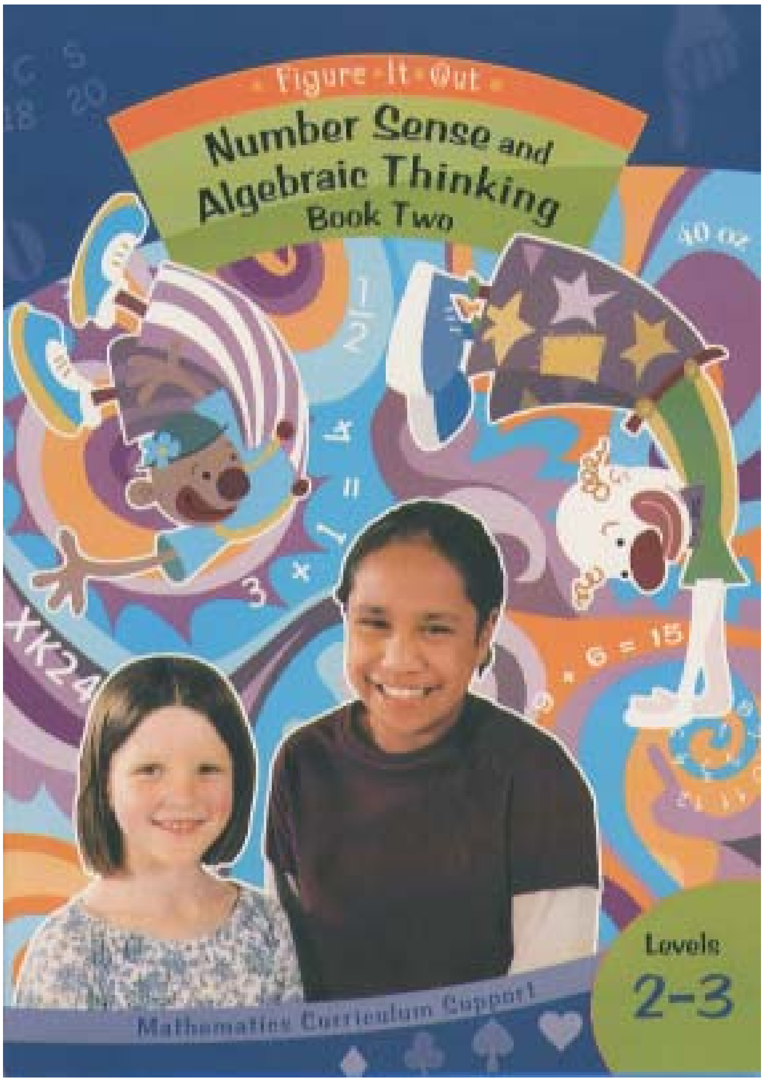
Answers and Teachers' Notes



MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

The books for levels 2–3 in the Figure It Out series are issued by the Ministry of Education to provide support material for use in New Zealand classrooms. These books are most suitable for students in year 4, but you should use your judgment as to whether to use the books with older or younger students who are also working at levels 2–3.

Student books

The activities in the student books are set in meaningful contexts, including real-life and imaginary scenarios. The books have been written for New Zealand students, and the contexts reflect their ethnic and cultural diversity and the life experiences that are meaningful to students in year 4.

The activities can be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. Also, the activities can be used to fill knowledge gaps (hot spots), to reinforce knowledge that has just been taught, to help students develop mental strategies, or to provide further opportunities for students moving between strategy stages of the Number Framework.

Answers and Teachers' Notes

The Answers section of the *Answers and Teachers' Notes* that accompany each of the *Number Sense and Algebraic Thinking* student books includes full answers and explanatory notes. Students can use this section for self-marking, or you can use it for teacher-directed marking. The teachers' notes for each activity, game, or investigation include relevant achievement objectives, Number Framework links, comments on mathematical ideas, processes, and principles, and suggestions on teaching approaches. The *Answers and Teachers' Notes* are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure

Using Figure It Out in the classroom

Where applicable, each page starts with a list of equipment that the students will need in order to do the activities. Encourage the students to be responsible for collecting the equipment they need and returning it at the end of the session.

Many of the activities suggest different ways of recording the solution to the problem. Encourage your students to write down as much as they can about how they did investigations or found solutions, including drawing diagrams. Discussion and oral presentation of answers is encouraged in many activities, and you may wish to ask the students to do this even where the suggested instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum,* page 7

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider other ways of solving a particular problem.

Figure It Out

Number Sense and Algebraic Thinking Levels 2–3, Book Two Answers

Page 1: Quick Thinking

Game

A game using basic facts. A completed game board might look like this:

Statement	Dice numbers		Point (✓)
Add to 10	6	4	✓
Difference of 2	5	3	✓
Two even numbers	6	2	✓
Sum of 8	4	✗	
Twice the first number	3	✗	
Sum less than 5	1	2	✓
Two odd numbers	3	5	✓
Two numbers greater than 3	5	6	✓
Sum greater than 10	6	4	
Difference of 3	5	1	
	Total points		6

(Wrong statements can result from entering a wrong number, which you cannot change, or having empty boxes towards the end that you have to fill with an ✗.)

Pages 2–3: An Odd Spell of Maths

Activity

1. a. Other possible words include:
ME: 1 + 9 = 10 or 11 + 9 = 20
IT: 17 + 5 = 22 or 17 + 15 = 32
AS: 3 + 21 = 24 or 13 + 21 = 34
HI: 7 + 17 = 24
AM: 3 + 1 = 4, 3 + 11 = 14, 13 + 1 = 14, or 13 + 11 = 24
AT: 3 + 5 = 8, 3 + 15 = 18, 13 + 5 = 18, or 13 + 15 = 28

b. The totals are all even. When 2 odd numbers are added together, the result is always even.

c. No. Odd + odd always equal even, regardless of what the digits in the tens column (or any other column except ones) are.

2. a. THE: 5 + 7 + 9 = 21 or 15 + 7 + 9 = 31
HEM: 7 + 9 + 1 = 17 or 7 + 9 + 11 = 27

b. Totals will vary, depending on the words chosen. Possible words include: AIM, ASH, ATE, CAT, EAT, HAM, HAS, HAT, HIM, HIS, HIT, ICE, ITS, MAT, MET, SAT, SEA, SHE, SIT, TEA, TIE.

c. The totals are all odd. When 3 odd numbers are added, the total is always odd. 2 odds make an even, and this even number and the next odd number make an odd.

d. i. There are four possible totals for MAT:
1 + 3 + 5 = 9
1 + 13 + 5 = 19
1 + 13 + 15 = 29
11 + 3 + 5 = 19
11 + 13 + 5 = 29
11 + 13 + 15 = 39

ii. They all add up to a number ending in 9, which is an odd number.

iii. The ones digits in the numbers that correspond to each letter are always 1, 3, and 5, which add up to 9.

3. a. Yes, Iosefo is correct. One possible word and its totals is TIME:
5 + 17 + 1 + 9 = 32, 5 + 17 + 11 + 9 = 42,
15 + 17 + 1 + 9 = 42, or
15 + 17 + 11 + 9 = 52
There are more than 40 possible words, including MEAT (18, 28, 38, 48), HEMS (38, 48), and CAME (32, 42, 52).

b. When 4 odd numbers are added, the total is always even. 2 odd numbers added together result in an even number. 2 sets of 2 odd numbers equals even + even.
Even + even = even.

4. The total for MATHEMATICS will be odd because it has an odd number of letters (11): 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 121

5. Some possible words and one of their totals include:
HE: 8 + 10 = 18
MAT: 2 + 4 + 6 = 12
THAT: 6 + 8 + 4 + 16 = 34
MATHS: 2 + 4 + 6 + 8 + 22 = 42
When even numbers are added, they always make an even sum.

6. Discussion will vary. You should have found out that odd + odd = even, odd + even = odd, and even + even = even. If you used counters or cubes to explore the numbers, you should have found that showing even numbers with counters or cubes will always make a complete rectangle of pairs, whereas odd numbers need 1 more or 1 less to make a rectangle of pairs.

Pages 4–5: Serious Circus Sums

Activity

1. **a.** $18. ($8 + $10)
   b. $36. ($14 + $22)

2. Sara: 3 children and 2 adults. ($11 + $16 = $27)
Nio: 1 child and 1 adult. ($5 + $10 = $15)
Siaosi: 2 children and 3 adults. ($8 + $22 = $30)

3. Sara: $45. ($15 + $30)
Nio: $20. ($5 + $15)
Siaosi: $55. ($10 + $45)

4. **a.** Answers and working will vary.
   b. Answers will vary.

5. **a.** 6
   b. $128. (One way to work this out is: $14 for each group of 4 children, so 6 x $14; plus $28 for 4 adults and $16 for 2 adults. [6 x 14] + 28 + 16 = $128)
   c. No. 6 x $20 + $60 + $30 = $210. This is $10 more than $200.

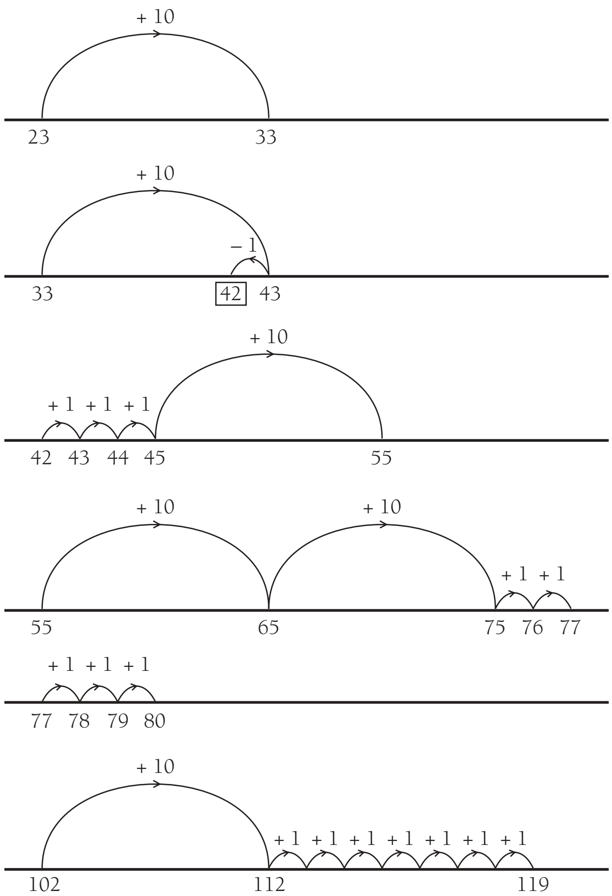
6. Answers will vary. Note that the pattern allows for 1–4 children and 1–4 adults. It doesn't extend beyond 4.

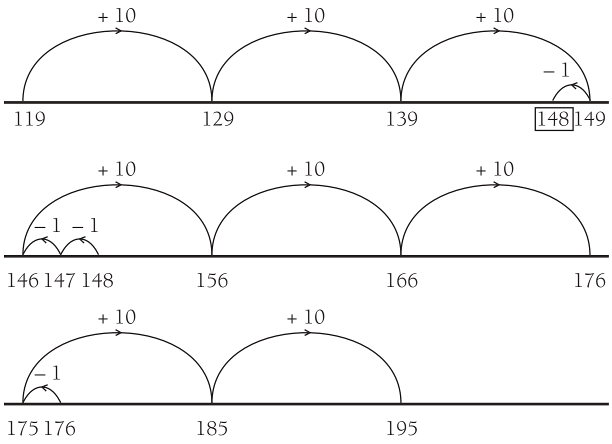
7. **a.** The afternoon prices for children begin at $5 for 1 child and add $3 for each extra child. The afternoon prices for adults are double these. They begin at $10, with $6 for each extra adult.
   b. The evening prices are multiples of $5 for children. Adult prices are 3 times those for the children, in multiples of $15.

Pages 6–7: Trimming Trees

Activity

1. **a.** Answers may vary in order and combinations. Possible number stories include:
23 + 10 = 33
33 + 10 – 1 = 42
42 + 1 + 1 + 1 + 10 = 55
55 + 10 + 10 + 1 + 1 = 77
77 + 1 + 1 + 1 = 80
102 + 10 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 119
119 + 10 + 10 + 10 – 1 = 148
148 – 1 – 1 + 10 + 10 + 10 = 176
176 – 1 + 10 + 10 = 195
   b. Number lines may vary. Number lines for the number stories given for **1a** are:





2. Problems will vary.
(The number story in the example shown is 56 + 10 + 10 + 1 + 1 + 100 = 178.)

3. a. Yes, because the + and – numbers in this expression cancel each other out. For example, 10 + 10 – 10 = 10

b. Paths will vary. Each path should still have the same solution as before.

Pages 8–10: Pick a Plate

Activity One

1. No. Explanations may vary. Caleb will often get more than Jessica when he doubles the odd numbers (for example, for the 9 in a number plate such as TS9696, Caleb would score 2 x 9 = 18 and Jessica would score 9 + 5 = 14). But Caleb has to halve the even numbers (for example, $\frac{1}{2}$ of 6 = 3), whereas Jessica only subtracts 2 (for example, 6 – 2 = 4). So with TS9696, Caleb would score 18 + 18 + 3 + 3 = 42 and Jessica would score 14 + 14 + 4 + 4 = 36.

2. a. Jessica has the highest total score. The table could look like this:

	Caleb		Jessica	
Number plate	Score	Score difference	Score	Score difference
RR4578	30		30	
ZQ4731	24		28	+4
SZ8827	23		24	+1
TG9915	48	+4	44	
WY4328	13		16	+3
YQ8888	16		24	+8
PJ2564	16		16	
RB3695	37	+1	36	
UG7006	17	+1	16	
XK2468	10		12	+2
TOTAL	234	+6	246	+18

One way to score ZQ4731 is:

	Caleb	Jessica
Odd:	2 x 7 = 14, 2 x 3 = 6, 2 x 1 = 2, 14 + 6 + 2 = 22	7 + 5 = 12, 3 + 5 = 8, 1 + 5 = 6, 12 + 8 + 6 = 26
Even:	$\frac{1}{2}$ of 4 = 2	4 – 2 = 2
Total:	22 + 2 = 24	26 + 2 = 28 (difference: +4)

b. Methods will vary. Possible ways include:
Caleb: Add up the odd numbers and double that total, then add up the even numbers and halve that total. Add up the 2 new totals to get the final total.
Jessica: Add up the odd numbers, then count the number of odd digits and multiply that by 5. Then add both totals together to get your odds total. Add up the evens, count the number of even digits and multiply that by 2, then subtract that from your evens total. Add up the new odds and evens totals to get the final total.

To find the overall winner, add up the positive score differences. (For example, in the answer table for question **2a**, Caleb's positive differences add up to 6 and Jessica's to 18, so Jessica wins by 12.)

3. a. Caleb usually scores higher on number plates that have at least 1 odd digit higher than 5 and that have lower even digits (if there are any).

b. Jessica usually scores more on number plates that have 2 or more of the higher even digits and that have lower odd digits (if there are any).

c. Caleb scores more when he doubles 7 or 9 than Jessica does when she simply adds 5. Jessica scores more when she subtracts 2 from 6 or 8 than Caleb does when he halves them.

d. 5 and 4

Activity Two

1. Practical activity. Answers will vary.

2. Rules will vary. A rule that will give both players the same chance of winning is: one player could multiply odd numbers by 3 and even numbers by 2, and the other player could add 10 to odd numbers and add 5 to even numbers.

You could show this on a chart:

Odd numbers	x 3	+ 10
1	3	11
3	9	13
5	15	15
7	21	17
9	27	19
Even numbers	x 2	+ 5
2	4	7
4	8	9
6	12	11
8	16	13
Total	115	115

Note that, although this gives both players the same chance of winning, the actual winner still depends on the number plates used. For example, for Caleb and Jessica's number plates in **Activity One**, Caleb would score 506 and Jessica would score 480, a difference of 26.

Page 11: Fair Mix

Activity

1. All the trays are arranged in multiples of 7.
Coconut ice = 7 x 4
Fudge = 7 x 8
Toffee = 7 x 6

2. **a.** They used the other factor for each tray:
4 coconut ice + 8 fudge + 6 toffee = 18 pieces.

b. There are 7 lots of 4, 6, and 8 altogether, so they could make 7 bags of 18 pieces out of each set of trays.

c. 70. (10 x 7)

3. **a.** $2.50. (175 ÷ 70)

b. Strategies will vary. For example,
70 + 70 = 140. 175 – 140 = 35, which is $\frac{1}{2}$ of 70. So 175 ÷ 70 is $2\frac{1}{2}$ or $2.50.

Pages 12–13: Good as Gold

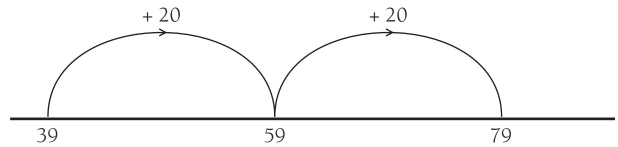
Activity

1. **a.**

	1st find	2nd find	Total for day
Mon.	32 oz	32 oz	2 x 32 oz = 64 oz
Tue.	40 oz	20 oz	90 oz – 30 oz = 60 oz
Wed.	30 oz	54 oz	64 oz + 20 oz = 84 oz
Thu.	39 oz	40 oz	89 oz – 10 oz = 79 oz
Fri.	15 oz	35 oz	200 oz ÷ 4 = 50 oz
Sat.	42 oz	30 oz	100 oz – 28 oz = 72 oz

Methods will vary. Strategies could include number lines, rounding and compensation (for example, 89 – 10 = 90 – 10 – 1), or using known facts (9 – 3 = 6 and 4 + 2 = 6 for Tuesday).
For example, Thursday's first finds could be worked out like this:
89 – 10 = 90 – 10 – 1
= 80 – 1
= 79
Or: 89 – 10 = 80 – 10 + 9
= 79
79 – 39 = 40

So 39 + 40 = 79
Or:



So 39 + 40 = 79

b. **i.** He found more gold in the second finds. (First finds: 198; second finds: 211)

ii. Strategies will vary. See the examples above.

2. **a.** 0. Explanations may vary. It can only be 0 because two of Monday's finds already total 50 (and you cannot have less than a 0 find).

b. Monday: 25 + 0 + 25 = 50 – 0
Tuesday: 37 + 0 + 33 = 70 – 0
Wednesday: 62 + 0 + 20 = 82 – 0
Thursday: 29 + 0 + 32 = 61 – 0
Friday: 15 + 0 + 45 = 60 – 0
Saturday: 0 + 36 + 15 = 51 – 0

3. Problems and discussion will vary.

4. Explanations will vary. For example:
= means there is the same value on both sides.
= means that the sides balance.

Pages 14–15: Make 28

Activity

1. Practical activity
2. a. Strategies may vary, but the best winning strategy is to aim for multiples of 7. Each time your classmate plays a number, play a card that makes a total of 7 or a multiple of 7. Your aim is to get to 21 (which is at least 1 short of your classmate being able to make 28 with their highest available card). They must then play a card to which you can play the last card to total 28. (If you are the first player, you will have to wait until your second turn to make a multiple of 7.)

   b. It does not make any difference what the starting number is as long as you can make a multiple of 7 as soon as you can.

   c. If you play second and keep to your strategy, you will always win. If you play first and can make a multiple of 7 on your second turn, then you will also win. But if both players try to use the "multiples of 7" strategy, the player who goes second will always win.

Page 16: Tidying Up

Activity

1. Practical activity. The compatible numbers are: 19 + 1, 18 + 2, 17 + 3, 16 + 4, 15 + 5, 14 + 6, 13 + 7, 12 + 8, 11 + 9, and 10 + 10.
2. 24 + 27 because 4 + 7 makes more than 10; and 17 + 43 because there are too many tens
3. a. 78

   b. 16

   c. 37

   d. 52

   e. 35

   f. 75

   g. 9

   h. 51
4. a. Answers will vary. For 20, only 1 of the digits you are adding will have a 10, otherwise the result will go beyond 20. For 50, either the tens digits add up to 5 or the tens digits add up to 4 tens and the ones digits add to 10. For example, 27 + 23 = 4 tens + 10. For 100, either the tens digits add up to 100 or the tens digits add up to 9 tens and the ones digits add up to 10. For example, 82 + 18 = 9 tens + 10.

   b. Yes, the strategy for 100 works for 1 000. However, it is slightly different because it is adding to 1 000, not 100. For example, 9 + 1 = 10, 90 + 10 = 100, 900 + 100 = 1 000. Two 3-digit numbers should add up to 9 hundreds + 100. For example, 345 + 655 = 900 + 100, or you may say 9 hundreds and 9 tens and 10 (900 + 90 + 10).

Page 17: Finding a Balance

Activity

Scale pictures will vary, but the equation you give for each set of scales must balance.

1. Possible equations include:

   a. 8 = 3 + 5, 8 = 3 + 3 + 2, 8 = 3 + 3 + 1 + 1, and so on. On your scales, you must have 8 objects on the left-hand side and 8 objects on the right-hand side.

   b. 3 = 3, 3 = 1 + 1 + 1, or 3 = 2 + 1

   c. The obvious one is 15 + 8 = 15 + 8, but there are other possible answers, including: 20 + 8 = 15 + 13, 8 + 8 = 15 + 1, 9 + 8 = 15 + 2, and so on.

   d. 8 + 16 = 12 + 12, 8 + 16 = 6 + 6 + 12, and so on.
2. 2 potatoes. Discussion will vary. The second scales will tell you that $\frac{1}{2}$ swede = 2 kūmara. You can put these kūmara into the first scales and take out the swede. Now the first scales say that 6 potatoes = 3 kūmara. This means that 2 potatoes will equal 1 kūmara.
3. Problems will vary.

Page 18–19: Curling

Activity

1. a. Statements may vary. The team whose stone is closest to the pot lid gets a point for that stone and for any other of their stones within the circles that are closer than the nearest opposition stone. (For example, Naseby won the first end with 3 shots. Kyeburn had 1 stone inside the outer circle, but it wasn't counted. Only one team scores from an end.)

   b. Kyeburn won, with 20 points.
   Your scorecard should look like this:

Naseby			Kyeburn		
Ends	Shots	Running total	Ends	Shots	Running total
1	3	3	1	0	0
2	0	3	2	3	3
3	0	3	3	4	7
4	1	4	4	0	7
5	4	8	5	0	7
6	0	8	6	3	10
7	0	8	7	1	11
8	2	10	8	0	11
9	0	10	9	3	14
10	0	10	10	6	20
11	4	14	11	0	20

2. 28 games. Strategies will vary. For example, you could make a table, look for a pattern, act it out, or draw a diagram.

   A round-robin table for 8 teams would look like this:

v	A	B	C	D	E	F	G	H
A		A v B	A v C	A v D	A v E	A v F	A v G	A v H
B	B v A		B v C	B v D	B v E	B v F	B v G	B v H
C	C v A	C v B		C v D	C v E	C v F	C v G	C v H
D	D v A	D v B	D v C		D v E	D v F	D v G	D v H
E	E v A	E v B	E v C	E v D		E v F	E v G	E v H
F	F v A	F v B	F v C	F v D	F v E		F v G	F v H
G	G v A	G v B	G v C	G v D	G v E	G v F		G v H
H	H v A	H v B	H v C	H v D	H v E	H v F	H v G	

   Note that all the games on the left side of the centre diagonal are repeats. For example, A v B is the same game as B v A.

3. 56 games. (28 x 4 ÷ 2. You have to divide by 2 because 2 teams are involved in each game.)

Pages 20–21: Slip-sliding Away

Activity

1. a. 260

   b. Diagrams will vary, but you need to focus on thirds. For example, you might use the box diagram shown on the page, or you might split the 390 into 300 and 90 and use a diagram like this:

100 30	100 30	100 30

   c. Strategies will vary.

2. a. 300. (260 + 40)

   b. 6 buses. (300 ÷ 50)

3. a. $2,600. (260 x 10)

   b. $9,400. Strategies will vary. (You need to work out 20 x 60, 260 x 30, 5 x 50, and 5 x 30 and then add those answers together.)

4. a. i. Kākāriki: 9 000 people;
   Whero: 3 000 people (9 000 ÷ 3);
   Mā: 4 500 people (3 000 + 1 500).

   ii. 16 500 people

   b. i. Answers will vary. Kākāriki's chairs hold the most people, and Mā's chairs hold more people than Whero's.

   ii. Mā: 3; Kākāriki: 6. Methods will vary. Kākāriki carries 3 times more people than Whero, so its chairs must be 3 times bigger (3 x 2 = 6). Mā carries $\frac{1}{3}$ of Kākāriki plus $\frac{1}{2}$ of Whero, so its chairs must carry $\frac{1}{3}$ of 6 + $\frac{1}{2}$ of 2.
   2 + 1 = 3 people on each Mā chair.

Pages 22–24: Ewe Scan

Activity One

1. a. i. 1 600

   ii. 2 400. (1 200 x 2)

   iii. 450. (150 x 3)

   b. 4 450. 1 600 + 2 400 (twins) + 450 (triplets) = 4 450 lambs

Strategies for question **1** will vary. First, you need to double or treble, by addition or multiplication, to find each individual amount. You might look for compatible numbers (for example, 600 + 400) when adding up all the lambs.

2. 668 lambs (rounded). Strategies will vary. For example, 4 450 lambs is 44.5 lots of 100 lambs. 10 x 44.5 = 445. 5 x 44.5 or half of 445 is 222.5. 445 + 222.5 = 667.5. You can't have half a dead lamb, so you need to round your answer.

3. **a.** 54 bales. Strategies will vary. One way is to see 1 350 (the number of ewes involved) as 13 hundreds and $\frac{1}{2}$ a hundred. So that's 13 x 4 bales + 2 bales (for the extra 50 sheep), which is 52 + 2 = 54.

b. 810 bales. 30 ÷ 2 = 15 lots of 2 days (as in question **3a**). 54 x 15 can be seen as 54 x 10 = 540 and 54 x 5 (or 540 ÷ 2) = 270. 540 + 270 = 810.

4. **a.** 135 000 g. (1 350 x 100)

b. 2 025 kg. (135 000 g x 15 can be seen as 135 000 x 10 = 1 350 000 and 135 000 x 5 [or 1 350 000 ÷ 2] = 675 000. 1 350 000 + 675 000 = 2 025 000. 2 025 000 g ÷ 1 000 = 2 025 kg.)

Activity Two

1. Descriptions may vary, but the following information needs to be included:

- 984 lambing ewes (534 [rounded] + 400 + 50)
- 1 484 lambs (534 + 800 + 150)
- 450 ewes that need extra feed
- 18 bales of hay every 2 days ($\frac{1}{3}$ of **Activity One**, question **3a**'s answer)
- 45 000 g of grain ($\frac{1}{3}$ of **Activity One**, question **4a**'s answer) every 2 days

2. **a.** 270 bales of hay and 675 kg of grain. (450 x 3 = 1 350, so 810 ÷ 3 = 270 and 2 025 ÷ 3 = 675)

b. 540 bales of hay and 1 350 kg of grain. (2 x 450 = 900, so 2 x 270 = 540 and 2 x 675 = 1 350)

Figure It Out

Number Sense and Algebraic Thinking

Teachers' Notes

Overview of Levels 2–3: Book Two

Introduction to Number Sense and Algebraic Thinking

The *Number Sense and Algebraic Thinking* books in the Figure It Out series provide teachers with material to support them in developing these two key abilities with their students. The books are companion resources to Book 8 in the Numeracy Project series: *Teaching Number Sense and Algebraic Thinking*.

Number sense

Number sense involves the intelligent application of number knowledge and strategies to a broad range of contexts. Therefore, developing students' number sense is about helping them gain an understanding of numbers and operations and of how to apply them flexibly and appropriately in a range of situations. Number sense skills include estimating, using mental strategies, recognising the reasonableness of answers, and using benchmarks. Students with good number sense can choose the best strategy for solving a problem and communicate their strategies and solutions to others.

The teaching of number sense has become increasingly important worldwide. This emphasis has been motivated by a number of factors. Firstly, traditional approaches to teaching number have focused on preparing students to be reliable human calculators. This has frequently resulted in their having the ability to calculate answers without gaining any real understanding of the concepts behind the calculations.

Secondly, technologies – particularly calculators and computers – have changed the face of calculation. Now that machines in society can calculate everything from supermarket change to bank balances, the emphasis on calculation has changed. In order to make the most of these technologies, students need to develop efficient mental strategies, understand which operations to use, and have good estimation skills that help them to recognise the appropriateness of answers.

Thirdly, students are being educated in an environment that is rich in information. Students need to develop number skills that will help them make sense of this information. Interpreting information in a range of representations is critical to making effective decisions throughout one's life, from arranging mortgages to planning trips.

Algebraic thinking

Although some argue that algebra only begins when a set of symbols stands for an object or situation, there is a growing consensus that the ideas of algebra have a place at every level of the mathematics curriculum and that the foundations for symbolic algebra lie in students' understanding of arithmetic. Good understanding of arithmetic requires much more than the ability to get answers quickly and accurately, important as this is. Finding patterns in the process of arithmetic is as important as finding answers.

The term "algebraic thinking" refers to reasoning that involves making generalisations or finding patterns that apply to *all* examples of a given set of numbers and/or an arithmetic operation. For example, students might investigate adding, subtracting, and multiplying odd and even numbers. This activity would involve algebraic thinking at the point where students discover and describe patterns such as "If you add two odd numbers, the answer is always even." This pattern applies to *all* odd numbers, so it is a generalisation.

Students make these generalisations through the process of problem solving, which allows them to connect ideas and to apply number properties to other related problems. You can promote process-oriented learning by discussing the mental strategies that your students are using to solve problems. This discussion has two important functions: it gives you a window into your students' thinking, and it effectively changes the focus of problem solving from the outcome to the process.

Although the term "algebraic thinking" suggests that generalisations could be expressed using algebraic symbols, these Figure It Out *Number Sense and Algebraic Thinking* books (which are aimed at levels 2–3, 3, and 3–4) seldom use such symbols. Symbolic expression needs to be developed cautiously with students as a sequel to helping them to recognise patterns and to describe them in words. For example, students must first realise and be able to explain that moving objects from one set to another does not change the total number in the two sets before they can learn to write the generalisation $a + b = (a + n) + (b - n)$, where n is the number of objects that are moved. There is scope in the books to develop algebraic notation if you think your students are ready for it.

The Figure It Out *Number Sense and Algebraic Thinking* books

The learning experiences in these books attempt to capture the key principles of sense-making and generalisation. The contexts used vary from everyday situations to the imaginary and from problems that are exclusively number based to those that use geometry, measurement, and statistics as vehicles for number work. Teachers' notes are provided to help you to extend the ideas contained in the activities and to provide guidance to your students in developing their number sense and algebraic thinking.

There are six *Number Sense and Algebraic Thinking* books in this series:
Levels 2–3 (Book One)
Levels 2–3 (Book Two)
Level 3 (Book One)
Level 3 (Book Two)
Levels 3–4 (Book One)
Levels 3–4 (Book Two)

Page 1: Quick Thinking

Achievement Objectives

- recall the basic addition and subtraction facts (Number, level 2)
- order any set of three or more whole numbers (up to 99) (Number, level 2)

Other mathematical ideas and processes

Students will also:

- use and interpret mathematical language
- recognise odd and even numbers.

AC
EA
AA
AM
AP

Number Framework Links

Use this game to give your students practice in recalling basic facts. It will be suitable for students using advanced counting strategies (stage 4) and early additive strategies (stage 5).

Game

In this game, the students need to make strategic decisions about the best line on which to put given pairs of dice numbers. The game gives them practice in recognising and recalling important mathematical terms and some addition and subtraction basic facts.

Carefully review the How to Play section with the students. They need to realise that each throw of the dice must be checked against any of the 10 statements. To do this quickly in the 15 second time limit, they will need to be familiar with the meanings of the terms in each statement. Check that they know the meaning of "sum", "difference", and "twice". You many need to pay special attention to the combination of terms in "sum less than" and "sum greater than". Reading the statement lines is likely to be an issue for some students.

Playing the game strategically involves thinking about which statements are hardest to satisfy. For example, getting two even numbers is much more likely than getting one number that is twice that of the other. The students should try to fill the "hard to get" statements as early as possible.

Extension

Have the students make up 10 statements that they could use in a game that requires them to multiply the 2 dice numbers together. They can test them out by playing a few games with a classmate before presenting them to the class. This would be suitable for students at the advanced additive stage.

Pages 2–3: An Odd Spell of Maths

Achievement Objectives

- state the general rule for a set of similar practical problems (Algebra, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the domain of addition and subtraction.

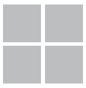
The students using advanced counting strategies (stage 4) could solve the calculations in these problems using materials, but they are unlikely to make the generalisations.

Activity

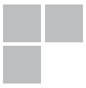
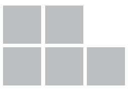
This activity focuses on exploring and generalising patterns made by combining odd numbers and even numbers in different ways. Another outcome is using strategies to find totals of varying numbers of addends.

Encourage the students in a guided teaching group to explore the interesting and important features in the spelling of the word mathematics. For example, before they even get to question **1c**, they may notice that the letters M, A, and T occur twice in the word and that each has a different value. Ask: *Could there be more than one way to make a total from the words that use these letters?* Have the students compare the value of the first M with the second M, the first A with the second A, and the first T with the second T. They should see that the second letter is always exactly 10 more than the first. They should also note that all the letter values are odd numbers.

The students who need to use materials could answer question **1b** with a visual model. Start with even numbers and represent them by pairs rectangles, such as the ones below, or with groupings of counters or cubes.

Even number	Shapes
2	
4	

Now show the pattern that odd numbers make when we group in pairs (such as a class lining up in twos):

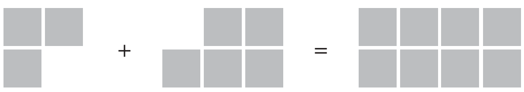
Odd number	Shapes
1	
3	
5	

Pattern boards and cubes are ideal materials to use to show these patterns. When the students add 2 odd numbers, they can join these patterns and see that an even number pattern must be the result.

Questions **2** and **3** ask the students to use 3 and then 4 odd number addends. Help your students to make a chart of the findings for questions **1**, **2**, and **3**. It may look like this:

Number of odd number addends	Result
1	Odd
2	Even
3	Odd
4	Even

A visual model for seeing the pattern in 3 + 5 = 8 is:



3 + 5 = 8

Ask: *Can you find a general rule that connects the number of odd number addends with the result?*
The students may say: "When you add an even number of odd number addends, you always get an even number. When you add an odd number of odd number addends, you always get an odd number."

The students need to apply this understanding in question **4**, which can be used to evaluate their understanding of this pattern.

When the students make up words with the letters M, A, or T in them, have them compare the results when they choose the first or second of those letters. They should notice that the difference is always an exact number of tens.

In question **5**, the students could use groupings of counters or cubes or the even numbers pattern on a pattern board to explore the problem.

A matrix table is a good way of recording the generalisations for adding odd and even numbers in question **6**.

+	Odd	Even
Odd	= even	= odd
Even	= odd	= even

Pages 4–5: Serious Circus Sums

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage
- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6) (using the later questions).

Activity

This activity presents many opportunities for addition and for using some multiplication strategies. Students can also find and describe some sequential number patterns. The variation in prices between children and adults provides extra opportunities for the students to record equations involving multiplication with addition.

With a guided teaching group, introduce the students to the activity and then have them attempt question **1** in small groups. Use their reporting back on question **1b** as an opportunity to highlight up to three different strategies for solving this problem. Record their strategies on the whiteboard as they report back. For example: "I added $20 to $10 to make $30, then $2 to $4 to make $6. So $30 + $6 makes $36" can be recorded as:

+ 20   + 2

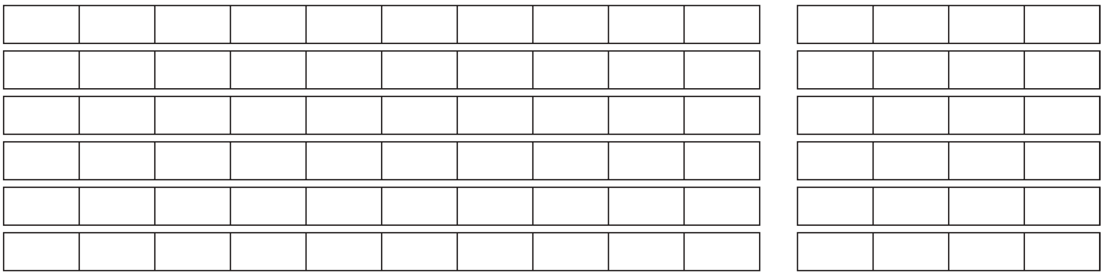
$10 ⌒ $30   then $4 ⌒ $6   So $30 + $6 = $36

Question **2** can be solved by trial-and-improvement strategies, but it gives the students a chance to model equations in a variety of ways, for example, 5 + ☐ = 27 for Sara and ☐ + 22 = 30 for Siaosi. There is also considerable opportunity for the students to reason logically. For example, the cost for Sara's family is $27, so the students might assume 2 parents, at $16, with the remainder of $11 being the cost of 3 children. An alternative scenario, 3 adults ($22) and 1 child ($5), does not fit with the information given that Sara has a younger sister.

In questions **5b** and **5c**, the students may need help to record their strategies. Use prompts such as:
How many adults will you have?
So what is the cheapest way to buy their tickets?

The Numeracy Project material master 4-9, Money, (available at www.nzmaths.co.nz/numeracy/materialmasters.htm) is an excellent material aid for students who need to represent the problems concretely. Have the students work in their small groups to find strategies to solve each question. Use the reporting-back stage to share and improve strategies. Take particular note of the strategies used to work out the total for the 24 children because this can be solved by adding the cost of a group of 4 six times or by multiplying 14 by 6.

After the students have recognised that 14 x 6 solves the problem, they could use Animal Strips (material master 5-2) to represent it. Using 6 fourteen-strips, the students could look at ways to partition the array to make the calculation easier. One way is to use place value partitioning, that is, 10 x 6 + 4 x 6. Another way is to use doubling, that is, 14 + 14 = 28, 28 + 28 = 56; 56 + 28 = 84.



Where possible, encourage the students to use multiplicative strategies rather than additive ones. Some students may record their working out of the adult costs separately from the children's costs. Help them record it all in one equation, for example, (2 x $22) + (6 x $14) = $128. (Note that for the adults, 2 times 3 adults for $22 is the same price as 4 adults for $28 plus 2 adults for $16.)

Question **7** asks the students to look beyond the simple relationship (the fact that the evening tickets cost more than the afternoon tickets) to find patterns within each show as the number increases and patterns between the adults' and children's prices. The afternoon show children's price increases by $3 per child, whereas the adults' price increases twice as much, that is, by $6 per adult. But in the evening show, the children's price increases by $5, whereas the adults' price grows 3 times that amount, in steps of $15. Ask the students to describe generalisations for each of these patterns in their own words. For example, the afternoon generalisation may be described as "the children's price increases by $3 for each child". The generalisation for the afternoon adults' price is "the adults' price increases by $6 for each new adult".

Pages 6–7: Trimming Trees

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

Other mathematical ideas and processes

Students may also:

- use arrow diagrams to express relationships in addition and subtraction patterns
- record equations in the form of 33 + □ = 42.

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- develop the students' knowledge of tens in numbers under 1 000 (stages 4 and 5)
- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the domain of addition and subtraction.

Students can solve the calculations in these problems using advanced counting strategies (stage 4).

Activity

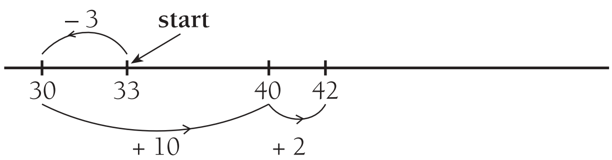
A hundreds board structure allows students to calculate addition and subtraction of ones, tens, and hundreds using stage 4 or stage 5 strategies. The students can use this structure to observe some directional patterns and form generalisations.

With a guided teaching group, model the context of this activity using a hundreds board. (See Numeracy Project material master 4-4 [available at www.nzmaths.co.nz/numeracy/materialmasters.htm] and the Thousands Book, material master 4-7.)

For question **1**, the students who use imaging can use a blank hundreds grid to work out the steps that Hamish can take to each tree. Those who need to use materials can use a hundreds board. The Thousands Book provides some number clues and also encourages imaging. As the students report back, demonstrate how to record a sequence of moves. For example, for 33 to 42, the arrow sequence ← ↓ records the calculation "subtract 1 and add 10".

You can use question **1b** to check that the students recognise the difference between the starting-point value and the size of the moves needed to reach the target. Use language such as: *1 and how many more makes 23 (that is, 1 + □ = 23)?* Place value material will help the students to recognise the quantities involved rather than just seeing the number pattern.

Show the students how to record a negative movement along a number line. For example, to move from 33 to 42, Hamish could start at 33, move back 3, then down 10 and across 2. This could be recorded as:



With the aid of place value materials, challenge the students to establish the combined effect, that is, □ – 3 + 10 + 2 = □ + 9. It is important to do this with materials until number properties are established.

In question **2**, highlight the special arrow symbol for 100 and make sure the students understand that it is the direction of the arrow that shows adding or subtracting.

Question **3** is a great chance for students to see that adding 10 and then subtracting 10 makes no difference to the result. This can lead to a discussion on the net effect of addition and subtraction, for example, + 10 – 2 is the same as + 8.

Numeracy Project materials (see www.nzmaths.co.nz/numeracy/project_material.htm)

- *Book 4: Teaching Number Knowledge*
  Hundreds Board and Thousands Board, page 16

Pages 8–10: Pick a Plate

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- write and solve story problems which involve halves, quarters, thirds, and fifths (Number, level 2)
- use a systematic approach to count a set of possible outcomes (Statistics, level 3)

Other mathematical ideas and processes

Students will also:

- apply and generalise understanding of number relationships to solve problems
- understand odd and even numbers.

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the domains of addition and subtraction and multiplication and division
- develop the students' knowledge of basic facts to stage 6.

Activities

This interesting calculation game will be useful as an ongoing maintenance activity both at and outside school. The activities use the number-plate context to encourage a variety of strategies for calculating. They also encourage the students to compare the effects on large and small digits of addition and subtraction and multiplication and division. The students then have an opportunity to generalise the effect of these operations on the numbers involved.

Activity One

Some students will be able to work through this activity independently, but do ensure that they understand the structure of the game. They may want to discuss whether the highest or lowest score wins. Check that they realise that the letters on the plates are not important.

With a guided teaching group, ensure that the students notice that both Jessica and Caleb got the same result for the first number plate, even though Caleb multiplied and divided and Jessica added and subtracted.

Question **1** makes the students focus on exploring the effects of the different strategies that Caleb and Jessica use. Use questions such as:
Which strategy would you expect to get the highest number most of the time?
Will there always be the same number of odd numbers and even numbers on the plate?
Is there a strategy that is better for a plate with more digits that are large numbers?
There are more odd number digits than even ones. Will this make a difference? (No. It depends more on whether the digits are high or low.)

In question **2**, the students will probably work out the odd numbers and the even numbers separately and then combine them. Caleb multiplies all the odd numbers by 2, so he can either multiply each separately and then add them together or add them all first and then multiply by 2. He also has two options for even digits, but this time he would divide by 2.

A model for recording each of these two strategies on the plate ZQ4731 for Caleb could be:

Odd		Even
2 x 7 + 2 x 3 + 2 x 1	+	4 ÷ 2 = □
14 + 6 + 2	+	2 = 24

Or

(7 + 3 + 1) x 2	+	4 ÷ 2 = □
11 x 2	+	2 = 24

Another model is:

	4	7	3	1	Total
Score	2	14	6	2	24

Question **3** is the place for an in-depth comparison of each set of rules. Have the students develop a chart that they can use to explore the effect of the rules on each digit. For example:

Odd numbers	**Caleb's rule**	**Result**	**Jessica's rule**	**Result**
1	x 2	2	+ 5	6
3	x 2	6	+ 5	8
5	x 2	10	+ 5	10
7	x 2	14	+ 5	12
9	x 2	18	+ 5	14

Even numbers	**Caleb's rule**	**Result**	**Jessica's rule**	**Result**
2	÷ 2	1	– 2	0
4	÷ 2	2	– 2	2
6	÷ 2	3	– 2	4
8	÷ 2	4	– 2	6

The students should explore who gains and by how much for each digit:

	Number	**Caleb's gain**	**Jessica's gain**
	1		4
	3		2
	5	0	0
	7	2	
	9	4	
	2	1	
	4	0	0
	6		1
	8		2
Total gain		**7**	**9**

Overall gain: Jessica will gain 2 more than Caleb, so she should win more often if the rule is that the highest score wins.

Activity Two

Strategies that will work for a new number-plate game may include alternating the highest total and the lowest total as the winning scenario. The students should focus on strategies for their games where both people will have a close-to-equal chance of winning. For example, have the students explore the scenario where one player adds 1 to odd numbers and subtracts 1 off even numbers and the other player doubles the odd numbers and halves the even numbers. A chart for this rule would be:

Plate number	+ 1 – 1	x 2 ÷ 2
1	2	2
3	4	6
5	6	10
7	8	14
9	10	18
2	1	1
4	3	2
6	5	3
8	7	4

Although the +1, – 1 player wins on 3 out of 9 and loses on 4 out of 9, the effect of the differences is greater for the x 2, ÷ 2 player (+ 20, compared to + 6 for the + 1, – 1 player).

The rule suggested in the Answers for question **2** is to change the calculations for odd and even numbers, which will give both players the same chance of winning. If the students use the chart approach suggested there or above, they will be able to quickly see whether their suggested rules are fair to each player.

A good discussion point is the fact that even a "fair" rule such as that in the Answers is no guarantee of success. For example, that rule applied to Caleb and Jessica's number plates gives Caleb a difference of + 28. In the end, it all depends on the range of digits in the number plates used.

Extension

Using the rule given in the Answers, have the students make a chart to examine the effect the calculations have on each number and discuss the outcomes.

A different game using number-plate numbers is to try to use all the numbers to reach a target number, say 10. The players can use the numbers once only and can combine them using any operation. For example, in WZ7283, 8 – 3 = 5, then 7 + 5 = 12, and finally 12 – 2 = 10.

Page 11: Fair Mix

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

Other mathematical ideas and processes

Students will also develop an understanding of the idea of a common factor (number).

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students to develop advanced additive strategies (stage 6) in the domain of multiplication and division
- help the students using advanced additive strategies (stage 6) to develop factor thinking to assist them in their transition to the advanced multiplicative stage
- develop the students' knowledge of basic facts.

Activity

The trays of sweets model the array concept for multiplication, in which two factors define the size of the array. The width of each tray is the same. This is an important element in the solution to the problems.

If necessary, use materials to introduce the activity by having the students cut out a model of each tray using grid paper or the Fraction Grid, Numeracy Project material master 4-27 (available at www.nzmaths.co.nz/numeracy/materialmasters.htm). The students can use these models to explore question **1** and see the common factor of 7. Try these questions to focus the exploration:
Compare the size of the trays. Do you notice anything similar?
How would you stack the trays so that they look tidy and fit together?
If you know there are 28 squares of coconut ice, can you use that to work out the number of squares of fudge and toffee?
Check that the students recognise that the tray of fudge holds twice as much as the coconut ice tray and that the toffee tray is $1\frac{1}{2}$ times the size of the coconut ice tray.

Before the students attempt question **2**, you may need to explain the meaning of the phrase "same mix". Each bag needs to have the same ratio of fudge to toffee to coconut ice. A ratio is a multiplicative recipe for combining different things, for example, 2 coconut ice : 3 toffee : 4 fudge.

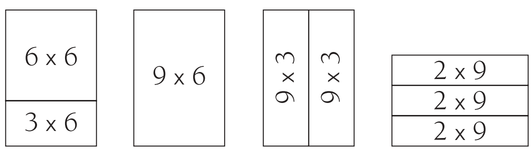
Send the students into small groups to work out the mix for each bag of sweets. When they succeed, ask them to compare the amount of each type of sweet per bag with the length of each side of the tray for each type. By recording each tray as factors, they should see that the mix is the same as the different factor for each tray:

Type	Array	Number in mix
Coconut ice	7 x 4	4
Fudge	7 x 8	8
Toffee	7 x 6	6

Have the students use a variety of ways to estimate the solution for question **3**. A double number line may be useful, for example:

Ask:
Where would $175 be on the line?
How far away from $140 and $210 is $175?
So what would be the price per bag at that point?

Extension
Explore other tray sizes that have proportional relationships, for example, 6 x 6, 9 x 3, 9 x 6, and 2 x 9. Encourage the students to use both spatial and number sense to find the relationships. For example:



Pages 12–13: Good as Gold

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)
- solve problems of the type $\square + 15 = 39$ (Algebra, level 3)

Other mathematical ideas and processes

Students will also:

- understand and use equations with the unknown in a variety of positions
- use a symbol to represent a variable.

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6) in the domain of addition and subtraction
- help the students who are using advanced additive strategies (stage 6) in the domain of addition and subtraction to develop parallel multiplicative strategies.

Activity

By changing the position of the unknown value in the equations, this activity challenges students to think carefully about the meaning of each situation and the strategy used to solve it. It extends the challenge by having students solve equations that also contain a symbol that represents the same amount. The historical context provides interest and an opportunity to integrate mathematics within essential learning areas such as social studies and technology.

You could introduce the activity by showing the students some goldmining areas on a map of New Zealand. In addition, discuss the ounce as a historical unit of mass. The students could get a feel for 1 ounce by putting 2 full tablespoons (30 millilitres) of water in a plastic bag.

For question **1**, model ways to read each equation with an emphasis on the place of the unknown. For example: *What plus 32 ounces will equal 2 times 32 ounces? 40 ounces plus what equals 30 ounces less than 90 ounces?*

Let the students solve each equation and report on their strategies. Emphasise the importance of deciding which calculation to do first. Discuss with them the distinctive features of each day's equation and what mental strategies the students used:

- Monday: This is a doubles situation.
- Tuesday: The total for the day must be between 40 and 90.
- Wednesday: This uses a – 10 and + 10 compensation strategy. (64 – 10 = 54; 20 + 10 = 30)
- Thursday: This can be solved using + 50 – 10 to connect the 39 with 89 – 10.
- Friday: 200 ÷ 4 is the same as 100 ÷ 2. This is an equal adjustment using division.
- Saturday: What compatible number goes with 28 to make 100?

To solve question **1b**, the students will add 6 numbers, so they could use a place value strategy where they add the tens and the ones separately and then add them together. It doesn't matter if they add the tens before the ones, for example:

32
(3 + 4 + 3) tens = 100 — 40 — (2 + 9) ones = 11
30
39
(3 + 1 + 4) tens = 80 — 15 — (5 + 2) ones = 7
42

The tens equal 180. The ones equal 18. So the total equals 198.

Introduce question **2** by focusing on the code used for the same amount with a question such as: *Does it matter what number the "bag of gold" is worth?* To work this out with number properties, have the students look carefully at Monday's situation to find the value of the bag of gold. Use questions such as *What number can you add to the left side and take from the right side that will keep the equation balanced?* The students should realise that the bag of gold must be worth 0.

The students need to see that an equation is a statement of balance and that this is indicated by the equals sign. Use question **4** to ensure that the students appreciate this point.

Pages 14–15: Make 28

Achievement Objectives

- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

Other mathematical ideas and processes

Students will also use logic and reasoning to form a winning strategy.

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the domain of addition and subtraction
- give the students practice in recalling basic facts.

Activity

This is another version of the traditional nim game and will help students practise addition and subtraction facts. More importantly, it also provides a context for using logical reasoning and number patterns to devise a winning strategy.

Before using this as an independent activity, explain the rules and have the students play the game in guided teaching groups. Make sure that the students understand the game and the recording sheet. The whole class could play the game if you have enough decks of cards.

An alternative to using cards would be to have each student write the numbers 1 to 6 out twice and cross off the numbers as they use them.

Make sure that the students play a number of games before discussing some winning strategies. Have them test the theories they come up with by playing the game. Resist the urge to tell them the winning strategy as long as possible so that it becomes the major goal of playing the game.

When the winning strategy is becoming well known in the class, challenge the students to explain why the player who goes second should always add the number that makes a multiple of 7 to the number played by the first player. A table may make the pattern clearer:

Round	Player 1	Player 2	Running total
1	2	5	7
2	6	1	14
3	4	3	21
4	1	6	28 wins!

It may be necessary to list the card numbers in order to show the 7s pattern:

1 2 3 4 5 6 So 1 + 6 = 7
2 + 5 = 7 and
3 + 4 = 7

The largest card number is 6, so the key numbers that control hitting 28 for the player who goes second are 21, 14, and 7. These are all multiples of 7.

Extension

Extend the digit cards to include 7 and make the winning target number 32 or 40. Another variation of the game would be to make the first person to reach the target number or higher be the loser. Challenge the students to work out a strategy rather than just playing the game.

Page 16: Tidying Up

Achievement Objective

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)

Other mathematical ideas and processes

Students will also generalise rules for finding compatible numbers.

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in the domain of addition and subtraction
- develop the students' knowledge of tens and hundreds in numbers under 1 000 (place value, stage 5).

Activity

This activity builds vital knowledge about compatible numbers that students can use as one of their part–whole strategies for solving addition and subtraction problems.

With a guided teaching group, materials such as 20- and 100-bead strings are ideal for showing the patterns in compatible numbers. Help the students to understand compatible numbers that add to 20 with questions such as: *Separate the string of 20 into halves. How many beads are there in each half?*

Begin a chart on the whiteboard, starting from the middle pair on the string, and describe the groups of beads as compatible numbers: 10 + 10 = 20. Have a student move 1 bead from one side to the other. Ask: *What two compatible numbers have we made by shifting 1 bead?* (9 + 11 = 20)
Continue this to make a chart of compatible numbers that add up to 20:
10 + 10 = 20
9 + 11 = 20
8 + 12 = 20
7 + 13 = 20
6 + 14 = 20
Ask: *How much does the difference between the two numbers change as we move 1 bead?*

Use a 100-bead string to do the same with compatible numbers for 100, starting with 50 + 50 = 100, then having the students move the beads and record on a chart:
50 + 50 = 100
51 + 49 = 100
52 + 48 = 100
53 + 47 = 100
Ask: *How much will the difference between the two numbers change if we move over 10 beads, and what compatible numbers will we make?* (The difference will change by 20 as you move 10 beads.)

When the students are ready to move onto using imaging, have them imagine a 50-bead string. Get them to make a chart starting with half the beads on each side:
25 + 25 = 50
24 + 26 = 50
23 + 27 = 50
22 + 28 = 50, and so on.

To generalise number properties, have the students examine the numbers in the ones and tens columns for each pair of compatible numbers they find. Ask them to describe any patterns they find. They should see that the ones column has numbers that always make 10 exactly. They may also see that the tens column nearly always adds up to one 10 less than the number of tens we want to have in the target number. For example, if the target tidy number is 50, the tens digits must add up to 40. If the target tidy number is 100, the tens digits add up to 90. The exception is when the compatible numbers are themselves tidy numbers. For example, in 20 + 30 = 50 or in 30 + 70 = 100, the tens make the target tidy number because there is nothing in the ones place.

Question **4b** extends the rule to include the hundreds column so that when the target number is 1 000, the hundreds must add up to 900 (unless, as explained above, the compatible numbers are themselves tidy numbers).

Page 17: Finding a Balance

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- use the mathematical symbols =, <, > for the relationships "is equal to", "is less than", and "is greater than" (Algebra, level 2)

Other mathematical ideas and processes

Students will also use "if ... then" statements to communicate logic and reasoning (mathematical processes).

Number Framework Links

AC
EA
AA
AM
AP

Use this activity to help the students who are beginning to use early additive strategies (stage 5) to become confident at this stage in all domains.

Activity

The concept that an equation is a statement of balance or equality is vital to the development of algebraic thinking. Inequalities, shown by "more than" or "less than" situations, need to be adjusted to become equations. Helping students to come up with strategies to create equations from unbalanced situations is the purpose of this activity.

Question **2** extends the rebalancing concept by adding an "if … then" situation, which is the basis for using logical substitution strategies.

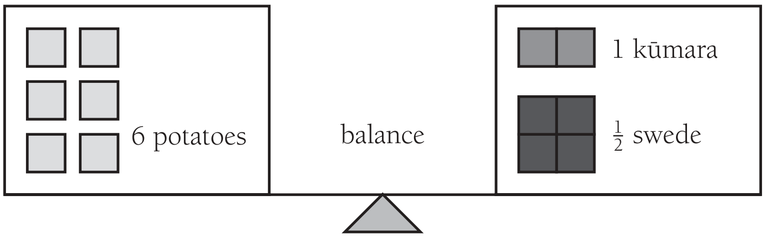
To introduce the activity, use materials like bucket balances and marbles or bolts to model the situations represented by the diagrams. For question **1a**, the buckets would hold 8 and 3 respectively, and the balance arm would dip towards the 8 and show an unbalanced situation. (Make sure that the marbles or bolts have the same mass.)

The key imaging question to use here would be: *How many marbles will we need to put in the right-hand side to create a balance?* Check the students' responses and find the correct number. It's important that you record this as the equation 8 = 3 + 5. Make it explicit to the students that the = sign is used to represent the idea of "balances" in the statement "8 balances 3 plus 5".

For question **2**, the students will need to assume that each potato weighs the same and each kūmara weighs the same. Have the students turn each illustration into a statement or question:
"6 potatoes balance 1 kūmara and half a swede."
"2 kūmara balances half a swede."
"1 kūmara balances how many potatoes?"

Ask the students to make "if … then" statements to make connections between the statements, for example:
"If 2 kūmara weigh the same as half a swede, then 6 potatoes weigh the same as 3 kūmara."
"If 6 potatoes weigh the same as 3 kūmara, then 2 potatoes weigh the same as 1 kūmara."

A model of each vegetable can be made with multilink cubes to check out each statement. Use 1 brown cube to represent each potato. Click together 2 red cubes to represent a kūmara and 8 yellow cubes to represent a swede. If you want to confirm the situations for the students by trying the combinations shown by the illustrations in a bucket balance, you will need to use heavier materials than multilink cubes, such as the marbles or bolts suggested earlier.



Pages 18–19: Curling

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- describe in words, rules for continuing number and spatial sequential patterns (Algebra, level 3)
- devise and use problem-solving strategies to explore situations mathematically (Mathematical Processes, problem solving, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage transition from early additive strategies (stage 5) to advanced additive strategies (stage 6) in the domains of addition and subtraction and multiplication and division
- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in the domain of addition and subtraction.

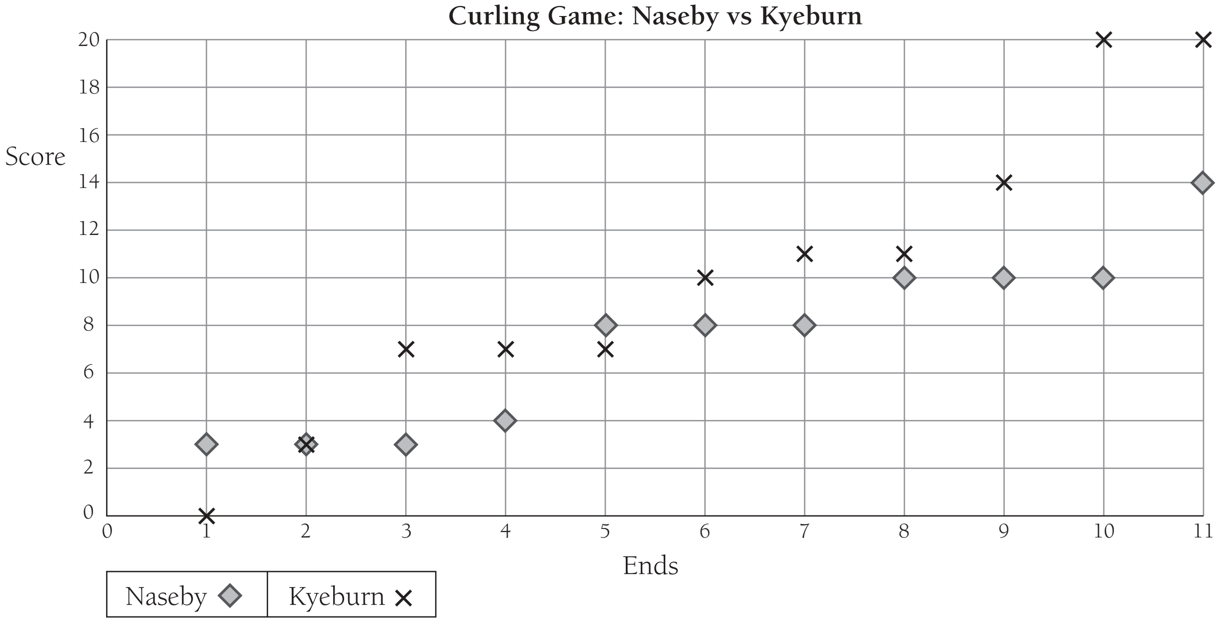
Activity

This unusual context is an opportunity to link mathematics with a unique aspect of life in New Zealand. The problems involved will challenge your students to find and describe number patterns. They will need to develop a systematic approach to recording the relationships involved in setting up a tournament draw.

Introduce the activity using a New Zealand map to identify the towns in the Central Otago region where curling is popular. Information and photographs of outdoor curling can be found at www.curling.org.nz/crampit/htm

Question **1** consolidates students' understanding of the game of curling and presents an opportunity for building basic addition strategies using the running total column of the scorecard.

The scorecard is a great context for developing the understanding and use of relationship graphs. Model the necessary axes and labels and have the students complete the graph. For example:



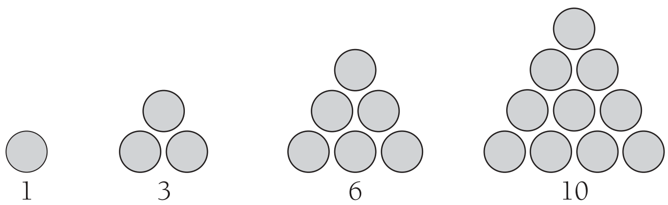
As an extension to this question, the students could develop a tabletop version of curling, using counters as stones and delivering the stone with a flick of their finger, aiming at a 5 cent coin as the pot lid. They could then practise graphing their scorecards as relationship graphs.

To solve question **2**, the students explore a real-life application of algebra skills. The use of a table showing the results of the "solve a simpler problem" strategy illustrated in the text will help the students to find a pattern and use it to solve the problem.

Number of teams	1	2	3	4	5	6	7	8
Number of games	0	1	3	6	10	?	?	?

+1 +2 +3 +4

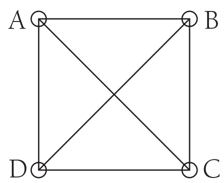
The students may recognise the pattern in the number of games table as the triangular number pattern that occurs in many other problems, such as the handshake problem (the pattern generated when asking for the number of handshakes done by pairs of people in any sized group). You can generate the triangular numbers pattern by forming triangles with counters:



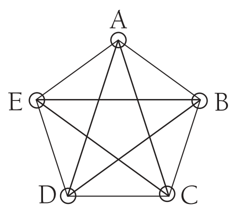
and so on.

You may need to suggest some ways for the students to work out the number of games played in questions **2** and **3**. One way is to use network diagrams.

For example, for 4 teams and 6 games:



and for 5 teams and 10 games:



Model the round-robin table structure (shown in the Answers) and help your students' understanding with questions such as:

How many squares are inside the 8 by 8 table?

How many squares can show 2 different teams against each other? (64 – 8 = 56)

Point out that this is 1 fewer 8 than the 8 by 8 of the whole square, so it is 7 x 8 = 56.

Each game is repeated on the table, so how many actual games are there in total? (56 ÷ 2 = 28)

Extension

Re-explain the use of the round-robin table to solve the problem: *Let's see what's happened from the start. We had 8 x 8, which became 8 x 7. Then we divided by 2, so we have $\frac{1}{2}$ of 8 x 7 = 28.*

Have students make a round-robin table to work out the total number of games with 2, 4, 5, 6, and 10 teams. Ask them to record the games for each number of teams in one equation:

2 teams: $\frac{1}{2}(2 \times 1) = 1$
4 teams: $\frac{1}{2}(4 \times 3) = 6$
5 teams: $\frac{1}{2}(5 \times 4) = 10$
6 teams: $\frac{1}{2}(6 \times 5) = 15$
10 teams: $\frac{1}{2}(10 \times 9) = 45$

Ask the students to describe the pattern in these equations. They may see that they always find a half of the number of teams multiplied by 1 fewer than the number of teams.

For question **3**, check that the students realise that each team is restricted to 4 games so this tournament is not a round-robin. (See the comment in the Answers.)

Pages 20–21: Slip-sliding Away

Achievement Objectives

- write and solve story problems which involve whole numbers, using addition, subtraction, multiplication, or division (Number, level 2)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- solve problems of the type $\square + 15 = 39$ (Algebra, level 3)

AC
EA
AA
AM
AP

Number Framework Links

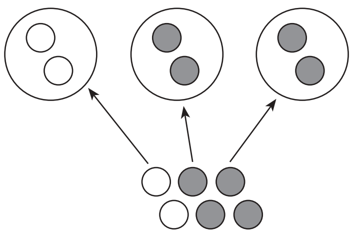
Use this activity to:

- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in the domains of addition and subtraction, multiplication and division, and proportions and ratios
- encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7) in the domains of multiplication and division and proportions and ratio
- develop the students' knowledge of fractional numbers to stage 6.

Activity

In this activity, the students look for strategies to solve problems that involve finding a fraction of a set using simple proportions.

Use simple denominators and numerators greater than 1 to check that the students know the meaning of fractions. They should be able to explain that $\frac{2}{3}$ means "2 out of every set of 3" in the context of a fraction of a set. $\frac{2}{3}$ also means that 1 set is divided into 3 equal subsets, and 2 are selected. So, for a set of 6:



Use some simple examples as a warm-up mental activity, for example: $\frac{1}{3} \times 3 = \square$, $\frac{1}{3} \times 30 = \square$ and $\frac{2}{3} \times 30 = \square$. Highlight the connections in the sequence of equations by asking: *Can you use one equation to help you work out the next one?*

For question **1a**, the students who need to use materials could model 390 with place value materials (3 hundreds and 9 tens). They can then organise the set into thirds to find $\frac{2}{3}$.

The students can group together two lots of the $\frac{1}{3}$ x 390 piles to find $\frac{2}{3}$ of 390. Encourage them to image the problem by asking them to visualise breaking up 390 into smaller numbers that are easy to break into thirds: 390 ÷ 3 = (300 ÷ 3) + (90 ÷ 3)

= 100 + 30

= 130

130 x 2 = 260

Suggestions for possible diagrams for questions **1b** and **1c** are given in the Answers.
Another possible diagram based on splitting 390 into 300 and 90 is:

300				**90**			
100	100	100		30	30	30	
200			+	60			= 260

In question **2b**, check that the students can interpret the "How many 50-seater buses …" as the division equation 300 ÷ 50 = □ and as □ x 50 = 300. Discuss some strategies for solving these. For example: 100 ÷ 50 = 2 so 300 ÷ 50 = 2 x 3 = 6 or 300 ÷ 100 = 3 so 300 ÷ 50 = 3 x 2 = 6.

For question **3b**, have the students record the equation for each cost category, for example:
20 x 60 = □
260 x 30 = □
5 x 50 = □
5 x 30 = □

Students who are using materials could use the Large Dotty Array, Numeracy Project material master 6-9 (available at www.nzmaths.co.nz/numeracy/materialmasters.htm) to cut out a rectangle to model a strategy for each equation. For 20 x 60 = □, suggest that they cut out a 20 by 6 rectangle and calculate the number of dots. Ask: *How can 20 x 6 = 120 help you solve 20 x 60?* For 260 x 30 = □, the students could cut out a 25 by 30 rectangle and add a 1 by 30 rectangle to make 26 x 30 = 780. Ask: *How can 26 x 30 = 780 help you solve 260 x 30?*

For students using number properties, there is an alternative strategy using factors that would be worth sharing with them. Ask: *Can you break the numbers into easy factors to help you calculate the equations?* Work with the students to get a response such as:
20 x 60 = 2 x 10 x 6 x 10
= 2 x 6 x 10 x 10
= 12 x 100
= 1 200
(This is a key idea in the transition to advanced multiplicative strategies.)

Have the students apply this strategy to 260 x 30 = □:
260 x 30 = 26 x 10 x 3 x 10
= 26 x 3 x 10 x 10
= (20 x 3 + 6 x 3) x 100
= (60 + 18) x 100
= 78 x 100
= 7 800

The final addition of each cost category is another opportunity to discuss efficient addition strategies. For example:

1 200 + 7 800 + 250 + 150 = □

9 000 + 400 = 9 400

For question **4b i**, highlight the key facts:

- all 3 chairlifts have the same number of chairs
- they are full each day
- they make the same number of trips in a day.

Ask: *Given that these facts are the same for each lift, why do the chairlifts carry different numbers of people?* When the students realise that the chairs on each lift must hold different numbers, have them complete the chart for question **4b ii**. One strategy may be to extend the chart to include the total number of people carried by each chair and find a relationship between them. You may need to prompt the students who find an additive relationship to look for a multiplication relationship (for example, Whero x 1.5 = Mā; Mā x 2 = Kākāriki).

Chairlift	Total people carried	People in each chair
Kākāriki	9 000	3 x 2 = 6
Whero	3 000	2
Mā	4 500	2 x 1.5 = 3

Alternatively, the students may find that Whero x 3 = Kākāriki and Kākāriki ÷ 2 = Mā. So 2 x 3 = 6 for Kākāriki, then 6 ÷ 2 = 3 for Mā.

Pages 22–24: Ewe Scan

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)

Other mathematical ideas and processes

Students will also apply simple proportions to solve problems.

AC
EA
AA
AM
AP

Number Framework Links

Use these activities to:

- help the students who are beginning to use advanced additive strategies (stage 6) to become confident at this stage in the domains of multiplication and division and proportions and ratios
- encourage transition from advanced additive strategies (stage 6) to advanced multiplicative strategies (stage 7) in the domains of multiplication and division and proportions and ratios.

Activity One

The connections between the various quantities of ewes, lambs, and feed used in this context allow a variety of strategies to be applied to the problems. During the reporting back time, encourage the students to think of some simple proportional strategies. These strategies involve finding a multiplicative relationship between two or more different quantities.

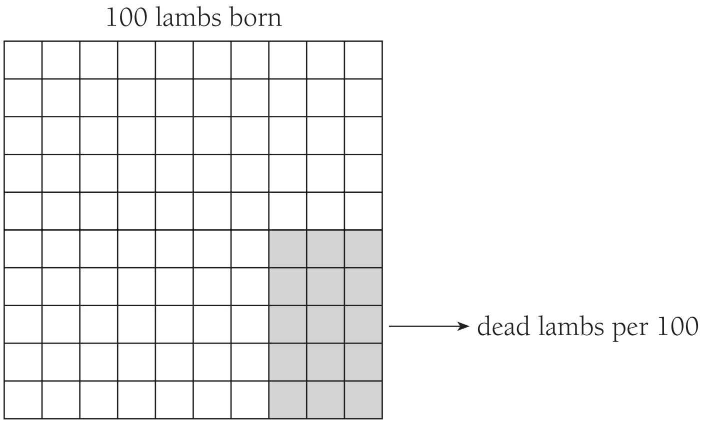
The students may need to revise the number of grams in a kilogram as well as multiplying and dividing by 10, 100, and 1 000. You could use series of mental questions such as:

- 3 000 grams = ☐ kg
- 5 x 100 = ☐
- 10 x 100 = ☐
- 15 x 100 = ☐
- 6 kilograms = ☐ grams
- 5 000 ÷ 100 = ☐

- 5 000 ÷ 10 = ☐
- 5 000 ÷ 1 000 = ☐
- 45 000 ÷ 1 000 = ☐.

Introduce the context and ensure that the students understand the chart showing drys, singles, twins, and triplets. Have the students move into small groups to attempt question **1**.

Materials may be useful for question **2**. After the students have found the total number of expected lambs in question **1b**, help them to understand question **2** by showing them how to use place value blocks, multilink cubes, bead frames, or similar materials to represent the groups of 100 lambs and use counters or 1 centimetre cubes to show the dead lambs.



Help the students to use number properties by modelling a ratio table to record the results shown by the materials:

Lambs born	50	100	200	300	400	4 000	4 450
Lambs lost	7 or 8	15	30	45	?	?	?

Focus on the column showing 100 lambs. Challenge the students to find the connections between that column and all the others up to 4 000. If they describe addition or subtraction connections, ask them to try duplicating the 100 lambs with materials to see what works instead.
Good scaffolding questions are:
300 is 3 x 100, so there will be 3 x 15 dead lambs.
Use a connection to work out the lambs lost in the 400 column.
What connection can we use between the 400 column and the 4 000 column?
How can you use what you have found out to work out the 4 450 column?

Question **3** can be approached in the same way with 4 bales of hay for every 100 ewes:

Ewes	50	100	200	300	500	1 000	1 300	1 300
Hay every 2 days	2	4	8	12	?	?	?	?
Hay every 30 days	30	60	120	180	?	?	?	?

Again, discuss the strategies used to complete the table with a focus on multiplicative connections, for example: *500 is 5 x 100, so there will be 5 x 4 = 20 bales.*

Make the most of question **4** as a measurement task by having the students find or describe objects that may weigh 100 grams and checking them on a balance or kitchen scale (for example, a small potato, a bag of peanuts, or a medium-sized chocolate bar). Another ratio table will help the students to see connections that can help the calculation strategies:

Twin and triplet ewes	1	50	100	200	300	500	1 000	1 300	1 350
Grain every 2 days (g)	100	5 000	10 000	20 000	30 000	?	?	?	?
Grain every 30 days (kg)	1.5	75	150	300	450	?	?	?	?

Activity Two

In the students' scenario, they use Donna's farm numbers as the guide for their farm. So in question **1**, they need to find one-third of the numbers of sheep that Donna has in each category and use this as the basis for working out the extra feed for 2 days.

Before the students attempt question **2a**, have them respond to the sheep's query in the speech bubble and encourage them to see the link to Donna's chart: *How can you use the fact that 450 x 3 = 1 350 to help you answer the question?*

An alternative strategy would be to use the appropriate columns in the ratio tables used for Donna's farm to make up the 450 ewes and match up the hay and grain.

If students do not see the connection between questions **2b** and **2a**, say: *Compare the number of ewes in questions **2a** and **2b**. What do you notice?*

Copymaster: Quick Thinking

Statement	Dice numbers		Point (✓)
Add to 10			
Difference of 2			
Two even numbers			
Sum of 8			
Twice the first number			
Sum less than 5			
Two odd numbers			
Two numbers greater than 3			
Sum greater than 10			
Difference of 3			
Total points			

Statement	Dice numbers		Point (✓)
Add to 10			
Difference of 2			
Two even numbers			
Sum of 8			
Twice the first number			
Sum less than 5			
Two odd numbers			
Two numbers greater than 3			
Sum greater than 10			
Difference of 3			
Total points			

Copymaster: Trimming Trees

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

Copymaster: Pick a Plate

	Caleb		Jessica	
Number plate	**Score**	**Score difference**	**Score**	**Score difference**
RR4578				
ZQ4731				
SZ8827				
TG9915				
WY4328				
YQ8888				
PJ2564				
RB3695				
UG7006				
XK2468				
TOTAL				

Number plate	**Score**	**Score difference**	**Score**	**Score difference**
TOTAL				

Copymaster: Make 28

		Running total

		Running total

		Running total

		Running total

		Running total

		Running total

Copymaster: Curling

Naseby			Kyeburn		
Ends	Shots	Running total	Ends	Shots	Running total
1	3	3	1	0	0
2	0	3	2	3	3
3	0	3	3	4	7
4	1	4	4	0	
5	4		5	0	
6	0		6	3	
7	0		7	1	
8	2		8	0	
9	0		9	3	
10	0		10	6	
11	4		11	0	

Naseby			Kyeburn		
Ends	Shots	Running total	Ends	Shots	Running total
1	3	3	1	0	0
2	0	3	2	3	3
3	0	3	3	4	7
4	1	4	4	0	
5	4		5	0	
6	0		6	3	
7	0		7	1	
8	2		8	0	
9	0		9	3	
10	0		10	6	
11	4		11	0	

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