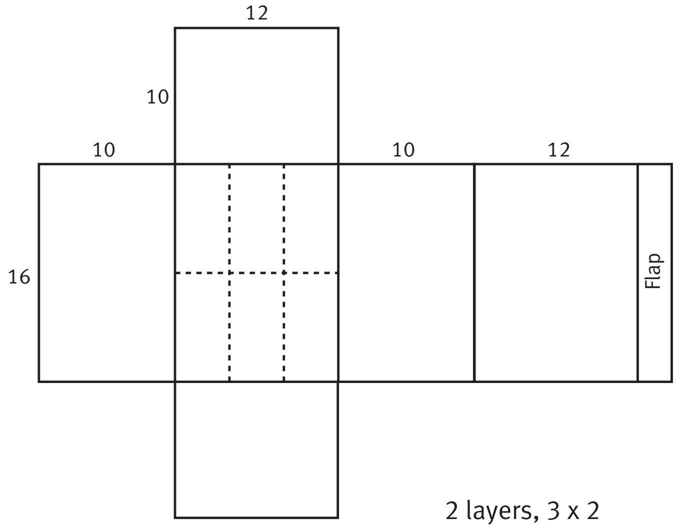
Page 12 Loads of Lamingtons

ACTIVITY

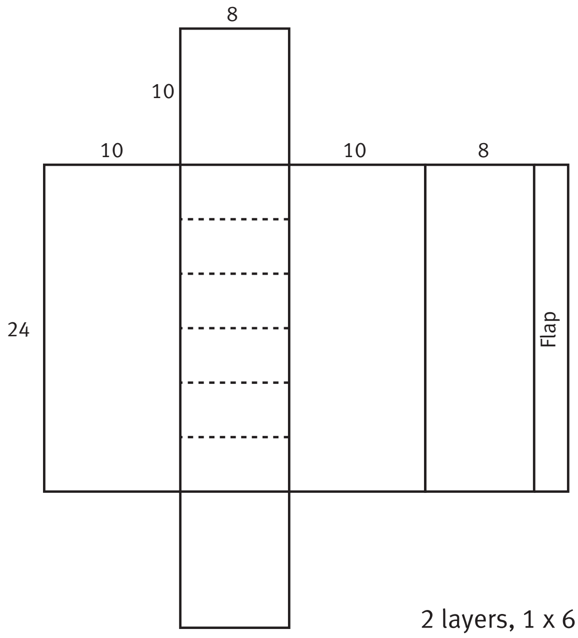
1. There are numerous possibilities, some of which are impractical. Three possible nets are:



1 layer, 2 x 6



2 layers, 3 x 2



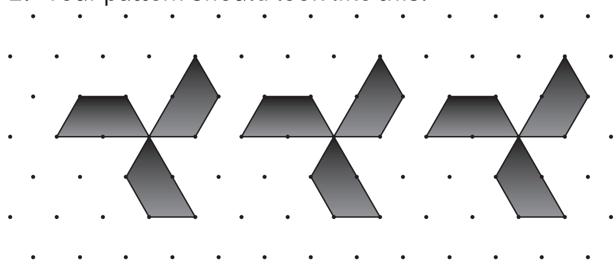
2 layers, 1 x 6

2. Answers will vary. Factors to consider include how easy it is to make the container, how effectively it will display the contents, and how easy it will be to pack and remove the lamingtons without damaging them.
3. Practical activity

Page 13 Shifting Shapes

ACTIVITY

1. Answers will vary. All of these patterns can be described in a number of different ways. Possible descriptions are:
   - **a.** Reflect the trapezium in a vertical axis. Move the axis of reflection $1^{1}/_{2}$ units to the right and reflect again. Repeat.
   - **b.** Translate it along 1 dot, up 1 dot, along 1 dot, and down 1 dot. Repeat.
   - **c.** Rotate it 60° anticlockwise about the top right-hand vertex and 60° clockwise about the bottom right-hand vertex. Repeat.
   - **d.** Reflect it in a horizontal axis and translate it 1 unit to the right. Repeat.
   - **e.** Reflect it in the axis that has a negative slope and makes an angle of 60° with the horizontal. Translate it 1 unit down that same axis. Repeat.
   - **f.** Rotate it 60° anticlockwise about the bottom right-hand vertex, 60° clockwise about the bottom left-hand vertex, 60° anticlockwise about the bottom right-hand vertex, and 60° anticlockwise about the bottom left-hand vertex. Reflect it in a vertical axis. Repeat.
   - **g.** Rotate it 60°clockwise about the bottom right-hand vertex and 60° anticlockwise about the same vertex. Translate it 2 units to the right. Repeat.
2. Your pattern should look like this:



3. Patterns and instructions will vary.

Pages 14–15 Kōwhaiwhai

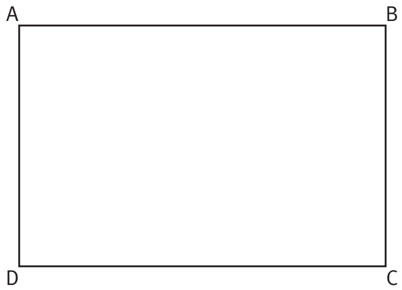
ACTIVITY

1.–2. Practical activities. Results will vary.

Page 16 Precision Flying

ACTIVITY ONE

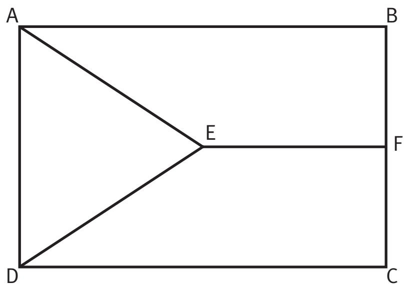
1. a. You should have a rectangle, labelled like this:



b. A rectangle

c. 90º

2. a. The finished design, before colouring, should look like this:



b. An isosceles triangle (not an equilateral triangle)

c. Practical activity

d. It is the flag of the Czech Republic.

ACTIVITY TWO

a. Practical activity

b. Descriptions will vary. Here is a suitable description:

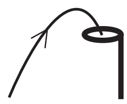
- I constructed a rectangle, ABCD, that was 16 cm by 10.8 cm.
- I drew the diagonal DB.
- I drew 2 lines that were parallel to, and 1.3 cm from, DB.
- I drew 2 further lines that were parallel to, and 2 cm from, DB.
- I coloured the top triangle dark green, the middle 2 strips black, the other 2 (thin) strips gold, and the bottom triangle blue.

Page 17 Ball Paths

ACTIVITY

1. The drawings for **a–j** will be similar to the ones below. They may also show the ball bouncing after it lands.

a. 

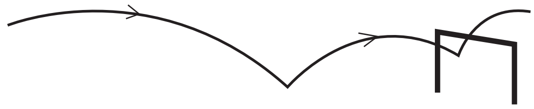
b. 

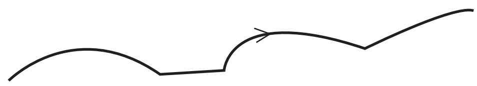
c. 

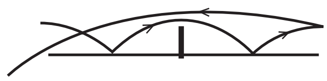
d. 

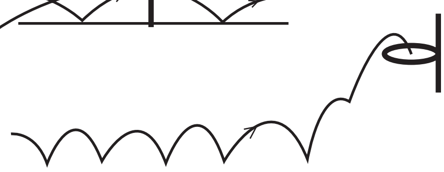
e. 

f. 

g. 

h. 

i. 

j. 

2. Discussion will vary.

3. Practical activity

Pages 18–19 Starry-eyed

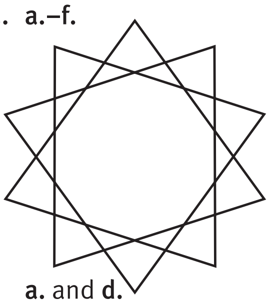
ACTIVITY ONE

1. a. Practical activity

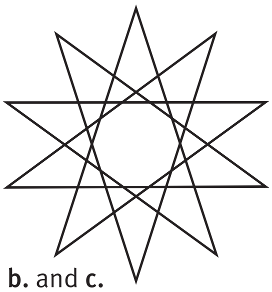
b. Practical activity

c. Although the lines are drawn in a different order, the two stars are identical. This is because 3 + 5 = 8.

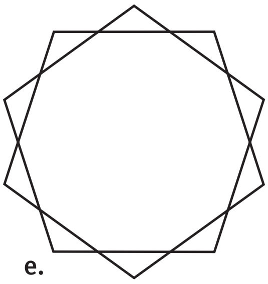
2. a.–f.



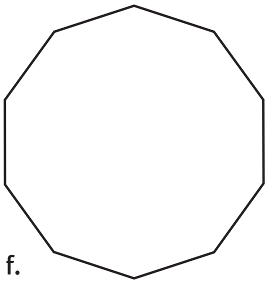
a. and d.



b. and c.



e.

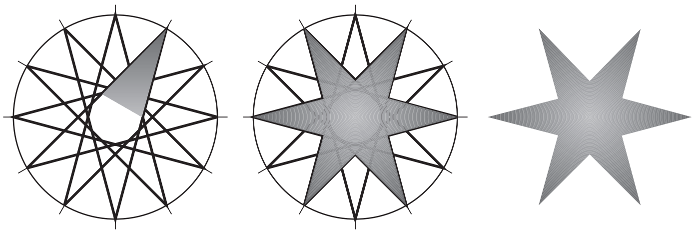


f.

3. There are 4 if you include the regular polygon, 9/1 (9/8). The others are 9/3 (9/6), 9/4 (9/5), and 9/2 (9/7).
4. a. i. A regular 16-sided polygon
   ii. An asterisk with 8 lines crossing at a central point. (If you don't lift your pencil, you will get a single straight line through the centre.)
   iii. 16/11
   iv. 16/2 (16/14), 16/4 (16/12), and 16/8.
   v. 8 if you include 16/1 and 16/8; 6 if you don't
   vi. 16/7 (16/9). 16/8 is not a polygon (because it does not enclose a space).
   b. Practical activity

ACTIVITY TWO

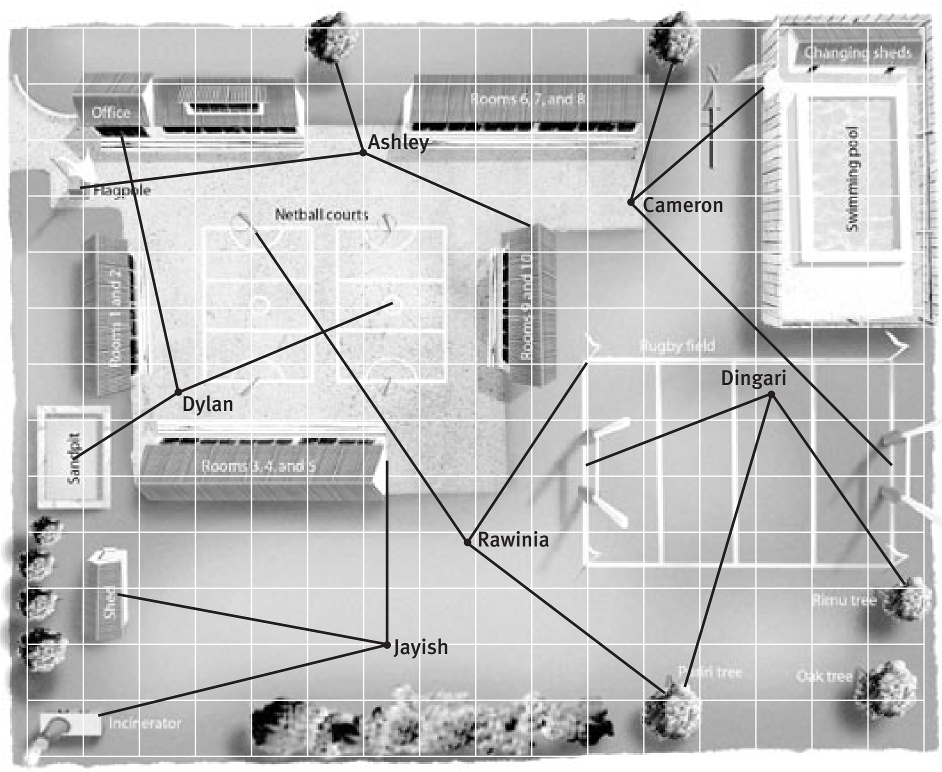
1. Practical activity
2. This diagram shows how the 6-pointed stars in the design were made. Begin with a cluster of 12/5 star polygons. Colour in every second point on each star polygon. Remove the construction lines.



Pages 20–21 Find the Spot

ACTIVITY

1. ⟶



2. a. Because we are given several bearings for each spot. The bearings all cross at a common point, so we know that it is the spot we are looking for. (If we had only 1 bearing, we would also need to know the distance if we were to find the spot.)
   b. 2
3. Practical activity. Results will vary.

Pages 22–23 Treasure Island

ACTIVITY

1. a.–b. Practical activity. Clues will vary.
2. Practical activity

Page 24 Blasting Bugs

GAME

A game that involves using compass directions

YEARS 7–8

Teachers' Notes

Overview Geometry: Book One

Page 1

Quad Queries

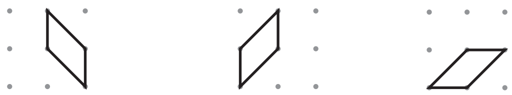
Achievement Objectives

- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, students create quadrilaterals, classify them, and then define the features of each kind in their own words. Those who have worked with quadrilaterals before will find that the activity revises and consolidates their knowledge. The fact that one kind of quadrilateral can be a special case of another will be a new concept for most students.

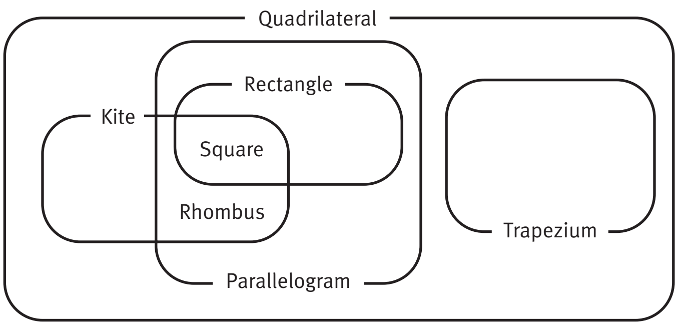
The students should be able to do question **1** with a minimum of instruction. All they need to know is that they are making 4-sided shapes within a 9-pin matrix. You may, however, need to clarify what is meant by 16 *different* quadrilaterals. Use a diagram like this and ask your students, "Are these rhombuses different?"



In this activity, it is only the *shape* that interests us, so the three rhombuses are the same. Your students may find it helpful to imagine each quadrilateral they make as the outline of a cardboard shape. Any other outline that they could draw by repositioning the cut-out or flipping it over counts as the same.

Questions **2** and **4** are deliberately circular. Question **2** creates the need for the definitions in question **4**, and the definitions in question **4** are developed as a result of working through question **2**. You could therefore leave whole-class discussion of question **2** until the students have completed their definitions in question **4**. The Answers suggest definitions that are acceptable from students working at level 4 of the curriculum.

The following Venn diagram may help explain the way in which the various quadrilaterals are related. From it, we can make statements such as: "some parallelograms are rectangles", "all rectangles are parallelograms", and "a square combines the properties of a rectangle and a rhombus".



Note: This diagram assumes (as does the student activity) that a trapezium has *exactly* one pair of parallel sides. An alternative definition of a trapezium is that it is a quadrilateral with *at least* one pair of parallel sides.

Polygons are closed figures (that is, the sides join to enclose a space) bounded by straight lines. Quadrilaterals are polygons with 4 sides. Although the square is the only *regular* quadrilateral, many quadrilaterals are classified according to features that distinguish them from the rest, and the term "irregular" is normally reserved for those that don't fit any other category. Note that, although the activity does not use the word "arrowhead" for a concave kite, you can do so if you wish. A *concave* quadrilateral is one that has an (internal) angle greater than 180 degrees. Note also that the term "isosceles trapezium" is sometimes used to describe trapeziums with an axis of symmetry.

You may like to discuss the concepts of *necessary* and *sufficient* conditions with your class. For example, for a quadrilateral to be square, it is necessary for it to have 4 right angles (or 4 equal angles), but this is not enough (sufficient) to *make* it a square (because a rectangle also meets this condition). To be a square, it must also have equal sides. Strictly speaking, for a quadrilateral to be a square, it needs equal sides and just *one* right angle. Have fun discussing this with your class!

You could also discuss the way in which the diagonals of each kind of quadrilateral intersect. Here are questions for your students to consider:
- Do the diagonals meet at right angles?
- Do they bisect each other (divide each other exactly in 2)?
- Is either of the diagonals an axis of symmetry?
- Does the arrowhead have 2 diagonals? (Yes, but they intersect outside the shape.)

As further activities, the students could find:
- the total number of squares (not different squares) that can be made on a 16-pin geoboard or even a 25-pin geoboard
- all the different quadrilaterals that can be made using a 16-pin geoboard
- how many degrees the (interior) angles of a quadrilateral add up to and a method of demonstrating this without a protractor.

Pages 2–3 Shaping Up

Achievement Objectives
- describe the features of 2-dimensional and 3-dimensional objects, using the language of geometry (Geometry, level 3)
- express quantities as fractions or percentages of a whole (Number, level 4)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

In these three activities, students explore the areas of pattern blocks relative to each other. They then consider the different ways in which a hexagon and a dodecagon can be made from pattern blocks. By doing this, they explore the behaviour of equivalent fractions.

ACTIVITY ONE

This activity involves regular hexagons. The students will soon discover that there are 7 ways of making a regular hexagon using the triangle, trapezium, and 60/120 pattern blocks (see the Answers), and from these, they should be able to write the various equations.

ACTIVITY TWO

In this activity, the students make regular dodecagons that have sides that match the edge of the square pattern block. They shouldn't need any introduction to question **1**. They will quickly find that there are many ways of using the various pattern blocks to complete the shape. The challenge for them will be to systematically record their solutions as tidy sketches.

In question **2a**, the students write the sum of the area of the dodecagon as the sum of 3 different shapes, but, as they will discover in the following parts of this question, they can express it as the sum of just 2 shapes.

To do this, they must first show (in question **2b**) that a square has the same area as 2 30/150 rhombuses. They may find it helpful to cut 2 rhombuses and a square from cardboard using the pattern blocks as templates. They can then overlay the rhombuses on the square. If they cut off the overlapping pieces, they can move these to fill the gaps on the square.

When doing questions **2c–f**, the students need to use the equivalences they discovered in **Activity One** and in question **2b**. (These are summarised in a panel in the Answers.) Using these equivalences, they can replace every 2 30/150 rhombuses with 1 square and every 6 triangles (or their equivalent) with 1 hexagon. For the first dodecagon in question **1**, the working could be set out like this:

Area = 4 squares + 4 30/150 rhombuses + 12 triangles
= 4 squares + 2 squares + 12 triangles
= 6 squares + 12 triangles
= 6 squares + 2 hexagons

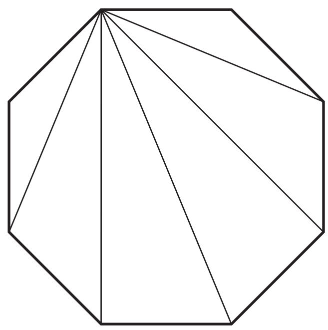
As they do questions **2e–f**, the students should come to see that the area of any of the dodecagons can be expressed as the sum of 6 squares and 2 hexagons. The area of each dodecagon is therefore the same.

ACTIVITY THREE

If we are trying to create a solid square (as implied), the answer to question **1** is "no". But it is possible to create a tessellation with square gaps in it. The Answers explain why and how.

When the students investigate question **1b**, they will find that it is not possible to make either a regular octagon or a regular decagon using the pattern blocks.

You may want to add to the explanation in the Answers by showing your class how to find the size of the interior angle of a regular polygon. One way is to first divide the polygon into triangles, as in the diagram. Because the 3 angles in a triangle have a sum of 180 degrees, you can multiply the number of triangles by 180 to get the sum of all the interior angles of the polygon. (This works because all the angles of the triangles together make up the angles of the polygon.) The polygon is regular, so divide this total by the number of angles in the polygon to get the size of each interior angle.



An octagon can be divided into 6 triangles, so the sum of its angles is 6 x 180 = 1 080 degrees. This means that, for a regular octagon, each of the 8 angles must be 1 080 ÷ 8 = 135 degrees.

Once again, for question **2**, a useful strategy is to trace and cut 6 squares and 2 hexagons from cardboard. The hexagons can then be cut in half to form 4 trapeziums. When the trapeziums are overlaid on the squares, they do not completely cover them. This shows that the area of the squares is greater than the area of the hexagons.

Pages 4–5 Pathway Patterns

Achievement Objectives

- apply the symmetries of regular polygons (Geometry, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

Tessellations can be found everywhere: in brick or concrete block walls, fences, and floor and wall tiles, on vinyl, carpet, and wallpaper, and in paving stones and cobblestones. The hexagonal honeycomb is an example from nature. Many of M. C. Escher's prints also feature tessellations. (See Escher Envy in *Geometry: Book Two,* Figure It Out, Years 7–8).

To do this activity, the students will need a good supply of the four regular shapes. If necessary, they can cut out their own. (The octagon is not a standard pattern block.) If they do this, they need to make sure that the length of the sides is the same for each kind of block.

Questions **1**, **2**, and **4** ask the students to find other patterns using various combinations of shapes. They will soon discover that there is only one possible pattern in each case. They may come up with what appears to be an alternative, but rotating the pattern will show that it is the same as one they already have. There are only 8 semi-regular tessellations. Questions **1**, **2**, and **4** cover 6 of these; the other 2 use a dodecagon (12-sided polygon).

Instead of using a protractor for question **3a**, the students should use their prior knowledge of geometrical facts, including:

- the sum of the angles in a triangle = 180 degrees
- each angle in a square = 90 degrees
- each angle in an equilateral triangle = 60 degrees
- the sum of the angles on a line = 180 degrees
- the sum of the angles at a point = 360 degrees
- each angle in a regular hexagon = 120 degrees.

As they work through the rest of question **3**, the students should see that the combined angles at any point add to 360 degrees. For example, the 4 angles that meet at C total 60 + 90 + 90 + 120 = 360 degrees. By contrast, the 3 tile angles at B total 120 + 120 + 90 = 330 degrees.

As an extension, you could show your students how to find the size of the angles in a regular polygon using the method shown in the notes for the previous activity. You could also ask them to investigate the tessellations they can make using dodecagons in combination with triangles, squares, and hexagons.

Totally Tessellated is an excellent website on tessellations. It can be found at http://library.thinkquest.org/16661 It includes information and examples of tessellations using both regular and non-regular polygons, and it has sections on Escher's tessellations and the history and applications of tessellations.

Page 6 Hexagon Hunt

Achievement Objectives

- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

GAME

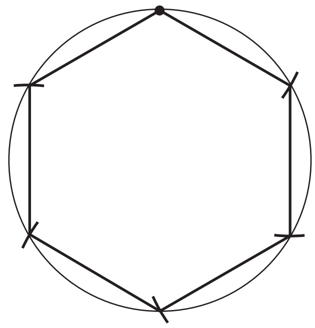
This game explores the different ways in which pattern blocks can be used to make a hexagon. It reinforces what students learned in **Activity One** of Shaping Up (page **2** of the students' book). It could also be used as an introduction to that activity.

Making the two 4-sided dice is an activity in its own right. The simplest method is to photocopy the copymaster outlines onto thin card. Alternatively, the nets for the dice are simple to construct using a compass and ruler. Encourage your students to make the best dice they can and to label them tidily.

The game board is modelled on a clock face, with 12 divisions. This is the smallest number that can be conveniently divided into quarters, thirds, and halves, following the instructions on the dice. Check that all your students know which direction is clockwise and which is anticlockwise.

When a 4-sided dice has been thrown, the only outcome that can be identified with certainty is which face is on the bottom, so this is the "winning" face of the throw. The students can, of course, lift the dice to find out what is on this face, but with a little practice, they will be able to "read" it without lifting the dice (by checking the 3 visible faces to find which action is missing).

As an extension, the students could construct their own hexagon within a circle, as in the diagram. To do this, they use a compass to draw a circle of suitable size, and then (without changing the radius) they use the compass to mark off equal arcs around the circumference. They then join the marked points on the circumference to complete the hexagon. If they make 2 hexagons of the same size, they can divide the second one into a trapezium, a rhombus, and an equilateral triangle so that they have a complete set of all the shapes used in the game.



Page 7 Alien Action

Achievement Objectives

- construct triangles and circles, using appropriate drawing instruments (Geometry, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

In this activity, the students create a design by following a set of instructions expressed in geometrical language.

The activity links to a level 4 suggested learning activity: "Students should be designing shapes comprising circles, rectangles, triangles, and other polygons, and talking about shapes they make using the language of geometry including words such as sector, arc, perimeter, circumference, semicircle, scalene, equilateral, isosceles, pyramid, vertical, horizontal." Formal construction techniques are a level 5 achievement objective, so the students should focus on understanding the language and ideas rather than on precision. You may find it useful to list the geometrical words used in this activity on the board and check that your students know and understand them.

The following instructions amplify those in the book:

i. Mark off a 10 centimetre line segment using a compass set to this radius. From each end of the line segment, without changing the radius, draw arcs that cross at a point. Join the two ends of the line segment to this point to create an equilateral triangle.

ii. Use a ruler to do this.

iii. At the 2 centimetre marks, draw lines at right angles using a protractor, a set square, or the corner of a piece of cardboard. Measure the point that is 1 centimetre along the lines you have just drawn. Set your compass to a radius of 1 centimetre and draw circles, using as centres the 2 points you have just marked.

iv. Draw 2 diameters of the circles at right angles to each other. Join the points where these intersect with the circumference to create the squares. Erase the diameter lines. You are not given the orientation of the squares, so they may not sit horizontally.

v. This instruction is self-explanatory.

vi. Use a compass set to 2 centimetres to draw the equilateral triangles.

vii. Mark points on the base side of the original big triangle, 1 centimetre either side of the centre. These points form 2 of the 3 vertices of another equilateral triangle. Use a compass set to 2 centimetres to complete this triangle.

viii. Join the top vertex of the original large equilateral triangle to the bottom vertex of the bottom "tooth" to create an axis of symmetry. Measure 2 centimetres down from the top of this axis. Set your compass to a radius of 2 centimetres, centre it on the point you have just marked, and draw an arc below it. Set your compass to a radius of about 2.8 centimetres, centre it on the axis of symmetry about 6.5 centimetres from the top, and draw an arc above. Join the points where the arcs intersect to the centres of the pairs of arcs so that you create a kite.

If question **3** is too hard for some of the students in your class, you could simplify it by removing the requirement that their instructions describe an alien face. They could design any simple combination of geometrical shapes and then write the instructions for a classmate to follow.

Page 8 Cute Cubes

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

At some stage, most students will have made a cube from a net. In this activity, they make a cube using 6 squares of paper that are folded origami-style and slid into each other. The result is an attractive mathematical model in which all the pieces interlock, so there is no need for glue or sticky tape. Memo cube squares are ideal for this activity: they are a good size and weight, they don't need cutting, they are cheap, and the colours are a bonus.

To complete their cube, the students will need to carefully follow each of the steps as described and illustrated. They will also need to fold their squares with precision. Give your students maximum opportunity to work the instructions out for themselves and to assist others who have trouble.

The critical step is instruction 5, where the students must flip the paper over and fold it back onto itself. If they don't flip it over, they will get a result that looks similar to diagram 5 but that won't interlock correctly with other pieces.

Tucking each of the corners under the centre fold (instruction 9) locks all the folds in place. The students then flip it over again so that all the folds are face down and then make two last folds. When this is done, there should be 2 triangles sticking up.

The hardest part of the task is assembling the cube from the 6 folded squares. Regard this as a problem-solving task for your students and resist the temptation to rescue them if their first attempts fail. Once some students have worked it out, their expertise will be in demand. You should try the task beforehand so that you know what you are asking your students to do.

To assemble the cube, start with 2 of the folded squares (folded side up) at right angles to each other. Slide the point of the second piece into one of the pockets of the first piece. (See the diagram on page **9** for an explanation of terms.) Take a third piece and slide one of its points into the remaining pocket of the first piece. Following the pattern you have now established, introduce the fourth, fifth, and sixth pieces.

Some students may take an hour or more to complete their cube, but they will be proud to take the finished product home.

Fantastic Folding (on the adjacent page) is an extension of this activity.

For origami techniques and ideas, see: www.paperfolding.com

Page 9 Fantastic Folding

Achievement Objectives

- model and describe 3-dimensional objects illustrated by diagrams or pictures (Geometry, level 3)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)
- devise and follow a set of instructions to carry out a mathematical activity (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

This extension to the previous activity is not easy, and you should first make the model yourself. It is, however, an excellent activity that requires students to follow instructions as well as to be precise and patient and work as a member of a team. The completed stellated icosahedron is a very attractive mathematical model.

This time, the students need 30 folded squares, so if they work in groups of 2 or 3, they can share the folding and speed up the process. Note that, this time, an extra fold is needed. The short diagonal of each parallelogram should be creased inwards on the side that has the slot down the middle. Each of these folds forms the valley between a pair of the triangular pyramids that make the icosahedron stellated.

The instructions tell the students to make 6 sets of 5 folded pieces, each set in a different colour. This makes it easy to explain the construction of the model as a series of stages, each represented by a different colour. The key relates the colours used in the photographs to the letters A–F used in the instructions. If these colours are available, your students could use them in the same sequence. If not, they should write down their own key, deciding which colour will be A and so on.

Once they have finished the folding, the students assemble the first set of 5 pieces in the rosette or pinwheel formation that can be seen in the top photograph in the side strip. To do this, they should push one of the points of piece 2 into one of the pockets of piece 1, one of the points of piece 3 into one of the pockets of piece 2 … and finally, one of the points of piece 1 into one of the pockets of piece 5 to complete the formation. They will need to be systematic with their choice of pockets. They should insert each new piece from the same direction and rotate the developing structure 90 degrees after doing this.

The students may find that their assembled pieces keep falling apart. They can solve this by putting a small amount of PVA glue where the label "pocket" appears in the diagram, pushing the appropriate point into that pocket, then pressing firmly.

The rest of the instructions should be self-explanatory, but the students should keep these points in mind:

- Be systematic. What they do with 1 piece, they do with all 5 pieces of that colour. It will help if they rotate their assembled structure one-fifth of a turn after each operation before carrying out the next.
- All the pieces fit together to form 5-sided rosettes and 3-sided pyramids.

The icosahedron is one of the 5 Platonic solids. It has 20 faces, each an equilateral triangle. The *stellated* icosahedron has 60 triangular faces because each of the 20 faces of the icosahedron is replaced by a 3-sided pyramid. You could make one by constructing an icosahedron and then sticking a triangular pyramid to each of its faces.

The Platonic solids are explored further in Tricky Truncations, pages 8–9, *Geometry: Book Two*, Figure It Out, Years 7–8. A website suitable for school students is:
www.mathacademy.com/pr/prime/articles/platsol/index.asp

Pages 10–11 A Different View

Achievement Objectives

- draw diagrams of solid objects made from cubes (Geometry, level 4)
- make a model of a solid object from diagrams which show views from the top, front, side, and back (Geometry, level 4)

In these activities, students have to visualise 2-dimensional drawings as the 3-dimensional objects they represent, not only from the given direction but also from various other directions. Some students (and adults) find such visualisation easy; others find it very difficult. Students can develop their visualising skills by practising with actual objects.

ACTIVITY ONE

In question **1**, the students work with symmetrical objects that have known shapes. This gives you an opportunity to discover which of your students are unable to visualise in 3 dimensions. If possible, have models of the different solids to discuss with these students. You can then talk about the visual clues that they need to recognise (for example, the dot in the centre of the bird's-eye view of object **b**, which is the only clue that separates it from the bird's-eye views of **a** and **e**). You could extend question **1** by asking your students which of the objects have other possible side views (**c**, **d**, and **f**) and why they have alternative views when the others don't. (They are not fully symmetrical about their vertical axis.)

Question **2** is more difficult than the first question because the solids are no longer regular. In each case, part of the object is hidden, and if the students were only given the isometric view, they couldn't be confident about the hidden views.

The important thing is to use all the information. In each case, the students are given 5 views of the solid, and there is only one shape that will fit all those views. The students should pay attention to the internal lines in views 1–4. These lines show that there is a change of plane at this point (rather than a single, continuous face).

Question **2** should be done as a practical task. Make sure that you have a good supply of multilink cubes so that your students can build the solids, perfecting them as they make them consistent with all the views.

You may wish to discuss the concept of isometric drawing with your class. Isometric drawing is one way to represent 3-dimensional objects in 2 dimensions. The technique is especially suited to objects with sides that are rectangular or square. The key ideas are:

- The object is turned so that you are looking at a vertical edge (not a face).
- You can choose which vertical edge is the leading (front) edge. Turn the object until you find the direction that reveals its structure best.
- Only the edges visible from the chosen direction are drawn, and in an object made from blocks, the joins between blocks are drawn only where there is a change of plane.
- The 3 dimensions (length, breadth, and height) are represented by 3 axes that form angles of 60 degrees with each other. For convenience, we often use isometric dot paper, which has rows of dots set at the correct angles and avoids the need to measure lengths.

Most students learn basic isometric drawing (using isometric dot paper) very quickly and enjoy it. For a challenging activity involving isometric drawing, see Two Views in *Geometry: Book 2*, Figure It Out, Years 7–8.

ACTIVITY TWO

In this activity the students work from objects to diagrams. While a ruler is appropriate for question **1**, questions **2** and **3** call for freehand drawing using a soft pencil (not felt-tip pens). Those who have artistic skills may wish to render (shade) their drawings. Rendering is especially useful for bringing out the 3-dimensionality of objects (as in question **2f**). Note that the students should use isometric dot paper only where specified, that is, in questions **1c** and **2d**.

Page 12 Loads of Lamingtons

Achievement Objectives

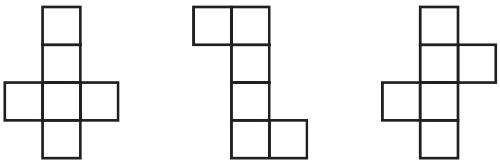
- design and make containers to specified requirements (Geometry, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)

ACTIVITY

This activity fits with a level 4 suggested learning experience: "Students should be investigating the construction and use of packaging and containers (including regular polyhedra) in the commercial world, for example, to minimise wastage, and for stacking (tessellating end faces)."

The design of nets for boxes and packets is a sophisticated skill that can make or break the success of a product. Consumers will usually choose an attractively packaged product ahead of a poorly packaged product, even if they know that they will throw the packaging away the moment they get home. You may be able to show your students examples of attractive and different packaging opened out to reveal the net.

If your students have had little prior experience designing nets, they could begin with nets for a cube. By doing this, they will see that there can be different ways of designing a net that will fold to give the same result. The net for a cube is made from 6 squares because a cube has 6 square faces. Your students may wonder if *any* combination of 6 squares will give the net of a cube. (It won't.) You may like to challenge them to find all the possible different nets. (Rotations and reflections don't count as different.)



If you begin with the net of a cube, you will also be able to demonstrate and discuss design and assembly techniques. You can talk about the importance of providing gluing tabs, cutting tidily, scoring lines before folding, and using sparing amounts of PVA to make a neat join. Your students should then be able to manage the more complex task.

When designing the boxes for the lamingtons, the students should remember that the box must have a lid with a flap, and although this is not part of the net, they may also consider what else could be part of the packaging. For example, should there be a paper liner, a separator between layers, or an outside wrapper?

Different-sized containers are possible, depending on how the lamingtons are arranged. The students could work in pairs on the design activity, beginning by spending time considering the different options, discarding all but three, and being able to justify their decisions.

The Answers contain nets for three possible boxes. Students will realise that the dimensions shown are the absolute minimum and would result in a very tight fit for the lamingtons. They may wish to design their nets with slightly larger dimensions in each case. Some products, such as biscuits, do not require extra space. In fact, they travel better if they are in boxes or packets where there is a tight fit. This is probably not the case with lamingtons.

Page 13 Shifting Shapes

Achievement Objectives

- describe patterns in terms of reflection and rotational symmetry, and translations (Geometry, level 3)
- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)

ACTIVITY

In this activity, students use the language of transformations to describe how a single pattern block can be used to generate frieze-type designs. The students may be surprised at the number of repeating patterns that can be created solely from one simple shape. Patterns generated in similar ways are found in the art, architecture, and artefacts of many cultures.

Transformations are an interesting and useful mathematical tool. Apart from the understandable confusion between the similar-sounding words “translation” and “transformation”, students quickly develop meaning for translation and the other three common transformations: reflection, rotation, and enlargement. But they are not likely to understand them fully or be able to describe them precisely until they reach senior high school.

In this activity, the students are asked to:
- name the transformation that shifts the trapezium from one position to the next
- define the transformation more precisely by giving the details listed in the panel.

Each pattern is generated by a single pattern block. The student “traces” the outline of the block, moves it in a defined way to a new position, traces its outline, and then moves it again. The students should use a trapezium block to help them understand the transformations involved.

Question **1a** involves repeated reflections in a series of vertical axes (mirror lines). Trapezium 1 is reflected in a vertical axis through its far right point to give the second trapezium. This is then reflected in a vertical axis through its far right point to give the third, and so on.

The pattern in question **1b** involves translation. The challenge is to describe it so that someone else could follow the instructions and get the same pattern. The Answers suggest describing the movement in terms of dots along and dots up or down.

Question **1c** involves a series of rotations, alternately anticlockwise and clockwise, about a centre that is different each time. The angles between the lines of dots on the isometric dot paper are all 60 degrees, so the angle of rotation can be defined precisely without measurement.

Two transformations are used in question **1d**. The trapezium is translated 1 unit to the right and then reflected in the horizontal axis that coincides with the short parallel side of the trapezium.

It will be obvious that both reflection and translation are involved in question **1e**, but because the axes are sloping (neither vertical nor horizontal), the students may not know how to define them. The Answers use the word “negative” to clarify which of the possible sloping axes they are referring to. A negative slope is one that goes downhill as your eyes move across the page from left to right.

Although it looks complicated, the pattern in question **1f** involves only a series of 60 degree rotations. The particular design is determined by the continually changing centre and whether the rotation is clockwise or anticlockwise.

It is probably best to visualise the pattern block in question **1g** as rotating 60 degrees clockwise and then 60 degrees anticlockwise about its bottom right vertex before being translated 2 units to the right. The second of these three transformations adds nothing to the pattern but makes it easier to define the following move.

Question **2** makes continued use of the fact that the angles defined by the isometric dot paper are all 60 degrees.

At this stage, the students will be doing well if they can describe what is going on in fairly simple terms and in such a way that someone else has a good chance of following their description. More rigorous definitions can be introduced at later levels.

You or your students may like to collect and display examples of patterns that repeat in a linear fashion. These could be in the form of pictures, photographs, drawings, or repeating patterns on actual objects. The students could try and describe *how* the patterns are repeated, using the language they have learned. In this way, you can help them make links between theory and practice.

Page 14–15 Kōwhaiwhai

Achievement Objectives

- design and make a pattern which involves translation, reflection, or rotation (Geometry, level 3)
- describe the reflection or rotational symmetry of a figure or object (Geometry, level 4)

ACTIVITY

This activity builds on the ideas of Shifting Shapes (page **13** in the students' book). The students create patterns using single transformations and combinations of transformations.

Kōwhaiwhai are often used as ornamentation on the uncarved heke (rafters) of a wharenui. Unlike carvings, kōwhaiwhai are not normally made by a master artist. However, their intricate, elegant curves require a designer's eye. Today, a cut-out stencil is sometimes used for the repeated design, and the painting is done as a team project involving young as well as old. All kōwhaiwhai have meanings and are not just ornamental.

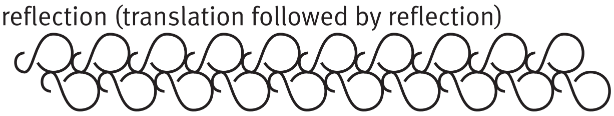
You could have your students study the geometry of kōwhaiwhai as a purely mathematical topic but, if possible, you should take them to a building or museum where they can see actual examples. You could ask someone knowledgeable to explain how they were made and what their significance is.

Both questions in this activity involve practical work, and the students may create very different patterns. Some may choose to modify the given designs, while others may try to create something quite different.

Question **2** says that the three basic transformations can be combined to produce more complex patterns. The complete list of transformations used for kōwhaiwhai is as follows:

- translation

- glide reflection (translation followed by reflection)

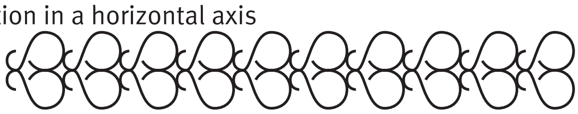


- reflection in a vertical axis

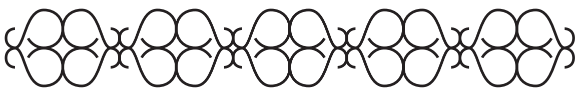
- rotation of 180 degrees

- rotation of 180 degrees followed by reflection in a vertical axis

- reflection in a horizontal axis



- reflection in a horizontal axis followed by reflection in a vertical axis.



Auckland Museum publishes an excellent educational kit, *Kōwhaiwhai Tuturu Māori*, which gives a background to the history and significance of kōwhaiwhai, examples of kōwhaiwhai, and an illustrated list of the mathematical transformations used. It is available online at www.akmuseum.org.nz/downloads/Kowhaiwhai.pdf

This site has a number of kōwhaiwhai in colour, together with their meanings: http://whakaahua.maori.org.nz/kowhai.htm

Precision Flying

Achievement Objectives

- construct triangles and circles, using appropriate drawing instruments (Geometry, level 4)
- share quantities in given ratios (Number, level 5)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

Rough drawing methods were sufficient for Alien Action (page **7** of the students' book), but in these activities, the students follow geometrical instructions and use some formal construction techniques to achieve a high level of accuracy. They should use only a sharp pencil for all parts of the drawing process, though the finished designs can be coloured. Make sure that the students know that they are expected to leave their construction lines in, so they should keep them tidy.

The geometrical terms used in **Activity One** include baseline, segment, arc, bisect, and midpoint.

ACTIVITY ONE

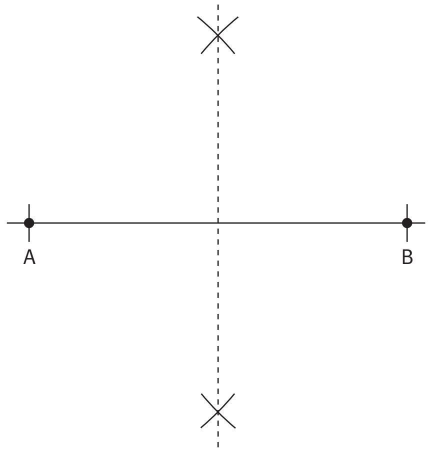
Question **1a** makes the distinction between a *line*, which is infinite in length, and a *line segment*, which is a finite part of a line. It also introduces the idea of using a compass not just to draw circles, but to find the point that is at a fixed distance from 2 other points (in this case, point B, which is 16 centimetres from A and 10.8 centimetres from C).

In question **1c**, the students use a compass and ruler to bisect the line BC. This method can give a more accurate result than measuring the length and splitting the result in two. It also creates a line through the midpoint at right angles. The method is illustrated in the diagram below, using the line AB.

The process is:

- Set the compass to a radius that is a little over half the length of the line segment to be bisected.
- Swing pairs of arcs above and below from A and B so that they intersect.
- Join the points of intersection to bisect the line segment.

The (dotted) line is known as the *perpendicular bisector* or *mediator* of AB.



There are a number of websites devoted to flags of the world (see below). Any of these will give the answer to question **2d**.

ACTIVITY TWO

This activity involves working with enlargement. The students need to work out how much larger the 16 centimetre by 10.8 centimetre rectangle is than the given illustration, then multiply the measurements in the illustration by this factor to find the measurements they need for their larger Tanzanian flag.

As an extension, groups of students could investigate the geometry of one or two other flags of the world and share their findings with the class. Where flags have features such as stars or crescents, the students could describe the geometry of the features even though they are unlikely to be able to give a complete set of instructions for drawing the flag.

Good websites include:
www.crwflags.com
www.flags.net/fullindex.htm
www.cia.gov/cia/publications/factbook/docs/flagsoftheworld.html

Page 17

Ball Paths

Achievement Objectives

- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 4)
- use their own language, and mathematical language and diagrams, to explain mathematical ideas (Mathematical Processes, communicating mathematical ideas, level 4)

ACTIVITY

Many students are keen sportspeople. This activity taps that interest and experience, tying it to the geometrical concept of *locus*, the path traced by a moving object. A level 4 suggested learning experience is "Students should be drawing loci (the paths of objects moving in space) including the flight path of balls, the locus of points which are a fixed distance from a given point (circle), a given straight line (a parallel line), and a circle (a concentric circle)."

Some students have trouble with the concept of locus, but you can refer them to familiar examples that will help explain it:

- the mark on the floor where a heavy object has been dragged across it
- the mark on the road where a car has skidded
- the path traced through the snow by a skier
- the patterns made by a child when doing a finger-painting
- the vapour trail that a jet plane has made through the sky
- the footprints left in the sand by a person walking along the beach.

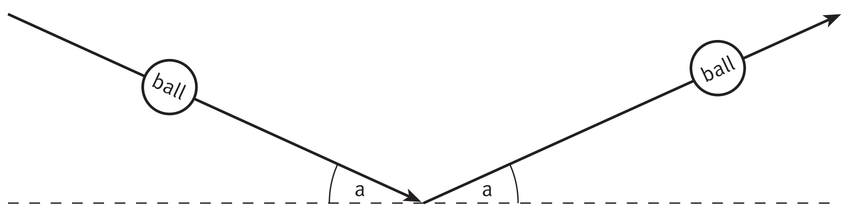
In each of these examples, the movement of an object or a person leaves a visible path that can be followed. A ball moving through the air doesn't leave a record, but it still has a locus.

Your class probably has "experts" in a number of the sports featured in question **1**, and they may be happy to talk to their classmates about how the ball moves in typical plays. They will need to show what they mean, using diagrams on the whiteboard. If you are not a sportsperson yourself, you may have a knowledgeable colleague who is willing to come into your class and contribute to the discussion.

In most of the situations listed, a person applies force to the ball, sending it up or along until gravity brings it back to earth. In the case of the tennis serve and the fast-bowled cricket ball, the sportsperson generally applies force in a downwards direction.

Sportspeople practise long hours to discover the best way to impart force to the ball, taking into account gravity and wind conditions. The degree of judgment involved in getting the ideal locus for the ball is amazing. The slightest deviation at the point of last human contact with the ball can make it fall short, go too far, or go wide of its mark. You could have a lively discussion about this with your students.

You could also consider in more detail what happens when a ball comes into contact with the ground, the backboard in basketball, or the cushion on a pool table. Those who play sports probably understand intuitively that the angle at which the ball strikes and leaves the ground (or backboard or cushion) is the same. This fact can be illustrated in a simple diagram:



Of course, this only works if the ground is reasonably hard and smooth. A golf ball landing in sand displays somewhat different behaviour. Spin also has an effect on the angle. What about a rugby ball? Well, that is something different again!

Question **3** gives students a chance to consider sports that don't appear in question **1**, such as volleyball and badminton.

Possible extension activities include:

- investigating the movement of a ball as it bounces successively off the cushion on different sides of a pool table
- finding a way of measuring the bounce of different balls when they hit a hard surface
- finding the angle of elevation that will give the greatest distance for a ball launched with constant force. This could be done with a cardboard tube, table tennis ball, and rubber band and could be a very good technology and mathematics project.

Pages 18–19 Starry-eyed

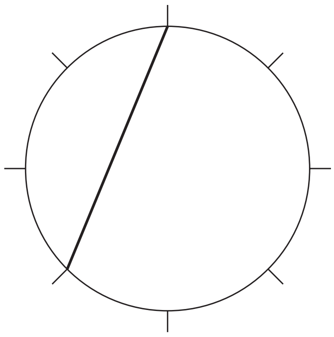
Achievement Objectives

- apply the symmetries of regular polygons (Geometry, level 4)
- construct triangles and circles, using appropriate drawing instruments (Geometry, level 4)
- use equipment appropriately when exploring mathematical ideas (Mathematical Processes, problem solving, level 4)

These activities involve investigating and creating patterns made by joining points around the circumferences of circles. Copymasters are provided to assist the students.

ACTIVITY ONE

The students are unlikely to have any problems with questions **1a–b**, but they may have trouble explaining (for question **1c**) why the 8/5 and 8/3 star polygons are the same. The reason can be seen from the diagram. Counting around 3 from one direction is the same as counting around 5 from the opposite direction. This is true because 3 + 5 = 8. The only difference, therefore, between the two star polygons is the order in which the chords (lines) are drawn.



Before they begin to draw the star polygons in question **2**, the students could predict which ones will be the same shape (10/3 and 10/7; 10/6 and 10/4) and why they expect that this will be the case (because 3 + 7 and 6 + 4 both equal 10).

When doing question **3**, your students could again predict which 9-point stars will be the same, and why, before they begin the drawings. If they are able to predict that 9/8 and 9/1, 9/7 and 9/2, 9/6 and 9/3, and 9/5 and 9/4 will each be the same, they can predict that there will be 4 different 9-point stars. They can confirm this by drawing them all.

Question **4** tests the students' understanding of the principles explored in the earlier questions as they predict the outcomes and then draw the stars to check their predictions.

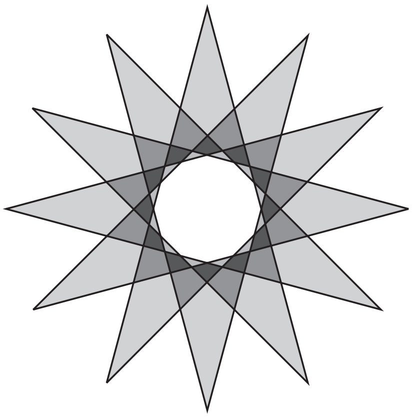
ACTIVITY TWO

It will take the students some time to complete this intricate design. The task is not difficult, but it demands a sharp pencil and a good degree of accuracy. If your students colour it as suggested, they may find that coloured pencils give them a finer result than felt-tip pens.

To answer question **2**, the students will need to experiment with various possibilities using another 12-point circle. If they do this, they should discover that the pattern is based on a 12/5 or 12/7 star polygon with every second point coloured to give a 6-pointed star. The students could make the pattern, starting with the same cluster of circles as for question **1**.

As a further activity, the students could draw a number of identical star polygons and colour the different repeated parts of the shape (perhaps using felt-tip pens this time) to reveal a variety of patterns. For example, they will discover a regular dodecagon in the centre of a 12/5 star polygon, surrounded by various triangles and quadrilaterals, all with their own reflective symmetry.

A website featuring star polygons is
http://mathworld.wolfram.com/StarPolygon.html



Pages 20–21 Find the Spot

Achievement Objectives

- specify location, using bearings or grid references (Geometry, level 4)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 4)

The ancient Babylonians believed that the year was made up of 12 moons, each lasting 30 days, which gave a total of 360 days in a year. The Babylonians are thought to have been the first serious astronomers, so they may have been the first to divide a circle into 360 degrees. 4000 years later, we still use degrees to measure directions and angles. Aircraft, ships, and orienteers all use compass bearings in degrees to describe their direction.

Navigation is the process of finding your way on land, at sea, or in the air. Students who are involved in sailing, orienteering, or tramping or who are interested in gaining their pilot's licence will already know that navigation skills are useful and sometimes vital. The two basic tools for navigation have long been a map and a compass. At sea, a chronometer and a sextant have also been essential tools for the last 250 years. Modern direction-finding devices that use global positioning systems (GPS) are now increasingly replacing the old methods.

ACTIVITY

In this activity, the students locate particular spots on an illustration using compass bearings. They then make up their own sets of clues, based on locations around their own school grounds.

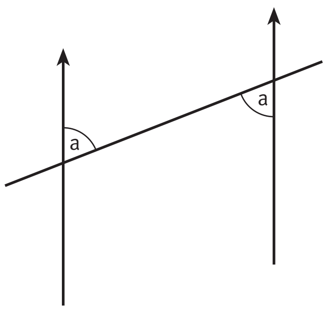
Before setting this activity, you may find it best to specifically teach your students how to use a protractor on a map. Students often get confused by the two competing scales on a protractor and have trouble using a protractor that goes only to 180 degrees to measure angles that are greater than this. (You may like to show them a circular protractor.)

You should also emphasise the two points made in the speech bubbles: bearings are always measured clockwise from north, and they are always written as 3-digit numbers.

Question **1** is best done using back bearings, as explained in the panel. If we know the bearing of B from A, the back bearing gives us the bearing of A from B. To find the back bearing:

- add 180 degrees to the bearing if it is less than 180 degrees (for example, the back bearing of 081° is 81 + 180 = 261°)
- subtract 180 degrees from the bearing if it is greater than 180 degrees (for example, the back bearing of 308° is 308 – 180 = 128°).

Back bearings make use of the fact that alternate angles are equal. Alternate angles are formed whenever a line cuts across a pair of parallel lines, as in this diagram:



If the students find the back bearings from two features, this will give them two lines that cross at the spot they are trying to find. The other clue is an "extra"; your students should use it to check that they have found the correct location. Question **2a** asks why distance is not needed. The reason is as above: 2 bearings always cross at a single point unless, of course, they are the same. If only 1 bearing is given, distance also must be given to fix a location.

Before starting this activity, find out if there is a school map available. It needs to be fairly accurate and to scale if it is to be used in question **3**. As an alternative, if your school has orienteering compasses, you may wish to do this task outside.

As extension activities, the students could investigate:
- the difference between true north and magnetic north
- the technologies that are used for modern navigation
- the history of navigation
- why the inventions of the chronometer and the sextant were so important.

Pages 22–23 Treasure Island

Achievement Objectives
- specify location, using bearings or grid references (Geometry, level 4)
- pose questions for mathematical exploration (Mathematical Processes, problem solving, level 4)

ACTIVITY

This activity goes beyond the previous activity in asking students to locate a series of connected points, using bearings and distances, as they would in orienteering. It gives them further practice using a protractor and stating bearings clockwise from north as well as working with scale distances. It also creates a context in which accuracy is very important. The cumulative effect of small inaccuracies in each clue will mean that the instructions are worthless and that the treasure will be lost forever!

When planning where to leave the clues, the students should:
- choose a resting place for the treasure that is some distance from the campsite
- remember that they are defining the location of the next clue, not describing how to get there
- zig-zag a bit to make treasure hunters work for their money
- use some of the landmarks, but for obvious reasons, they should not name these in their instructions.

Each clue must have:
- the bearing in degrees (measured clockwise from north and written using 3 digits)
- the distance from one point to the next as the crow flies.

Example:
- clue 1: bearing 190°; 2.75 km
- clue 2: bearing 305°; 2.5 km.

If your students enter into the spirit of the activity and want to invent cryptic verses or lines to go with their clues, that's fine.

Question **2** involves the students swapping their clues with a classmate and seeing if they can locate the spot where the treasure is buried. For them to succeed:
- the information they are given in the clues must be accurate
- they must interpret the details correctly
- they must carry out the instructions with a high level of accuracy.

As an extension activity, your students could investigate orienteering. They will find a useful introduction on this website: www.us.orienteering.org/OYoung

Page 24 Blasting Bugs

Achievement Objectives

- specify location, using bearings or grid references (Geometry, level 4)
- make conjectures in a mathematical context (Mathematical Processes, developing logic and reasoning, level 4)

GAME

No matter where a person stands (except at the poles), every other point on the face of the earth is either north or south of them or west or east of them. In other words, from their perspective, the world can be divided into 4 regions defined by direction: NE, NW, SE, and SW. These are known as the 4 quadrants of the compass.

The same concept is used in this activity. If a student guesses that the BB bug is in square D3 when it is really hiding in square G1, their classmate would tell them that the bug is in the quadrant SE of D3 (the area that is to the south and the east of D3).

Note that if a student's guess is in the correct north–south or east–west line, their classmate will respond by naming the correct direction (N, S, E, or W), not by naming a quadrant.

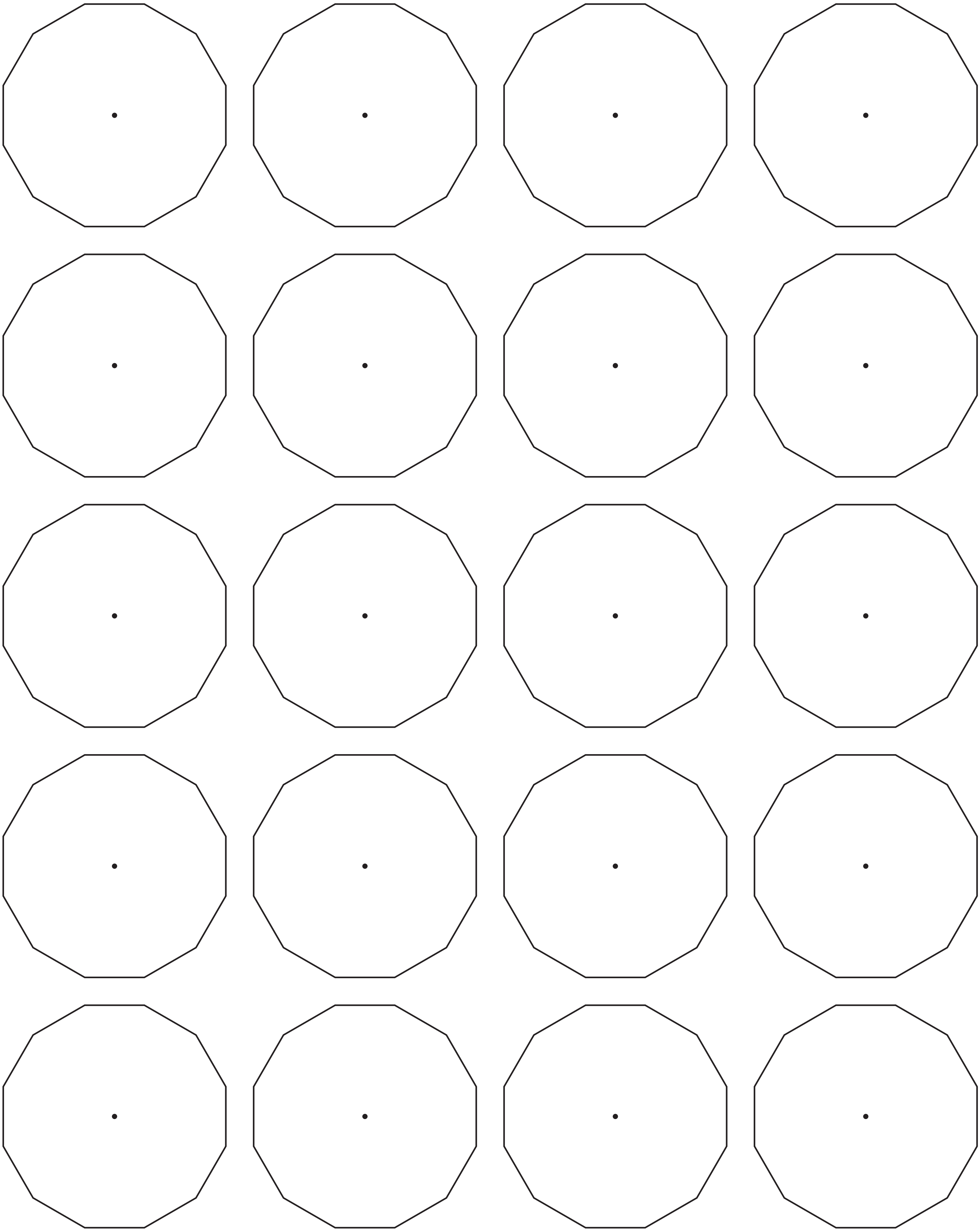
It is probably best not to try and link this grid too closely with the system of x and y axes that are the standard for algebraic functions. The two systems have these important differences that are likely to confuse students:

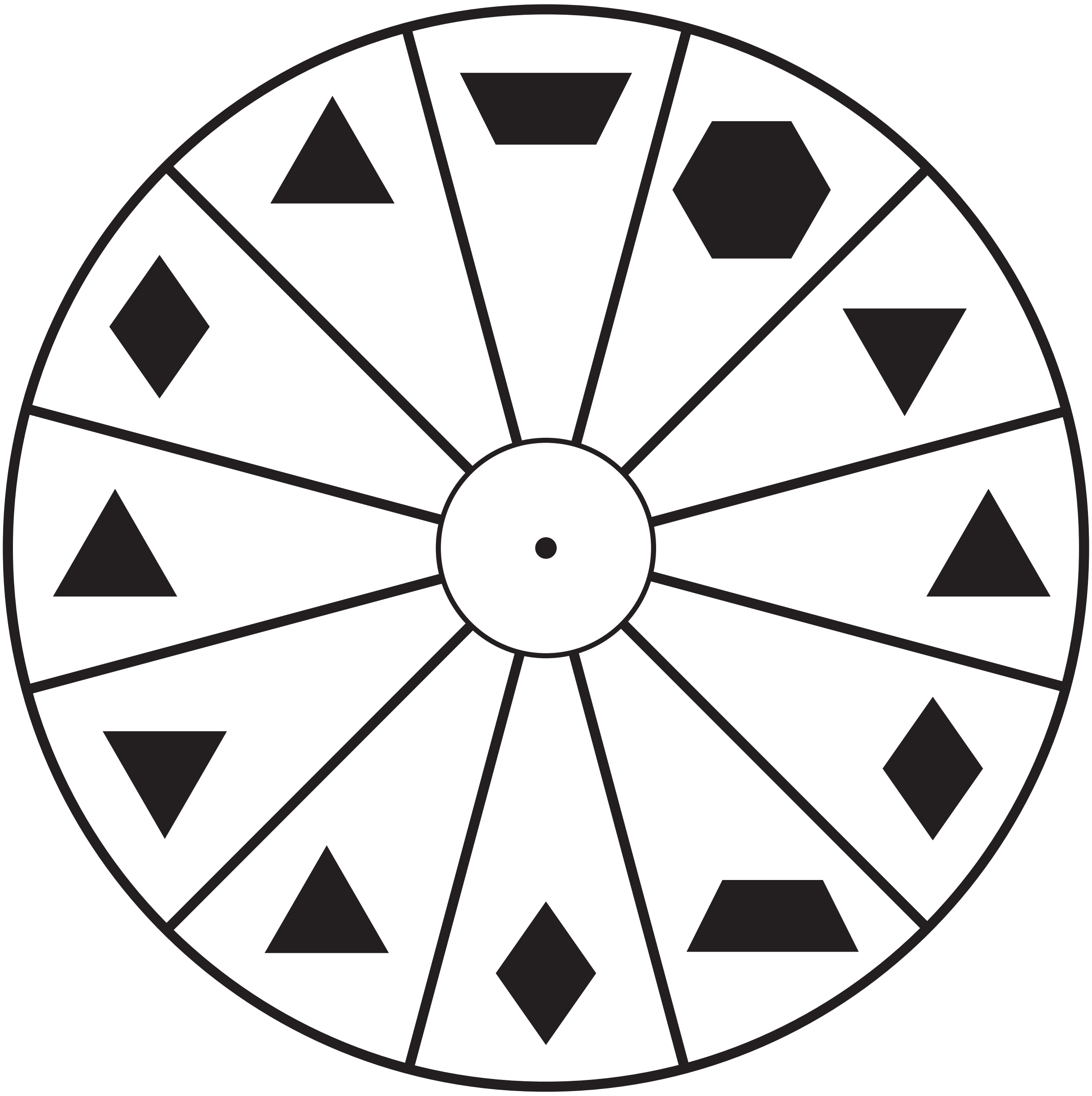
Standard x and y axes	Blasting Bugs grid
The grid reference defines a point.	The grid reference defines a cell.
The quadrants are defined from a fixed point ([0, 0], the origin).	The quadrant is redefined after every move.
The axes are labelled with integers.	The axes are labelled using the alphabet and whole numbers.

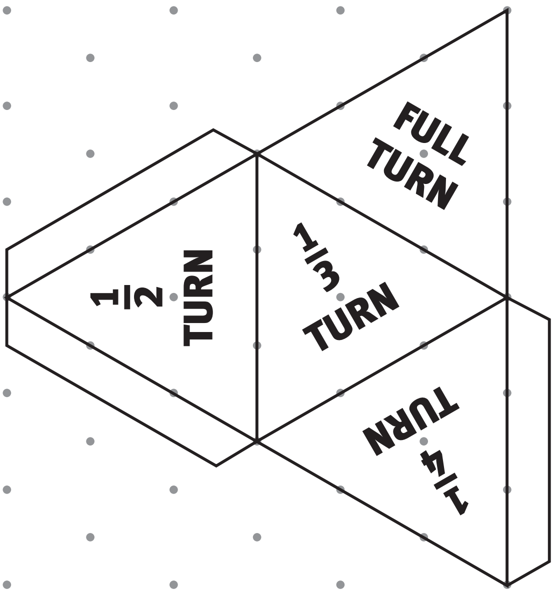
You could also discuss these ideas with your students:

- Although there is a northern hemisphere and a southern hemisphere, there is no true western or eastern hemisphere (these terms are, however, sometimes used). Why is this? What do we mean when we talk about "the West" and "the East"?
- Although NE (for example) is used in this activity to define a quadrant, it is also used as a precise compass direction. What does this mean, and what are the other directions marked on a traditional compass (for example, SSW)?
- What is the significance and history of the line of longitude that passes through Greenwich?

Shaping Up



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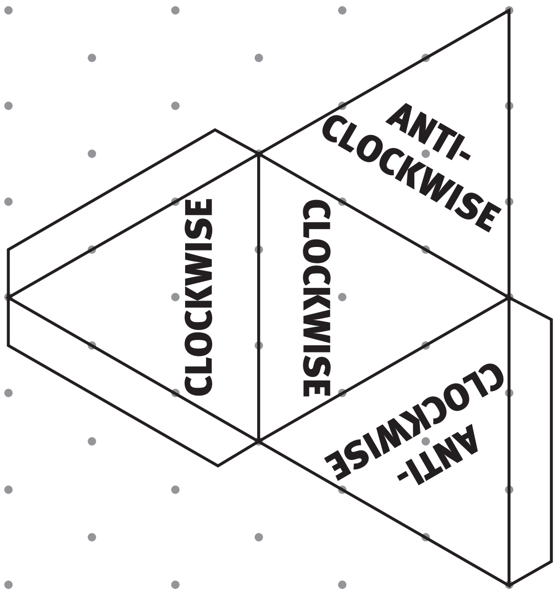
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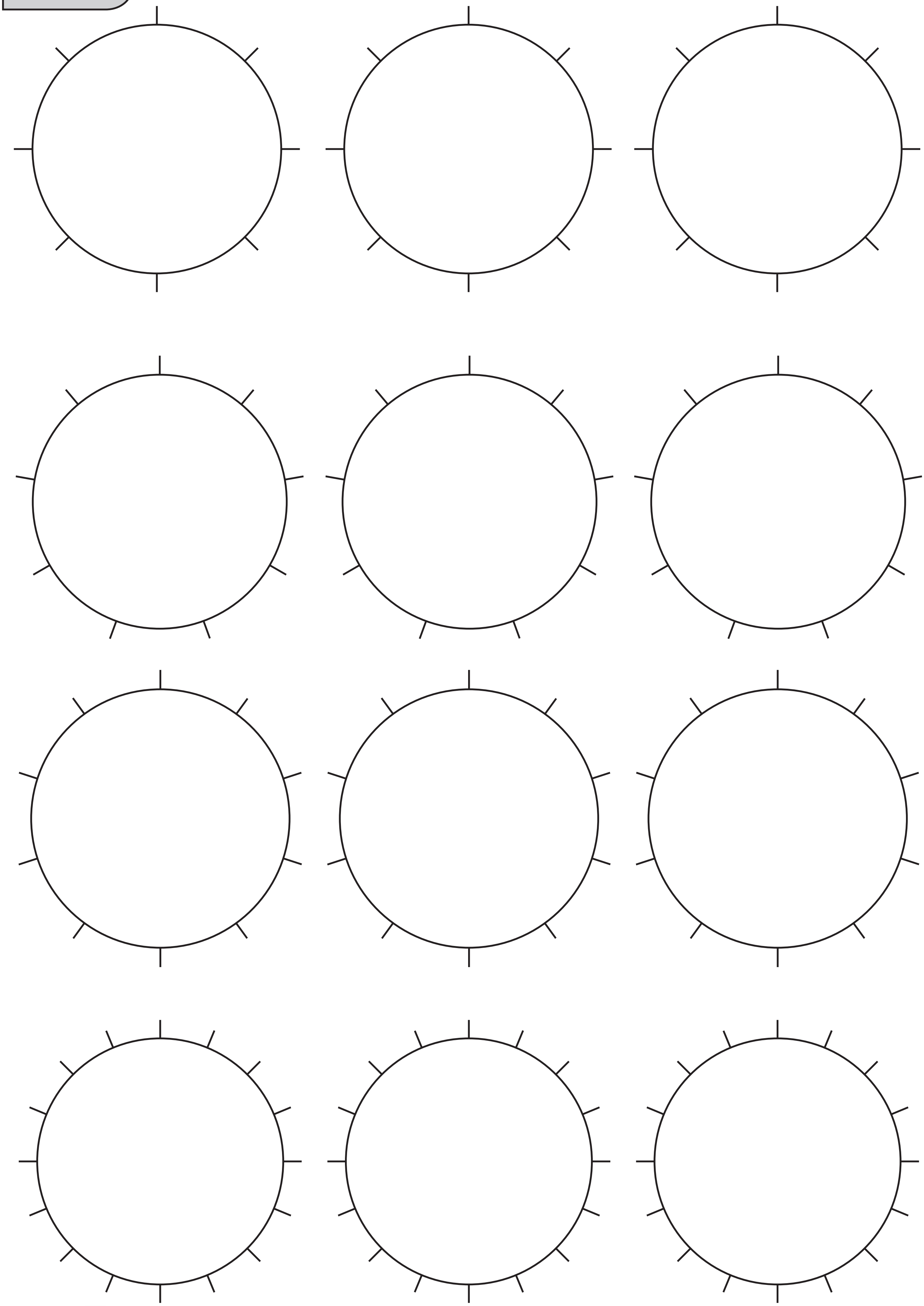
Starry-eyed



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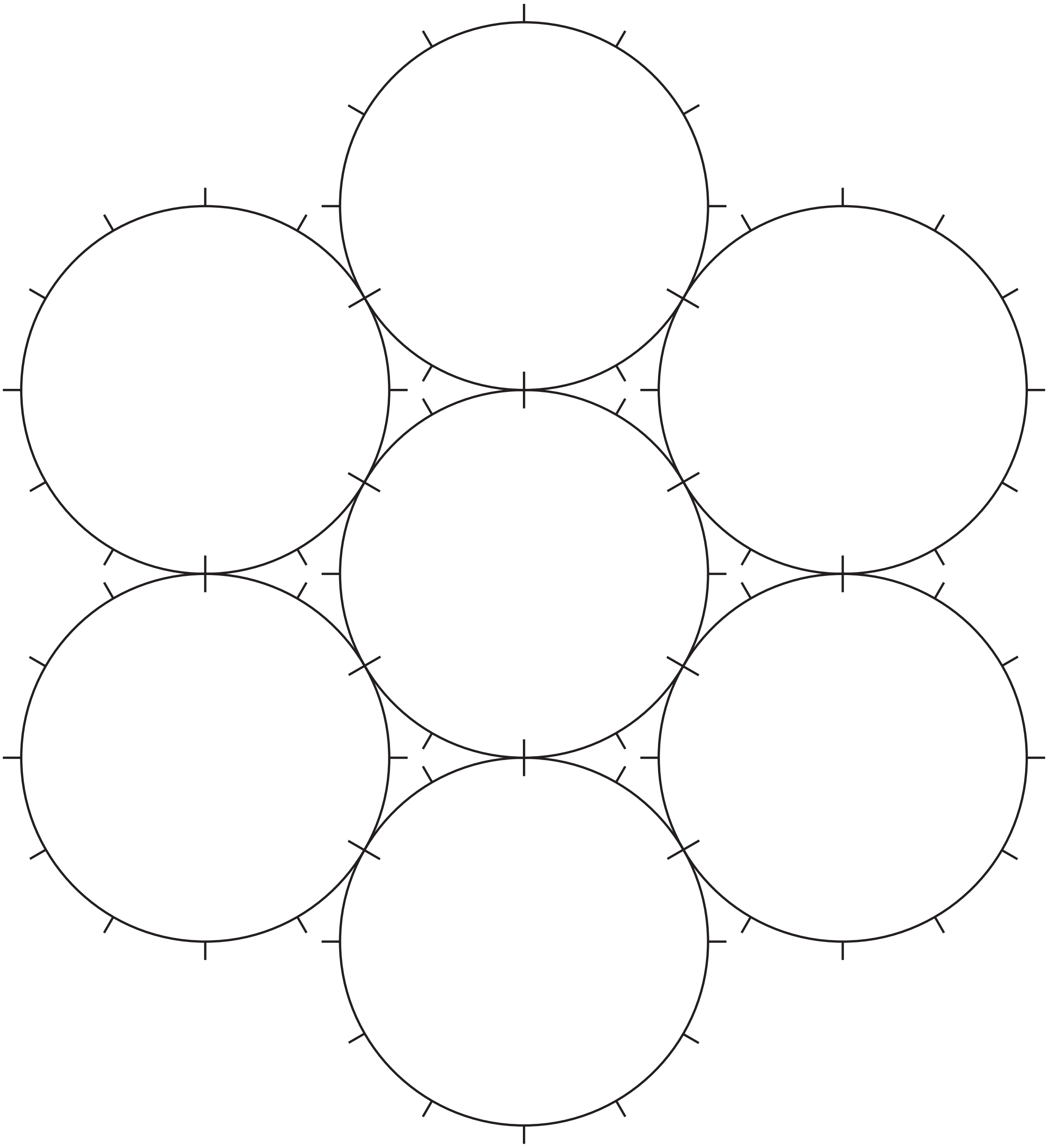
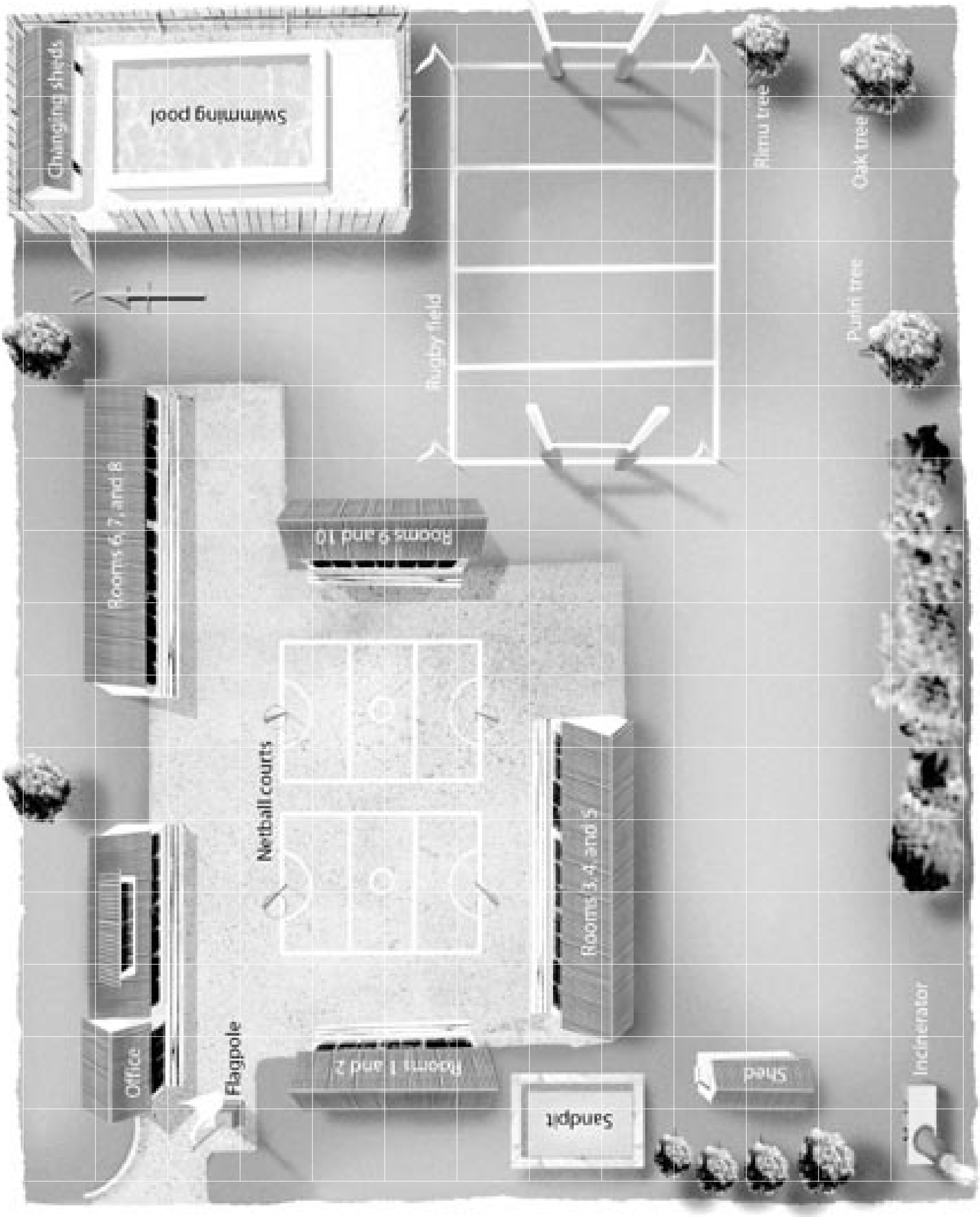
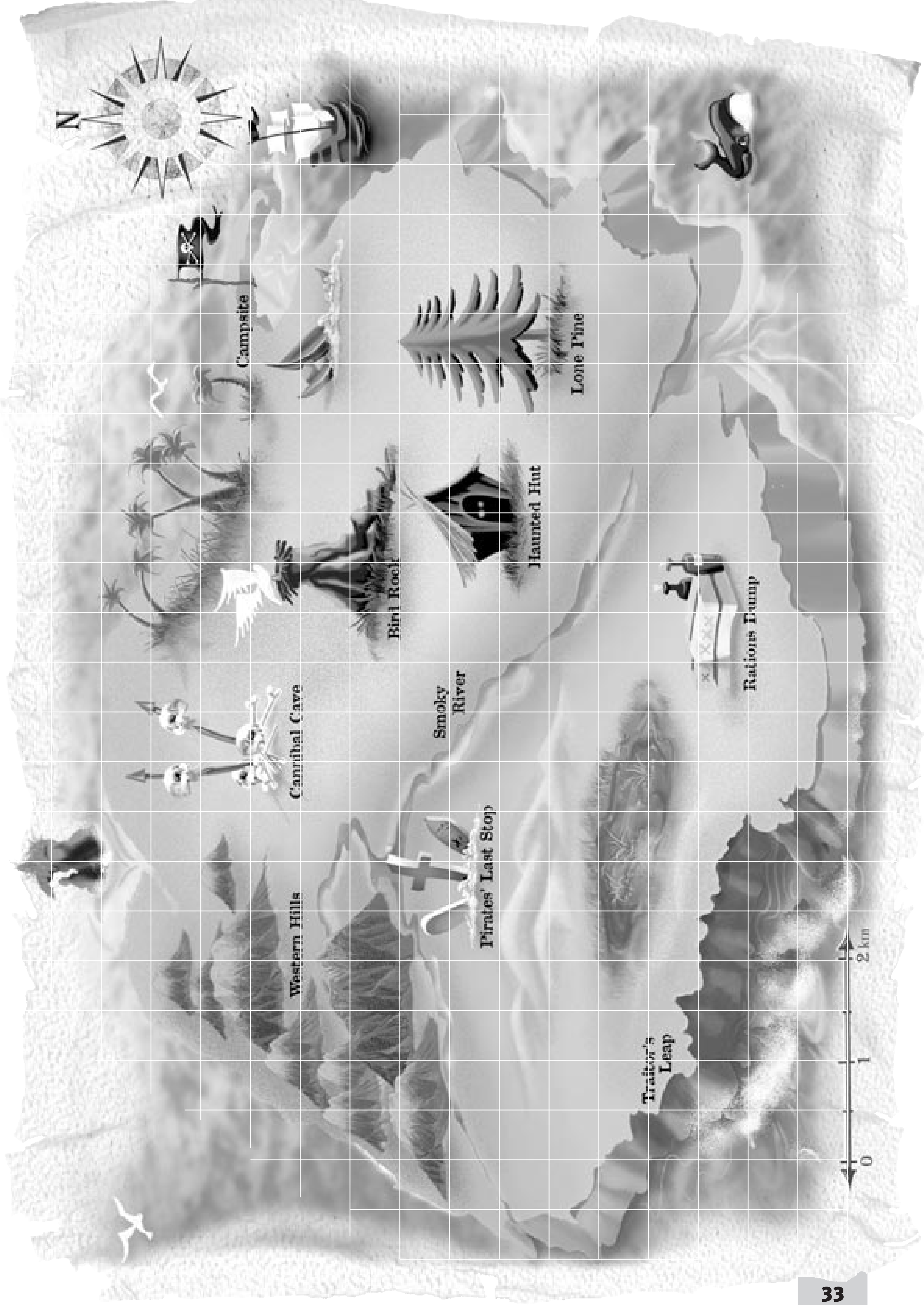


Copymaster **Find the Spot**



Treasure Island



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