Answers and Teachers' Notes

MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

Contents

Introduction

This book is one of three in the Figure It Out series that have proportional reasoning as their focus: *Proportional Reasoning*, Level 3+ and *Proportional Reasoning*, Level 3–4+ (Books One and Two). In these books, students explore the meaning of fractions and ratios and learn how to use them to make comparisons in a wide variety of contexts.

The books have been developed to support teachers whose students are moving onto the early proportional and advanced proportional stages of the number framework (stages 7 and 8). Like the other "plus" books in the Figure It Out series, these should be suitable for students needing extension. The level 3–4+ books are intended for use with students in year 6 but could be used at other levels at the discretion of the teacher.

The books aim to set activities in real-life and imaginary contexts that should appeal to students. The real-life contexts reflect many aspects of life in New Zealand and the young people portrayed in illustrations and photos reflect our ethnic and cultural diversity.

The activities may be used as the focus for teacher-led lessons, for students working in groups, or for independent activities. But bear in mind that the Figure It Out series is a resource, not a set of textbooks. This means that if you are setting an activity to be done independently, you should check that you have done whatever prior teaching is needed.

Teachers sometimes say that their students have difficulty understanding the words on the page. We are very mindful of this and try to keep written instructions as brief and as clear as possible, but to create a context and pose questions, some words must be used. It is important that mathematical language and terminology be deliberately taught.

The Answers section of the Answers and Teachers' Notes that accompany each student book includes full answers and explanatory notes. Students can use them for self-marking, or you can use them for teacher-directed marking. The teachers' notes for each activity include achievement objectives, a commentary on the mathematics involved, and suggestions on teaching approaches. **Although the notes are directed at teachers, able students can use them as a self-help resource**. The Answers and Teachers' Notes are also available on Te Kete Ipurangi (TKI) at www.tki.org.nz/r/maths/curriculum/figure/

Where applicable, each page starts with a list of the equipment needed. Encourage the students to be responsible for collecting this equipment and returning it at the end of the session.

Encourage your students to write down how they did their investigations or found solutions, drawing diagrams where appropriate. Discussion of strategies and answers is encouraged in many activities, and you may wish to ask your students to do this even where the instruction is to write down the answer.

> The ability to communicate findings and explanations, and the ability to work satisfactorily in team projects, have also been highlighted as important outcomes for education. Mathematics education provides many opportunities for students to develop communication skills and to participate in collaborative problem-solving situations.
>
> *Mathematics in the New Zealand Curriculum, page 7*

Students will have various ways of solving problems or presenting the process they have used and the solution. You should acknowledge successful ways of solving questions or problems, and where more effective or efficient processes can be used, encourage the students to consider their merits.

Figure It Out

Proportional Reasoning Level 3–4+, Book One Answers

Page 1: Galloping Greyhounds

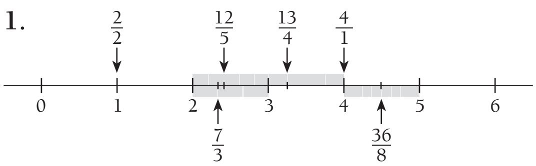
Activity

1. a. $\frac{3}{20}$. ($\frac{3}{5} + \frac{1}{4} = \frac{12}{20} + \frac{5}{20} = \frac{17}{20}$; $1 - \frac{17}{20} = \frac{3}{20}$)
   b. 12:5:3
2. a. \$360, \$150, \$90
   b. \$450, \$187.50, \$112.50
   c. \$540, \$225, \$135
   d. \$1140, \$475, \$285
   e. \$1500, \$625, \$375
3. a. \$48. ($\frac{3}{5} \times 800$ = \$480. 10% of \$480 = \$48)
   b. $\frac{3}{50}$. ($\frac{48}{800} = \frac{12}{200} = \frac{6}{100} = \frac{3}{50}$ or 6%)
4. a. $\frac{1}{40}$. ($\frac{1}{4} \times \frac{1}{10} = \frac{1}{40}$ or 2.5%)
   b. $\frac{3}{200}$. ($\frac{3}{20} \times \frac{1}{10} = \frac{3}{200}$ or 1.5%)

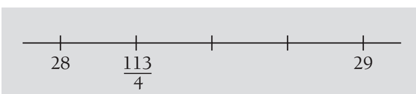
Pages 2–3: Fraction Line-up

Activity

1.

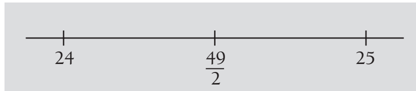


2. $\frac{113}{4} = 28\frac{1}{4}$

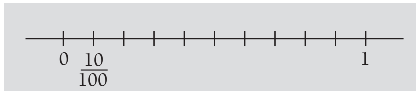


3. a. $\frac{1001}{5} = 200\frac{1}{5}$

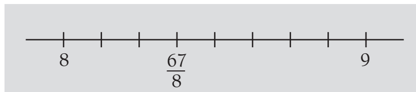
b. $\frac{49}{2} = 24\frac{1}{2}$



c. $\frac{10}{100} = \frac{1}{10}$

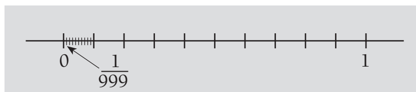


d. $\frac{67}{8} = 8\frac{3}{8}$

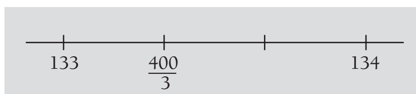


e. $\frac{55}{7} = 7\frac{6}{7}$

f. $\frac{1}{999}$ (very close to zero)

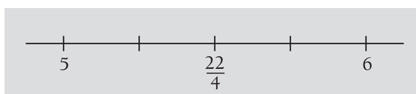


g. $\frac{400}{3} = 133\frac{1}{3}$

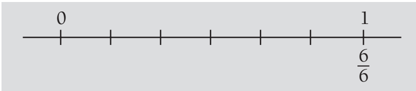


4. There are many other names, including $\frac{15}{12}$, $\frac{20}{16}$, $\frac{25}{20}$.
5. a. There are many names, including $5\frac{2}{3}$, $\frac{34}{6}$, $\frac{170}{30}$, $\frac{51}{9}$, $\frac{85}{15}$, 5.67, 567%.

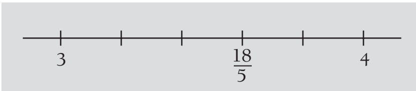
   b. There are many names, including $5\frac{2}{4}$, $5\frac{1}{2}$, 5.5, $\frac{11}{2}$, $\frac{33}{6}$, 550%.



c. There are many names, including $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{23}{23}$, $\frac{100}{100}$, 1, 1.0, 100%.



d. There are many names, including $3\frac{3}{5}$, $\frac{36}{10}$, 3.6, $\frac{54}{15}$, $3\frac{9}{15}$, 360%.



e. There are many names, including $2\frac{1}{2}$, $\frac{5}{2}$, $\frac{10}{4}$, $2\frac{3}{6}$, $\frac{250}{100}$, 2.5.

Pages 4-5: Tri Fractions

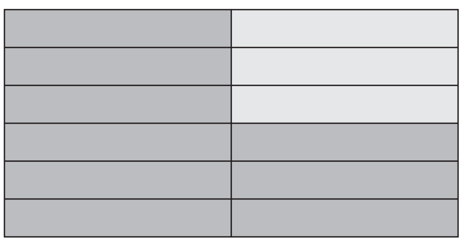
Games

Games for ordering fractions

Pages 6-7: Paper Partitions

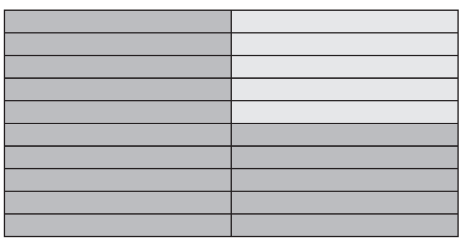
Activity One

1. a. 12 parts, 9 shaded



b. $\frac{3}{4} = \frac{9}{12}$

2. a. 20 parts, 15 shaded



b. $\frac{3}{4} = \frac{15}{20}$

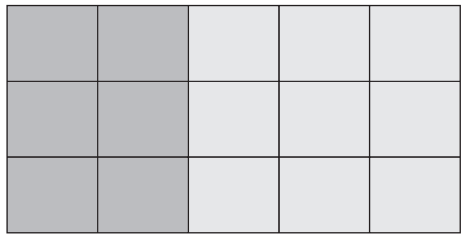
3. a. Fold the quarters in half: $\frac{3}{4} = \frac{6}{8}$

b. Fold the quarters into quarters: $\frac{3}{4} = \frac{12}{16}$

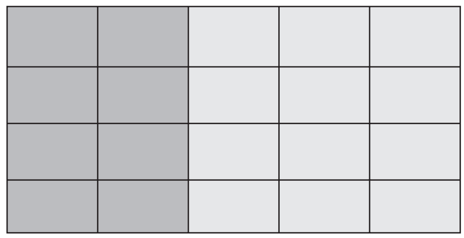
c. Fold the quarters into twenty-fifths: $\frac{3}{4} = \frac{75}{100}$

Activity Two

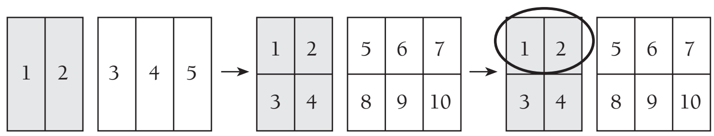
1. 15 parts, 6 shaded



2. 20 parts, 8 shaded



3. a. Discussion will vary. This diagram may help you make sense of the folds.



b. $\frac{2}{5} = \frac{6}{15}$; $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

c. $\frac{2}{5} = \frac{8}{20}$; $\frac{1}{4} \times \frac{2}{5} = \frac{2}{20}$

4. a. $\frac{5}{6} = \frac{10}{12}$

b. $\frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$

c. $\frac{5}{6} = \frac{15}{18}$

d. $\frac{3}{8} = \frac{9}{24}$

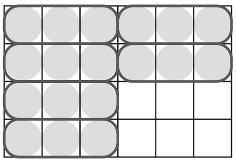
e. $\frac{1}{4} \times \frac{3}{8} = \frac{3}{32}$

f. $\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$

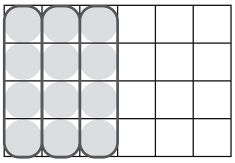
Pages 8–9: Fraction Extraction

Activity

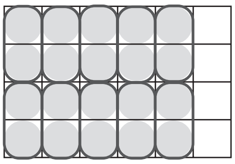
1. **a.** $\frac{1}{4}$ **b.** $\frac{1}{8}$ **c.** $\frac{1}{6}$
   d. $\frac{1}{3}$ **e.** $\frac{3}{4}$
2. **a.** $\frac{2}{3}$ **b.** $\frac{3}{8}$ **c.** $\frac{5}{6}$
3. $\frac{3}{4} \div \frac{1}{8} = 6$. Sue can give them six 3-packs.



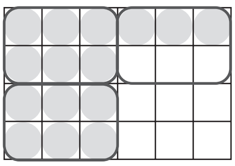
4. **a.** $\frac{1}{2} \div \frac{1}{6} = 3$. Sue can give them three 4-packs.



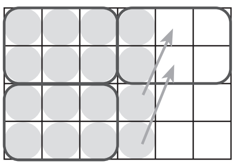
b. $\frac{5}{6} \div \frac{1}{12} = 10$. Sue can give them ten 2-packs.



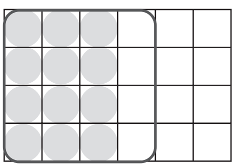
c. $\frac{5}{8} \div \frac{1}{4} = 2\frac{1}{2}$. Sue can give them two and a half 6-packs.



d. $\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3}$. Sue can give them two and two-thirds 6-packs.



e. $\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$. Sue can give them three-quarters of a 16-pack.



Page 10: Dot to Dot

Game

A game for practising adding and multiplying fractions

Solution:

12	2	2	$\frac{1}{6}$	$\frac{2}{8} = \frac{1}{4}$	13
42	4	16	1	$\frac{1}{8}$	18
13	$\frac{5}{8}$	$\frac{1}{4}$	28	14	32
$\frac{3}{4}$	16	$\frac{1}{2}$	$\frac{1}{16}$	18	23

Activity

Game boards will vary.

Page 11: Magnificent Models

Activity

1. **a.** 4.62 m. (10 x 46.2 cm = 462 cm = 4.62 m)
   b. Between 2.56 and 2.88 m
2. Measured to the nearest cm, the Minichamp racing car (43 x 10.75 = 462 cm = 4.62 m) and the V8 saloon (4.62 m) are the same length. The vintage racing cars are a lot smaller.
3. $420 \times \frac{1}{10} = 42$ cm (1:10 scale)
   $420 \times \frac{1}{8} = 52.5$ cm (1:8 scale)
   $420 \times \frac{1}{43} = 9.8$ cm (1:43 scale)
4. Answers will vary, depending on the cars or other objects chosen.

Pages 12–13: Planet Peewee

Activity

1. Rezel wants her height increased from 30 cm to 135 cm, an increase of $\frac{135}{30}$ = 4.5.
So her shoes will need to be 4.5 x 5 = 22.5 cm long.

2. a.

	Height (cm)		Shoe (cm)	
	Now	New	Now	New
Dixub	32	160	4.8	**24**
Wakog	18	126	2.7	**18.9**
Mujok	24	156	3.6	**23.4**

Dixub: 24 cm. ($\frac{160}{32}$ = 5; 4.8 x 5 = 24)

Wakog: 18.9 cm. ($\frac{126}{18}$ = 7; 2.7 x 7 = 18.9)

Mujok: 23.4 cm. ($\frac{156}{24}$ = 6.5; 3.6 x 6.5 = 23.4)

b. Answers will vary.

3. a.

	Height (cm)		Shoe (cm)	
	Now	New	Now	New
Pifal	28	**112**	4.2	16.8
Vazom	19	**114**	2.5	15

Pifal: 112 cm. ($\frac{16.8}{4.2}$ = 4; 28 x 4 = 112)

Vazom: 114 cm. ($\frac{15}{2.5}$ = 6; 19 x 6 = 114)

b. Answers will vary.

Pages 14–15: In Proportion?

Activity

1. 160 g is $\frac{1}{5}$ of 800 g. The amounts are all in proportion to each other:
$\frac{1}{5}$ of 400 is 80 g of rolled oats
$\frac{1}{5}$ of 250 is 50 g of bran flakes
$\frac{1}{5}$ of 150 is 30 g of dried fruit.

2. There is no way of knowing. The *theoretical* probabilities for each colour are: black $\frac{3}{10}$, white $\frac{1}{10}$, pink $\frac{1}{5}$, yellow $\frac{3}{20}$, blue $\frac{1}{4}$.
Using this information, you could predict that your 50 would be made up in this way: 15 black, 5 white, 10 pink, 7 or 8 yellow, and 12 or 13 blue. But it is extremely unlikely that this is exactly what you would get. You would get a different result every time you tried.

3. 10 days. (30 ÷ 3 = 10)

4. 61. (20 x 3 + 1 = 61)

5. 10 km is 100 times as far as 100 m, but André will not be able to run 10 km in 100 x 15 s (1 500 s = 25 min). This is because a runner can't keep up a sprint speed over a long distance. All that can be said for certain is that André will take longer than 25 min to run 10 km.

6. 32 g. (Twice the amount of sugar in the Feisty bottle is 48 g. Trim 'n' Slim has $\frac{2}{3}$ of this, that is, 32 g.)

Feisty: 24 g sugar	8 g	8 g	8 g	
Trim 'n' Slim: 16 g sugar	8 g	8 g		
	For a bottle the same size as Feisty			
Trim 'n' Slim: 32 g sugar	8 g	8 g	8 g	8 g
	For a bottle twice the size of Feisty			

7. $2.50. (If 6 oranges cost $1.50, 1 orange costs 25 cents, so 10 oranges cost 10 x 25 cents = $2.50.)

8. There is no way of knowing for sure. For one thing, the boats will have different lengths and weights. The pairs will probably be slower than the fours, but not twice as slow. The eights will probably be faster, but not twice as fast.

Pages 16–17: Skilful Skaters

Activity

1. Answers will vary, but the base must be less than 12 m in each case. Three possible answers (using only whole numbers) are: base 9 m, height 3 m; base 6 m, height 2 m; base 3 m, height 1 m.

2. a. There are an infinite (endless) number of possible ramps with the same slope.

b. To have the same slope, the height must always be one third of the base.

3. a. 5 m

b. 7 m

4. a. 2 m

b. 2 m

c. 4.5 m

d. 3 m

5. $10\frac{1}{2}$ or 10.5 m. (The base of the small ramp is $1\frac{1}{2}$ times its height, so the base of the larger ramp will be $1\frac{1}{2}$ times its height. $1\frac{1}{2}$ x 7 = $10\frac{1}{2}$ m.)

Pages 18–19: The Caves of Anawheku

Activity One

1. 3 times. (The stalactite is growing 0.06 mm/yr and the stalagmite 0.02 mm/yr. 0.02 x 3 = 0.06)

2. a. 2 106 mm. (2 100 + [0.06 x 100])

   b. 702 mm. (700 + [0.02 x 100])

   c. 8 mm. (6 + 2 = 8)

   d. 13 750 yrs.
   The gap between stalactite (i) and stalagmite (ii) is 1 100 mm (3 900 – [2 100 + 700]).
   1 100 ÷ 0.08 = 13 750 yrs.

3. (v) and (vi) will touch first.
   The gap between (i) and (ii) is 3 900 – 2 800 = 1 100, and this gap is closing by 0.06 + 0.02 = 0.08 mm/yr. 1 100 ÷ 0.08 = 13 750 yrs.
   The gap between (iii) and (iv) is 3 900 – 2 000 = 1 900, and this gap is closing by 0.09 + 0.07 = 0.16 mm/yr. 1 900 ÷ 0.16 = 11 875 yrs.
   The gap between (v) and (vi) is 3 900 – 2 700 = 1 200, and this gap is closing by 0.12 + 0.06 = 0.18 mm/yr. 1 200 ÷ 0.18 = 6 670 yrs (rounded to the nearest 10).

4. (i) and (ii) are the oldest pair; (iii) and (iv) are the youngest. (i) and (ii) have been growing for 2 100 ÷ 0.06 = 35 000 yrs. (iii) and (iv) have been growing for 1 125 ÷ 0.09 = 12 500 yrs. (v) and (vi) have been growing for 1 800 ÷ 0.12 = 15 000 yrs.

5. 2 500 yrs. At the moment, the difference is 900 – 875 = 25 mm. Each year, (iv) gains 0.01 mm (0.07 – 0.06 = 0.01) on (vi), so the two should be the same height after
   25 ÷ 0.01 = 2 500 yrs.

Activity Two

a. 0.05 mm. (i) and (ii) have been growing for 450 ÷ 0.02 = 22 500 yrs.
1 125 ÷ 22 500 = 0.05 mm.

b. 810 mm. (iii) and (iv) have been growing for 1 620 ÷ 0.06 = 27 000 yrs. 27 000 years of growing at 0.03 mm/yr =
27 000 x 0.03 = 810 mm.

c. 0.05 mm. (v) and (vi) have been growing for 15 500 yrs. 775 ÷ 15 500 = 0.05 mm.

Page 20: Fruit Proportions

Game

A game for comparing proportions

Page 21: Smart Sizes

Activity

1. a. The A3 sheet has twice the area of the A4 sheet.

   b. The length of the A4 sheet is the same as the width of the A3 sheet.

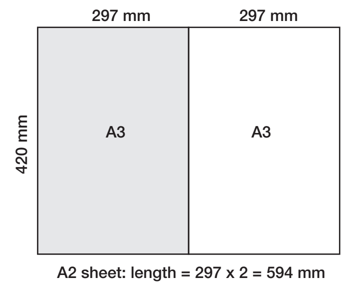
   The length of the A3 sheet is twice the width of the A4 sheet and about 1.4 times the length of the A4 sheet. ($\frac{420}{297}$ = 1.414)

2. a. 141%

   b. 71%

   c. If you enter 200%, you double the length and the width, which multiplies the area by 4. 50% halves the length and width and shrinks the picture to $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of the original size.

3. 594 x 420 mm:



4. 1 m^2. (This tidy measure is not an accident! Those who set up the system chose it for convenience.)

5. a. 5 times

   b. 32 sheets. ($2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$)

6. a. 37 x 26 mm

   b. The answer depends on the stamp, but it is most likely to be A9, A10, or A11.

Pages 22–23: The Right Mix

Activity One

1. a. Jenna: 12 + 12 + 12 + 60 = 96,
   96 ÷ 4 = 24°C.
   Toni: 3 + 3 + 3 + 15 = 24°C.

   b. Jenna: 12 + 12 + 12 + 12 + 60 = 108,
   108 ÷ 5 = 21.6°C.
   Toni: $2\frac{2}{5} + 2\frac{2}{5} + 2\frac{2}{5} + 2\frac{2}{5} + 12 = 9\frac{3}{5} + 12 =$ 21.6°C.

   c. Jenna: 12 + 12 + 12 + 60 + 60 = 156,
   156 ÷ 5 = 31.2°C.

   Toni: $2\frac{2}{5} + 2\frac{2}{5} + 2\frac{2}{5} + 12 + 12 = 7\frac{1}{5} + 24 =$ 31.2°C.

2. a. Discussion will vary. Where no fractions are involved (question **1a**) Toni's strategy is quick and easy. Where fractions are involved or a calculator is needed, Jenna's strategy will be best.

   b. If you add two or more numbers and take a fraction of the total, you get exactly the same result as first taking a fraction of each of those numbers and then adding them. This important mathematical idea is known as the *distributive principle*.
   Example: $(1 + 1) \div 4 = \frac{2}{4} = \frac{1}{2}$ is the same as $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

3. a. 30°C. (5 x 12 + 3 x 60) ÷ 8 = 30°C

   b. 32.6°C. (8 x 12 + 6 x 60) ÷ 14 = 32.6°C

4. a. We know from the girls' conversation that if there are equal quantities of cold and hot the temperature will be 36°C. Because the temperature is less than this, there must be more cold than hot.

   b. 6 cold and 4 hot. (Trying possible combinations)

Activity Two

a. Add 3c and 2h. This will fill the bath to the 150 L mark and keep the temperature at 31.2°C.

b. Add 2c and 3h. This will fill the bath to the right level and make the temperature 34.4°C.

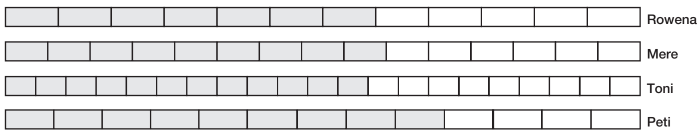
c. Add 1c and 4h. This will fill the bath to the right level and make the temperature 37.6°C.

Page 24: Top Shoot

Activity

1. Accuracy is measured by comparing the number of goals with the number of attempts. It can be expressed using a fraction, decimal, percentage, ratio, or using a diagram (for example, a strip diagram).

2. It depends on what is meant by "best shooting record". Toni scored most goals, 3 more than her nearest rivals. But Peti was the most accurate shoot. The record for the four girls at the end of game 1 can be summarised like this:



3. Again, it depends on what the coach is looking for. If it is accuracy, Peti is the clear winner: 15 out of her 20 attempts scored goals. The other three girls couldn't match this level of accuracy, but all three of them scored more goals. Mere scored most goals (22) but came only third as far as accuracy goes.

   The record for the four girls at the end of both trial games can be summarised like this:

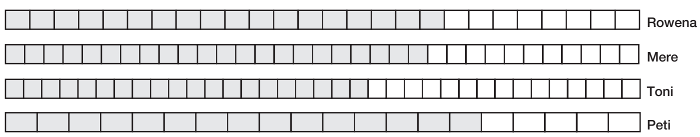


Figure It Out

Proportional Reasoning

Teachers' Notes

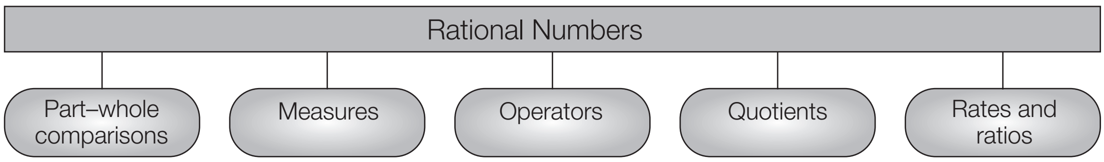
Overview of level 3–4+: Book One

Introduction

It is said that the rhinoceros beetle is the strongest animal on Earth; by some accounts, it can support up to 850 times its own weight on its back – the equivalent of a man supporting 75 cars. This does not mean that a rhinoceros beetle can lift heavier objects than any other animal; rather, that it is *proportionally* stronger than any other animal: the fairest measure of its strength is found by comparing what it can lift with its body weight.

Before they can make multiplicative comparisons of this kind, students need to extend their knowledge of numbers to include all rational numbers (those that can be written as fractions in the form $\frac{a}{b}$). With their two components (numerator and denominator), these numbers are able to express the relationship between two measures.

A difficulty for both teacher and student is that rational numbers can be used and interpreted in subtly different ways depending on the context. Kieran[1] suggested this helpful classification:



1. *Part–whole* comparisons involve finding the multiplicative relationship between part of a continuous space or of a set and the whole. For example, what fraction of a square has been shaded?

2. In a *measurement* context, a rational number is the answer to questions of the kind, "How many times does this fraction (or ratio) fit into that fraction (or ratio)?"

3. As *operators*, rational numbers perform operations on other numbers, for example, $\frac{1}{3} \times 12 = \square$.

4. As *quotients*, rational numbers provide the answers to sharing problems. It is important for students to recognise that $7 \div 4$ is an *operation* while $\frac{7}{4}$ (the quotient) is a *number* that is the result of that operation.

5. *Rates* involve a multiplicative relationship between two variables, each with a different unit of measurement (for example, kilometres and hours). Ratios are a special case of rates in that the units of measurement are the same for each variable (for example, 1 shovel of cement to every 5 shovels of builders' mix).

It is important that students are exposed to rational numbers in all their guises and that they learn to attribute different meanings to them, depending on the use and the context. It is also important that students learn a range of different ways of modelling situations that require proportional reasoning. This book will help in both areas. It should also help the teacher recognise that many everyday contexts can provide relevant and often intriguing rate and ratio challenges for their students.

[1]Kieran, T. (1976), "On the Mathematical, Cognitive, and Instructional Foundations of Rational Numbers", in Lesh, R. (ed.), *Number and Measurement: Papers from a Research Workshop*, Columbus, Ohio: ERIC/SMEAC.

Page 1: Galloping Greyhounds

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of equivalent fractions (stage 7)
- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

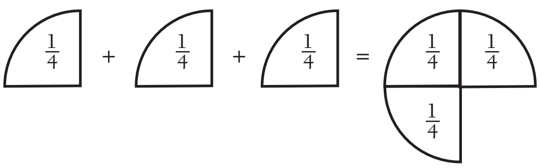
This activity provides a context in which students can see the need for a common denominator when adding or subtracting fractions. They use multiplication to find fractions of whole number amounts and make connections between fractions and ratios.

If the students are to do this activity independently, they will need to understand the significance of both the numerator and denominator, how a common denominator is used, and how to express parts of a whole as ratios.

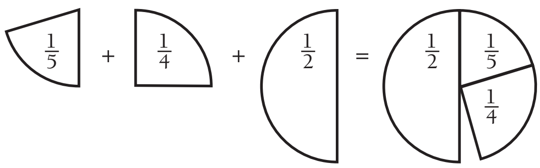
Denominator	The number on the bottom of a fraction is called the *denominator* because it specifies (or denominates) the unit value of the fraction. For example, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{9}{4}$, and $\frac{15}{4}$ are all fractions of the quarter kind (all multiples of $\frac{1}{4}$). Compare the use of the word *denomination* to specify the face value of coins or banknotes.
Numerator	The number on the top of a fraction is called the *numerator* because it tells us how many of that kind of unit fraction we have. $\frac{5}{4}$, $\frac{5}{5}$, $\frac{5}{6}$, $\frac{5}{12}$, and $\frac{5}{100}$ are all different kinds (denominations) of fractions, but in each case, we have 5 of them.

Use the speech bubbles in the student book as a starting point for discussion. Ask *"Why would this person choose to think of* $\frac{3}{5}$ *as* $\frac{12}{20}$ *?"* and *"Why do we need to use a common denominator when adding or subtracting fractions?"*

Ask your students to add together a number of fraction pieces that have the same denominator (are of the same denomination). For example:



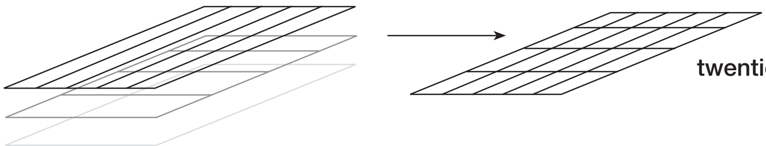
Then ask them to add some pieces with unrelated denominators (that are of different denominations). For example:



Through discussion, reinforce the fact that adding fractions with different denominators is a little trickier.

Students are used to the idea that multiplying increases size or quantity. For this reason, when they multiply the denominators of two fractions to get a common denominator and then rewrite the fractions with the new denominator, they may believe that these fractions are larger than the originals. Fraction models can be used to help them see that this isn't the case.

One method involves creating three overhead projector transparencies showing (i) a large square as a whole, (ii) the square divided into fifths, using four parallel lines, and (iii) the square divided into quarters, using three parallel lines. Superimposing the transparencies shows that twentieths are *smaller* pieces. You can use this grid to model all the fractions encountered in this activity.



A similar method involves paper folding (see Paper Partitions, pages **6–7** in the student book).

In question **1b**, students are asked to express the parts of the whole as a ratio instead of fractions. Fractions and ratios are closely related concepts, but students easily confuse the process by which the one can be expressed as the other. (The most common mistake is to think that $\frac{3}{4}$ can be written as 3:4, when in the case of the fraction, the whole is divided into 4 parts and in the case of the ratio, it is divided into 7 parts.)

Because the three fractions add up to a whole (the prize pool), their common denominator shows how many parts the whole must be split into before it can be shared out in a ratio. The numerators of the three fractions: $\frac{12}{20}$, $\frac{5}{20}$, and $\frac{3}{20}$, show the relative size of each share: 12, 5, and 3. This information is written as the ratio 12:5:3.

Your students may be puzzled as to why the two methods co-exist. Use the 5 by 4 grid illustrated above as the basis for discussion. Note:

- A fraction contains only information about itself: if Phil gets $\frac{3}{5}$ of the prize pool, this tells us nothing about how many others share the pool or what part of it they receive.
- A ratio contains complete information about the sharing process: if we know that the prize pool is divided in the ratio 12:5:3, we know that there are 3 claimants and that each receives $\frac{12}{20}$, $\frac{5}{20}$, or $\frac{3}{20}$ of the total. (Adding the parts of the ratio gives the denominator.)

Students need to learn to make sense of information expressed in either form.

Encourage your students to solve the parts of question **2** without a calculator. By doing this, they will further develop their proportional reasoning skills. Each of the prize pools divides neatly by 20 (as suggested by the speech bubble). Once the students have done this, all they need to do is multiply a single share (a twentieth) by the three numbers in the ratio to find what each person gets.

Suggest that the students look for short cuts when doing the calculations. For example, the answers to question **2c** will be half as much again as the answers to question **2a** because \$900 is half as much again as \$600. The \$1,900 in question **2d** may be thought of as \$2,000 – \$100 and the answers found easily in this way. The answers to question **2e** can be found by adding the answers to questions **2a** and **2d** because \$2,500 is \$600 + \$1,900.

In questions **3** and **4**, students learn to write basic percentages as fractions and to find the value of $\frac{1}{10}$ of the prizes for each place. Ensure that they can interpret question **3** and record a statement, an equation, or equations that will lead to a solution, for example: $\frac{1}{10} \times \frac{3}{5} \times \$800$.

To solve question **4**, students will need to be able to multiply two fractions together.

Pages 2–3: Fraction Line-up

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- express a fraction as a decimal, and vice versa (Number, level 4)
- express a decimal as a percentage, and vice versa (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help students consolidate and apply their knowledge of fractions through stages 5, 6, 7, and 8.

Activity

Questions **1** and **2** of this activity are best introduced in a guided teaching session. Once the students have completed and understood them, they may be able to work independently.

After reading through the introduction, ask *"What would be a good length for our unit strip?"* Discuss with your students how many folds will be needed and the lengths that can be measured and marked on the folds. The students may see that choosing a length that matches a common denominator for all seven fractions will make the task easier and aid accuracy. 120 mm is probably the shortest length suitable for folding.

Have the students make separate strips marked in thirds, quarters, fifths, and eighths so that they can choose the appropriate one to locate each of the different fractions in the question **1** challenge.

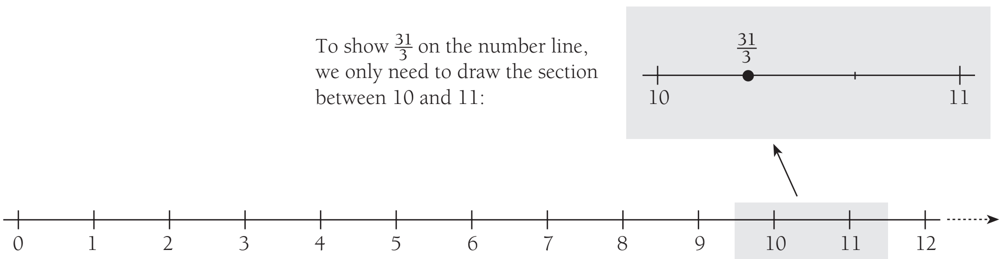
A feature of question **1** is that the numerators of all the fractions are greater than or equal to the denominator. This is to discourage the common misconception that fractions are always numbers less than 1.

Questions **2** and parts of **3** are designed to encourage students to extend by *imaging* the number line that they have made. Ask them to visualise the position for the fraction, tell a classmate where it should be, and explain their reasoning. The speech bubbles suggest how such an explanation might be made. Ask the students to be as precise as possible in their descriptions. Then get them to illustrate the location on a small section of the number line.

Your students may never have worked with a number line that doesn't start at 0, so you may need to explain what is meant by "a small section of number line". In reality, a number line extends infinitely in both directions (called *the* number line), so all we ever work with is "a small section" (called *a* number line). We draw as little as we need to show what we want to show, and if this means that we must image the location of 0 somewhere in the distance, that's fine.

To answer questions **3** and **5**, students need only show the part of the line that includes the whole number immediately above and below where the fraction belongs, as in this example:

To show $\frac{31}{3}$ on the number line, we only need to draw the section between 10 and 11:



Before your students start to answer questions **4** and **5**, they should read and understand the conversation that takes place in the speech bubbles. You will need to specifically teach or revise the equivalence relationship that exists between the fraction, decimal, and percentage expressions of a number. In everyday practice, we use the word *fraction* for numbers expressed in the form $\frac{3}{4}$ or $1\frac{1}{2}$. But 0.75 and 1.5, 75% and 150% are also fractions, differing only in their appearance. This means, of course, that $\frac{3}{4}$, 0.75, and 75% are all the same point on the number line. You could use Percentage Strips (material master 7–4, available on www.nzmaths.co.nz) and the associated activity in Numeracy Project *Book 7*: *Teaching Fractions, Decimals, and Percentages*, page 31, to help teach the relationship between fractions and percentages.

Pages 4–5: Tri Fractions

Achievement Objective

Not stated directly but implicit in this objective:

- order decimals with up to 3 decimal places (Number, level 3). Common fractions need ordering as well as decimal fractions.

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to help students to consolidate and apply their knowledge of equivalent fractions (stage 7).

Game

In this enjoyable game, students practise the ordering of fractions less than 1. Introduce the game by explaining the rules and then demonstrating it with a team of players while the rest of the class or group observes.

All the denominators in **Game One** relate as doubles. Discuss with your students how this feature helps to make it easy for them to select a fraction. They may see that all the denominators can easily be converted to 16 and the equivalent fraction found for each. The fractions on the cards can easily be converted to the same denominator.

Model a procedure for resolving disputes about the correctness of answers. The students should challenge any answer they think is incorrect but must explain their reasons. The player who placed the disputed counter needs either to justify their answer or remove the counter.

Students could use a set of fraction pieces or Fraction Strips (material master 7–7, available on www.nzmaths.co.nz) to help them argue their case.

Game Two is more challenging than **Game One** because the denominators of the fractions are multiples of 2 and 3, not just 2.

Ask your students *"What denominator would work for all the fractions in this game?"*

If they list the denominators in order of size: {2, 3, 4, 6, 8, 12, 24}, they may see that all these numbers are factors of 24, which means that this is the common denominator they are looking for. They should also check that all the fractions on the cards can be expressed using this denominator.

As an extension, students can make up their own game using a range of denominators appropriate to their level of understanding. They will need to make a set of cards to match their board.

When your students are confident with these games, you can use them as maintenance activities to be played independently while you work with other students.

Pages 6–7: Paper Partitions

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of equivalent fractions (stages 6 and 7)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

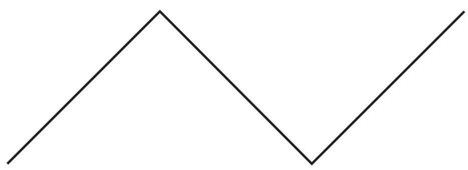
Activity One

This activity creates meaning for the concept of equivalent fractions. It is best used in a guided teaching situation. The students should follow the instructions carefully, particularly noting the need to refold the sheet into quarters after shading before they try to fold these into thirds. It is this folding process that turns the quarters and thirds into twelfths. The activity models the way a common denominator is found through multiplication.

Show the students how to record what is happening to the whole sheet through the folding process, using equations:

- The unfolded sheet is the "whole", which means that it can be represented by the number 1 (which can also be written as $\frac{1}{1}$).
- When the sheet is folded into quarters, this can be represented by the equation $\frac{1}{1} \times \frac{4}{4} = \frac{4}{4}$. The sheet shows 1 whole as 4 quarters.
- When the quarters are folded into thirds, this may be represented by the equation $\frac{4}{4} \times \frac{3}{3} = \frac{12}{12}$. The sheet shows 1 whole as 12 twelfths.

Students may find it difficult to fold a sheet or strip into thirds with a reasonable degree of accuracy. Demonstrate how the folding procedure produces a Z shape along the edge of the paper. Tell your students to try to get each of the three sections the same length before making their crease final.

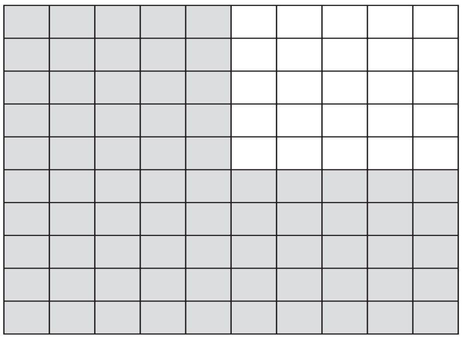


Students should aim to be as tidy and accurate as possible, but their folding doesn't need to be precise for them to understand the maths concepts involved.

In question **2**, the quarters are folded into fifths. If necessary, students could use a ruler to mark the shortest side of the quarter-folded paper at approximately 20 mm intervals to show where the fold lines should go.

Ask them to record the folding process and its effect on the shaded sections as an equation: $\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$.

The students should be able to do question **3c**, but because of the number of folds and the thickness of the paper, it's not easy. When opened out, the sheet should look like the following picture. By now, your students will see that paper folding has its limitations!



Ask them to solve the same problem using number properties: $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$. Then get them to summarise their findings, ensuring that they include some equivalent fractions for $\frac{3}{4}$ and that they explain that the folded sheets show the same fraction in different-sized pieces.

Activity Two

In this activity, the emphasis is on the writing of the equations modelled by the folding process, and students do not need to do the folding in questions **1–3** unless they still want the support of materials.

If they look at the shaded parts of the sheet in question **1**, they will see that $\frac{2}{5}$ reappears as $\frac{6}{15}$. Ask *"How can we show, using numbers, what has happened to the $\frac{2}{5}$?"*

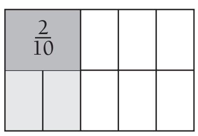
Encourage responses such as "When we folded the fifths into thirds, we multiplied fifths by thirds to make fifteenths (a new denominator), so 2 fifths became 6 fifteenths (the numerator changed too)." This can be recorded as $\frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$.

The equation for question **2** is $\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$. Ask "*$\frac{4}{4}$ is another name for what number?*" The students should realise that the number is 1 (the whole).

You could say *"That means we are multiplying $\frac{2}{5}$ by 1 in another form. What happens to a number when it is multiplied by 1?"* Multiplying a number by 1 can change its appearance but not its value. $\frac{8}{20}$ is still $\frac{2}{5}$. This leads to an important understanding that can be summarised like this: *"We make equivalent fractions by multiplying (or dividing) a fraction by equivalents of 1 (for example, $\frac{3}{3}$ or $\frac{5}{5}$)."*

Question **3** creates meaning for the concept "a fraction of a fraction", or the multiplication of one fraction by another. In this question, students are first asked to consider half of $\frac{2}{5}$ (not to rewrite $\frac{2}{5}$ in an equivalent form).

Once the students have folded the shaded $\frac{2}{5}$ in two to halve it, they can add further shading to the parts that amount to one half of the $\frac{2}{5}$. They will see that these parts make up $\frac{2}{10}$ of the whole (1).



Link this diagram to the equation $\frac{1}{2} \times \frac{2}{5} = \frac{2}{10}$. This equation provides a model for the answers to questions **3b–c**.

The students can continue to use paper folding or another strategy involving materials to help them solve the equations in question **4**, or they may be able to solve them using imaging or number properties.

Pages 8–9: Fraction Extraction

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find fractions equivalent to one given (Number, level 4)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

This activity creates meaning for the concept "division of a fraction by a fraction".

The carton uses a set (24 cans) as a unit or whole. Students should understand that this can be represented mathematically by the equation: $\frac{24}{24} = 1$.

The illustrations on the page provide good support for an introductory discussion leading into this activity. If further support from materials is needed, use counters to represent the cans.

Use question **1** to establish that your students understand fractions of a set and can express them as equivalent fractions in their simplest form.

Encourage your students to make statements such as "A 6-pack is 6 out of 24 so it is a quarter of a carton." They should record this as $\frac{6}{24} = \frac{1}{4}$. You could also ask them to record their statements as ratios, like this: 6:24 = 1:4.

Ask confident students to explain how to simplify the fraction using number properties: *"The only way we can divide a number without changing its value is to divide it by 1. So to simplify the fraction we need to divide it by a suitable equivalent of 1. Of all the possiblilities, which should we try?"* $\frac{6}{6}$ is the fraction they need. Students can record the division operation in this way: $\frac{6}{24} \div \frac{6}{6} = \frac{1}{4}$.

Question **2** gives the students further practice at finding fractions of a whole and expressing them in their simplest forms.

Question **3** introduces the idea that one fraction can be divided by another. The speech and thought bubbles give the students the clues they need to solve each problem. Guide them through the problem using numbers of cans: *"How many cans are in the $\frac{3}{4}$ carton?" "How many 3-packs are in the $\frac{3}{4}$ carton?" "Why is the question 'How many 3-packs are there in $\frac{3}{4}$ of a carton?' the same as 'How many $\frac{1}{8}$s are in the $\frac{3}{4}$ carton?'* "

It is vital that students connect these "How many … in?" questions so that when they read division equations, they can interpret them in ways that are meaningful to them. $\frac{3}{4} \div \frac{1}{8} = \square$ can be read as "$\frac{3}{4}$ divided by $\frac{1}{8}$ equals what?" but it means "How many $\frac{1}{8}$s are there in $\frac{3}{4}$?"

Another approach is to ask *"Can we use equivalent fractions to make it easier to see the answer?"* A response might be "If we change $\frac{3}{4}$ into $\frac{6}{8}$, we can see that there are 6 one-eighths in six-eighths. So $\frac{6}{8} \div \frac{1}{8} = 6$."

Questions **4c** and **4d** increase the challenge because both solutions are mixed numbers. If students reinterpret question **4c** as "How many $\frac{2}{8}$s are in $\frac{5}{8}$?", they will find it much easier to make sense of the problem.

Show your students how to set the problem down mathematically. Rewrite $\frac{5}{8} \div \frac{1}{4}$ in the form $\frac{5}{8} \div \frac{2}{8}$, making use of an equivalent fraction for $\frac{1}{4}$. Now divide one numerator by the other ($5 \div 2 = 2\frac{1}{2}$) and one denominator by the other ($8 \div 8 = 1$). The result is $2\frac{1}{2} \div 1 = 2\frac{1}{2}$. This makes meaningful use of number properties to solve a division-by-a-fraction problem instead of the traditional "rule": to divide by a fraction, multiply by its reciprocal (or, invert and multiply).

Question **4e** can easily be misinterpreted. Put students into mixed-ability problem-solving groups with not more than four members and have them find and justify an equation that matches the problem. The question is "How many two-thirds cartons can be made from one half carton?" Expressed slightly differently, this is "How much of $\frac{2}{3}$ is in $\frac{1}{2}$?" The appropriate equation is $\frac{1}{2} \div \frac{2}{3} = \square$. If some students can't see this, they should go back to any "goes into" equation that they do understand and see how this one follows the same pattern.

Using equivalent fractions, $\frac{1}{2} \div \frac{2}{3} = \square$ can be rewritten in the form $\frac{3}{6} \div \frac{4}{6} = \square$. Dividing one numerator by the other gives $\frac{3}{4}$. Dividing one denominator by the other gives 1. $\frac{3}{4} \div 1 = \frac{3}{4}$. This shows that a half carton is the same as three-quarters of two-thirds of a carton! Match this statement to a diagram of two thirds of a carton so that students can visualise it (as in the Answers).

Page 10: Dot to Dot

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of equivalent fractions (stage 7)
- develop confidence in students who are beginning to use advanced multiplicative strategies (stage 7).

Game

This variation on an old game will motivate students to apply their knowledge and strategies to calculations involving fractions. The challenge of devising a winning strategy will encourage them to come back and play the game again.

This game could become a valuable addition to your repertoire of practice and maintenance activities suitable for independent use.

Most of your students are likely to have met and played the traditional version of this game. When explaining the rules for this version, highlight the key difference that the player who completes a square does *not* get an extra turn. This means that the emphasis moves from completing as many squares as possible to trying to complete the squares that are worth most points. Some squares are worth very little; others are worth a lot.

Note that "of" appears as an operator in a number of squares. Compare the questions that use "of" with those that use the "x" sign. The game reinforces the fact that both operators have an identical effect.

After the students have played the game a couple of times, get them to discuss strategy. Questions such as these will focus the discussion:

- *Which squares are worth most?*
- *Is it more important to try for squares that are worth most or ones that you are sure to get correct?*
- *Which operation gives the most points?*
- *Which squares did you find most difficult?*

If players can't agree on an answer, they could ask a classmate or consult the answers master.

You may need to discuss with your students how they will total their points. It may be simplest if each player keeps a running total. Most of the answers are easily added whole numbers. The fractions will be a little more difficult, and if students have trouble with them, you could suggest that they record them as they win squares, but leave the addition until the end of the game. As this game is about learning to add and multiply fractions, players should not use calculators!

Activity

There is plenty of scope here for students to create their own game boards, using fraction problems that are within their range of understanding. The game board copymaster has no subtractions or divisions. Students could make a game that includes one or both of these operations.

You may like to make up further game boards yourself, at different levels of difficulty. If you make them as masters and copy them as needed, you will get good value out of the time spent.

Page 11: Magnificent Models

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

This activity explores direct proportion in a context involving scale models.

Some of your students may have had experience with scale models. If they have kept any of the boxes that contained their models, ask them to bring them and show the class. They may even be prepared to bring along the models.

Discuss and compare the scales on the boxes. Scales for car models are often 1:18, 1:24, and 1:64. Aeroplane models are often 1:48 or 1:87. Discuss with your students why such a range of scales exists. If you are able to obtain two models of the same object in different scales, you will be able to demonstrate the scaling effect in an interesting way.

Ensure that the students understand that (i) the notation "1:10" means that 1 unit of length on the model represents 10 units on the real-life object, and (ii) the ratio is true for any unit of length.

Question **1a** provides a context for exploring strategies for multiplying by 10. The old rule, "to multiply by 10, add a 0", is shown to be nonsense here: 46.2 becomes 46.20, which is mathematically no different. (Well, let's not get into limits of accuracy!) Encourage your students to see that when a number is multiplied by 10, the digits move 1 place to the left, and so 46.2 becomes 462.0.

Revise their knowledge of measurement by asking them to express the measurement in centimetres as a measurement in metres. Get them to compare the position of the digits after multiplying the centimetre length by 10 and then dividing the result by 100 to give a measurement in metres: 46.2 cm multiplied by 10 becomes 462.0 cm, which, when divided by 100, becomes 4.62 m. Encourage them to make observations such as "Multiplying by 10 moves the digits one place to the left of the decimal point, while dividing by 100 moves them two places to the right."

Question **1b** shows a range of model lengths rather than a specific length. Students will need to use the scale to express this range in metres.

When the students compare the Minichamp racing cars in question **2** with the V8 in question **1**, they will find that the Minichamp cars are 0.25 cm longer. You could ask your students about whether this amount "matters" in the circumstances. Remind them what a quarter of a centimetre looks like. The Answers suggest that it is sensible to round the measurement to the nearest centimetre.

In question **3**, students need to scale down instead of scale up as in the previous two questions. Point out that scale ratios are usually expressed as "1 to something", not "2 to something". As long as this is the case, it is a simple matter of multiplying by the larger number in the ratio to scale up, or dividing by the larger number to scale down. The students should be given every chance to deduce this for themselves.

Expect your students to solve the 1:10 and 1:8 scale problems mentally or on paper but to use the calculator option for the 1:43 scale. They should discuss their mental strategies for solving the problems. A possible strategy for the 1:8 problem may go like this: "I thought of 4.2 m as 420 cm. The model is 8 times smaller, so I needed to divide by 8. I split 420 into 400 + 16 + 4 to make it easy. $400 \div 8 = 50$; $16 \div 8 = 2$; $4 \div 8 = \frac{1}{2}$. So the model is $50 + 2 + \frac{1}{2} = 52\frac{1}{2}$ (or 52.5) cm."

In addition to calculating the lengths asked for in question **4**, the students could try drawing outlines of their chosen objects on square grid paper, using one or more of the three scales. They will need to use objects that are no longer than 3 m if they are to fit on A4 paper at a scale of 1:10. And they'll need to take some vertical measurements too.

As an extension, you could get your students to draw geometrical shapes (or more complex objects) using a suitable computer program and then experiment with the stretch or enlarge function to change their size. If they do this, they will need to express increases or decreases as percentages rather than ratios.

Pages 12–13: Planet Peewee

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

In this fantasy context, students learn how to express the relationship between two measurements as a ratio and then use this ratio to find a missing measurement.

Get the students to form small problem-solving groups to tackle question **1**. To help them focus, put these questions to them:

- *How many facts do we have?* (3)
- *How many details are we asked to find?* (1)
- *How are the three facts and the missing detail connected?*

Have the groups report their findings. One line of reasoning goes like this: "If we can find out what we need to multiply the first height by to get the new height, we could use the same number to multiply the first shoe length to get the new shoe length." Discuss strategies for solving the equation $30 \times \square = 135$.

Make sure that the students know that they can also write the new height / first height relationship in ratio format: $\frac{135}{30} = 135:30 = 4.5 : 1$

Another line of reasoning goes like this: "If we work out how many times taller the person is than the length of their shoe, we can divide their new height by this number to find their new shoe length." Once the students see that Rezel's height is 6 times her shoe length, they can solve the equation $6 \times \square = 135$.

The first height / shoe length relationship can also be recorded as a ratio, like this: $\frac{30}{5} = 30:5 = 6:1$

Although the header for this activity suggests that a calculator is necessary, you could challenge your able students to manage without. The calculations lend themselves to strategies such as partitioning and the multipliers are all whole numbers or whole numbers plus $\frac{1}{2}$.

Have the students work independently or in problem-solving groups on question **2a** and then report back before tackling question **2b**.

In question **3**, the relationships are reversed. This time, the students need to work out the multiplier for shoe length and then use it to work out each alien's new height.

By inventing their own aliens and measurements in questions **2b** and **3b**, students demonstrate that they understand how a ratio that applies to one set of measurements can be applied to another.

Once again, after your students have worked on question **3a**, get them to discuss the thinking process. For example: "To find the multiplier, I needed to divide 16.8 by 4.2. I asked myself '4.2 times what makes 16.8?' I could see that it was 4. So I multiplied the first height by 4. 28 x 4 = 112."

This activity provides an ideal opportunity for showing students how to set up a spreadsheet and use a formula. If they enter the data from question **2** in columns as in the table, the program will label the columns from the left as A, B, C, D, and E. Students will need to enter a formula in column E that connects the other data and calculates the entry. In MS Excel, the formula would be =C1/B1*D1.

Pages 14–15: In Proportion?

Achievement Objectives

- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

In this activity, students learn that they need to think carefully before deciding that change is proportional and whether proportional change is direct or inverse.

There are two main approaches that you could take when introducing the activity to your students.

- Put them into problem-solving groups and set them to work on it. Each of the scenarios is simple enough to generate discussion. It will be interesting to see how soon groups realise that proportional reasoning may not be appropriate in all cases.
- Discuss the activity with your students before they begin, alerting them to the learning outcome at the bottom of the page and emphasising the importance of using common sense before applying proportional reasoning to a problem.

The key question in each case is "Is it possible to solve this problem using proportion?"

In question **1**, the ingredients in the large and small quantities of muesli are directly proportional: for less muesli, you need less of the ingredients. Students may find it helps to use a ratio table:

Size	Rolled oats : bran : fruit	Comment
800 g	400 : 250 : 150	
80 g	40 : 25 : 15	Divide by 10
160 g	80 : 50 : 30	Double

Alternatively, they may reason "We need to know how many 160s there are in 800. Since 10 x 160 = 1 600 and 1 600 is double 800, it must be that 5 x 160 = 800. So we need to divide all the amounts by 5."

Get the students to report back with their strategies. Restate the explanations to help others understand them, and record them on the board.

There is no answer to question **2**. If the students are unable to see this, or are uncertain, they could use plastic beans, marbles, or similar materials to act out the scenario a number of times, recording their findings. (The logistics will be easier if they divide all numbers by 10.) They will see that each time they withdraw the required number of beans, the result is different.

You may like to discuss the fact that proportional reasoning can nevertheless be used in a case like this to work out the kinds of outcomes that are more likely or less likely. In other words, introduce probability as a mathematical tool that can be used to analyse the scenario. Given, for example, that there are twice as many blacks as yellows, it is probable that a blindfolded person who removes some beans from the jar will end up with more blacks than yellows. This trend would certainly become obvious if the experiment were conducted many times. It can be confirmed mathematically by looking at the ratios of the numbers of beans of the different colours:

Number of beans	Black : white : pink : yellow : blue	Comment
1 000	300 : 100 : 200 : 150 : 250	
10	3 : 1 : 2 : 1.5 : 2.5	Divide by 100

Students should not confuse what is probable with what will happen. If a blindfolded person took 50 jellybeans from the jar as described, they would probably get some of each colour, more blacks than any other colour, and fewer whites. But then again … all 50 could be black!

The “catch” in question **3** is that the relationship between consumption and duration is inverse: the big fish eats three times as much as the little fish, so the packet of food will last the big fish just one third as many days.

Question **4** encourages students to think algebraically as they try to find a rule to connect the number of eagles and the number of trapeziums. Suggest that they make a table connecting eagles and trapeziums:

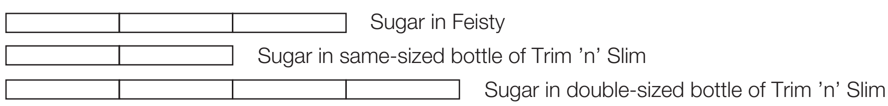
Eagles	1	2	3	4		20
Trapeziums	4	7	10	13		?

Ask *“What patterns can you see in the numbers?”* They should see that each time an eagle is added, the number of trapeziums increases by 3. Ask *“Why does the number needed go up by 3 trapeziums when there are 4 in each eagle?”* They should see that each eagle added to the first shares a trapezium.

This information can now go into a short cut: either (20 x 3) + 1 = 61 for twenty eagles, or (19 x 3) + 4 = 61. Encourage your students to verbalise a generalisation for this pattern and record it algebraically, for example: to find the number of trapeziums, multiply the number of eagles by 3 and add 1. Algebraically, this can be written as $t = (e \times 3) + 1$ (where t stands for the number of trapeziums and e for the number of eagles).

Most students will have had some experience of running and should recognise that no runner can sustain a sprint speed for a long distance. This means that question **5** cannot be solved using proportional reasoning.

Question **6** is not difficult, but like a lot of questions involving proportional reasoning, it may be hard for students to work out where to begin. A strip diagram can be a useful tool for visualising the problem and seeing the relationship between the various bits of information:



Question **7** is the most straightforward part of this activity: the cost is directly proportional to the number of oranges. Like question **5**, question **8** can’t be solved using proportional reasoning.

To round off the activity, bring the groups together to summarise the eight questions in terms of the learning outcome. Highlight the importance of thinking about the context and looking for answers that are sensible, instead of unthinkingly multiplying or dividing.

Pages 16–17: Skilful Skaters

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- find and justify a word formula which represents a given practical situation (Algebra, level 4)
- enlarge and reduce a 2-dimensional shape and identify the invariant properties (Geometry, level 4)

AC
EA
AA
AM
AP

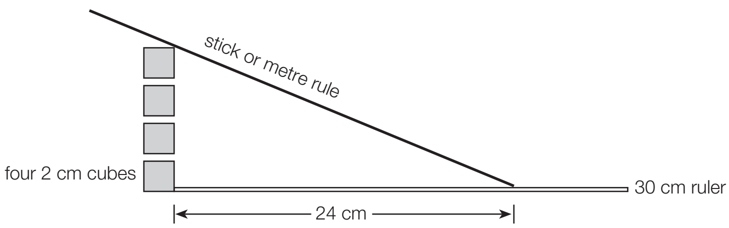
Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of equivalent fractions (stage 7)
- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8).

Activity

This activity explores the base–height–slope relationship and introduces students to similar triangles. Similar triangles are a further application of direct proportion. It's a good idea to introduce the activity using materials. Rulers (30 cm and 1 m) and cubes (1 cm and 2 cm) are useful:



Put the students into problem-solving groups. Tell them to represent the ramp with a ruler. Get them to keep its lower end 24 cm from the base of the pile of cubes and to explore what happens to its slope when its height is changed by adding or removing cubes.

Next they should leave their pile of 4 cubes in place but move the lower end of the ramp along the horizontal ruler and note what happens to the slope. They should then change both the number of cubes and the position of the bottom of the ramp and note how the slope changes.

Have the students set up two ramps. The first is their benchmark ramp. Working now with the second ramp, challenge the students to change both the height and the position of the lower end whilst keeping the slope the same as that of the benchmark ramp. They should see that there is a precise relationship between height, base, and slope, even if they can't put it into words yet.

Ask the students to solve questions **1** and **2** in their small groups and report back with their answers for question **2b**. Help them refine their statements into a formula. Two alternatives are:

- height of ramp = length of base divided by 3 ($h = \frac{b}{3}$)
- the base is 3 times as long as the height ($b = 3 \times h$).

Some may like to express the slope as a ratio, 1:3.

(Note that when expressing slope as a ratio, height always comes before base. This convention avoids confusion. Walking up a 1:5 slope is altogether different from walking up a 5:1 slope!)

Your students should now be able to solve question **3**.

When solving questions **4** and **5**, your students may find that a ratio table or double number line helps them to see how the height and base are related:

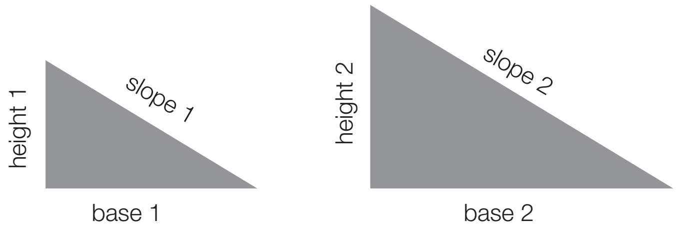
Height	4 m	? m
Base	6 m	3 m

They can then use multiplication or division to find the missing dimensions. Another important insight is that the height–base relationship for any slope can be seen as an (infinite) sequence of equivalent fractions, for example: $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$. Students can use this insight as a strategy for solving these problems.

As an extension, this activity could be used to introduce the concept of similar triangles (and other similar shapes). There are two key ideas to get across:

- In maths, *similar* means "has exactly the same shape as".
- Each pair of *corresponding* (matching) sides is in the same ratio.

Using the two triangles shown as an example, base 1 : base 2 = height 1 : height 2 = slope 1 : slope 2.



You could challenge your students to identify which of the four pairs of triangles in question **4** are similar. (Pairs **a**, **b**, and **d** are because the height:base ratio in each case is 2:3.)

Pages 18–19: The Caves of Anawheku

Achievement Objective

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of decimal place value (stage 8)
- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8).

Activity One

This activity involves exploring and comparing rates of change and requires students to think carefully about how different factors relate to each other.

Begin by revising what happens to the place value of the digits in a whole number or a decimal when it is multiplied or divided by 10, 100, and 1 000. Students need to know that when they multiply a number by 10, the digits move one place to the left in their place value columns and when they divide by 10, the opposite happens. Questions like these can be used to check understanding: 0.15 x 100 = □ and 10.6 ÷ 1 000 = □.

Check that your students can read the diagrams. Questions that you could use to do this include:

- *What is the height of the cave?*
- *How can you work out the size of the gap between a stalactite and its stalagmite?*
- *How does the diagram tell you the rate of growth of the stalactites and stalagmites?*
- *Why do the stalactites grow faster than the stalagmites?*

In question **1**, students need to understand that if stalactite (i) is growing at a rate of +0.06, this means that it is growing by $\frac{6}{100}$ of a millimetre each year (mm/yr) and that this is faster than $\frac{2}{100}$ of a millimetre per year, which is the growth rate of stalagmite (ii) below it. From there, it is a short move to comparing the rates of growth and answering the question *"How much faster …?"*

To solve question **2d**, students need to see that the stalactite and stalagmite are coming together simultaneously but at different rates, so they are getting closer by 0.06 + 0.02 = 0.08 mm/yr. If the gap is being reduced by this amount each year, the answer to the question *"How long will it be …?"* is answered by dividing the gap of 1 100 mm by 0.08.

You will need to make a judgment call as to whether you want your students to use a calculator to solve the different parts of this activity. Almost all of the calculations involve numbers that are fairly easy to work with, and they are especially suited to part–whole strategies. If you want your students to work without calculators, get them to use the think, pair, and share technique to help them find strategies. You may need to prompt them:

- *What do you have to find out to know how long it will take?*
- *What equation means the same as that?*
- *If dividing by 0.08 is awkward, what can you do to make it easier?*
- *Have you got an easy way to divide 1 100 by 8?*

One part–whole strategy would go like this: 800 ÷ 8 = 100; 240 ÷ 8 = 30. That leaves 60 ÷ 8 = 7.5. So we have 137.5 lots of 100 yrs, which makes 13 750 yrs.

Before the students start calculating in question **3**, get them to use proportional reasoning to predict the answer. They may see that (i) and (ii) are coming together at 0.08 mm/yr while the rate for (iii) and (iv) is 0.16 mm/yr, and for (v) and (vi), 0.18 mm/yr. This last pair is coming together twice as fast as the first pair but the gap is similar; and a little faster than the second pair but the gap is much smaller. Based on this reading, the third pair will join first. Students should then confirm this prediction by calculation and discuss a variety of strategies for doing so.

In question **4**, proportional reasoning suggests that the first pair must be the oldest because their total length is greater than that of the other two pairs, but they have been growing at no more than half the rate. To confirm that this is the case, students should simply divide the length of either the stalactite or the stalagmite by its rate of growth. This will give its age in years.

Question **5** should generate some interesting discussions and strategies. Most students will probably start with a trial-and-error method. Show them how to turn this into a trial-and-learn method: under the heading Trial, they could record the statement "(iv) starts at 875 and goes to 882 in 100 yrs, while (vi) goes from 900 to 906." Under the heading Learn, they could record "over 100 yrs, (iv) gained 1 mm on (vi)." Some will use this reasoning to see the solution immediately, while others will need to do a number of trials.

Activity Two

Have the students discuss this question in their problem-solving groups and report back to the class. Direct them to come up with a strategy for solving part **a**, based on the known facts. The vital connection they need to make is that stalagmite (ii) is the same age as stalactite (i). Once they realise this, they should be able to determine the growth rate of stalactite (i) by dividing its length by its age.

Encourage your students to share their calculation strategies. Assist them with recording by modelling their explanations on a chart or whiteboard.

Page 20: Fruit Proportions

Achievement Objectives

- express quantities as fractions or percentages of a whole (Number, level 4)
- find fractions equivalent to one given (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of equivalent fractions (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Game

Organise the playing cards and the dice. You will need a set of cards and a dice for each pair of students. Use 2 cm wooden or plastic cubes marked with yellow and orange permanent markers. Alternatively, use standard numbered dice and specify that even numbers represent lemons and odd numbers represent oranges.

To play this game, students need to understand how to compare two proportions or ratios.

The speech bubbles model one way to do this. The key is to match numbers for one of the two fruits. Alex sees that 5:10 is equivalent to 1:2. But there are 4 oranges on Pania's card, so he finds another equivalent, 2:4, that will give him a match on the oranges. He can now compare 2:4 with 3:4 and see that Pania's card has the highest proportion of lemons (more lemons for the same number of oranges). For an activity that explores this method ("cloning"), see Flavoursome (*Proportional Reasoning*: *Book Two*, levels 3–4+, pages 6–7.)

A second way to compare the ratios is to first express them as fractions. In the scenario that introduces the game, the dice shows a lemon, so the winner is the player with the highest proportion of lemons. Lemons make up 5 of the 15 fruit on Alex's card ($\frac{5}{15}$). They make up 3 of the 7 fruit on Pania's card ($\frac{3}{7}$). Which is greater: $\frac{5}{15}$ or $\frac{3}{7}$? Alex and Pania may see intuitively that $\frac{3}{7}$ is more than $\frac{1}{3}$, but they can prove that this is true using equivalent fractions with a common denominator: $\frac{1}{3} \times \frac{7}{7} = \frac{7}{21}$ and $\frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$, so Pania has the highest proportion of lemons.

Each pair of students should play the game a number of times. They may need teacher help at first, as they learn strategies. They may also need a pencil and paper handy. They should practise using both strategies described above.

Confident students will be able to use this game for independent skills maintenance while you work with a guided teaching group.

Page 21: Smart Sizes

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- express quantities as fractions or percentages of a whole (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

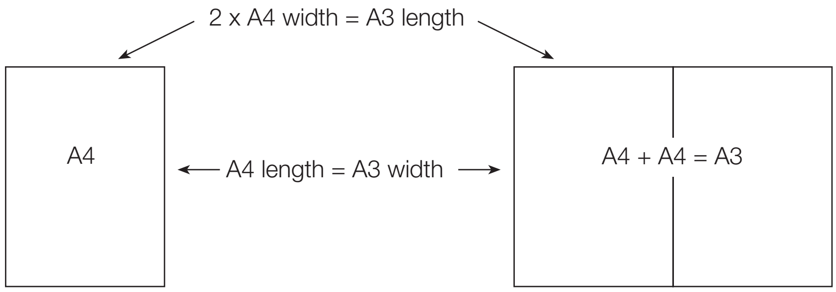
- help students consolidate and apply their knowledge of fractions at stage 8
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

This real-life context offers scope for a rich mathematical investigation of ratios. As long as the focus is on area, students can get a long way by doubling or halving. When the focus is on length, they face a greater challenge but will discover some surprising patterns.

Introduce the activity by explaining that the codes A3 and A4 are part of the ISO (International Organization for Standardization) system, set up to encourage countries to use the same units of measurement.

When doing question **1**, students should work with actual sheets of paper and discover these relationships for themselves:



As a next step, they can make a chart that compares the area of different-sized sheets, using A4 paper as the unit:

Code	A4	A3	A2	A1	A0
Area	1	2	4	8	16

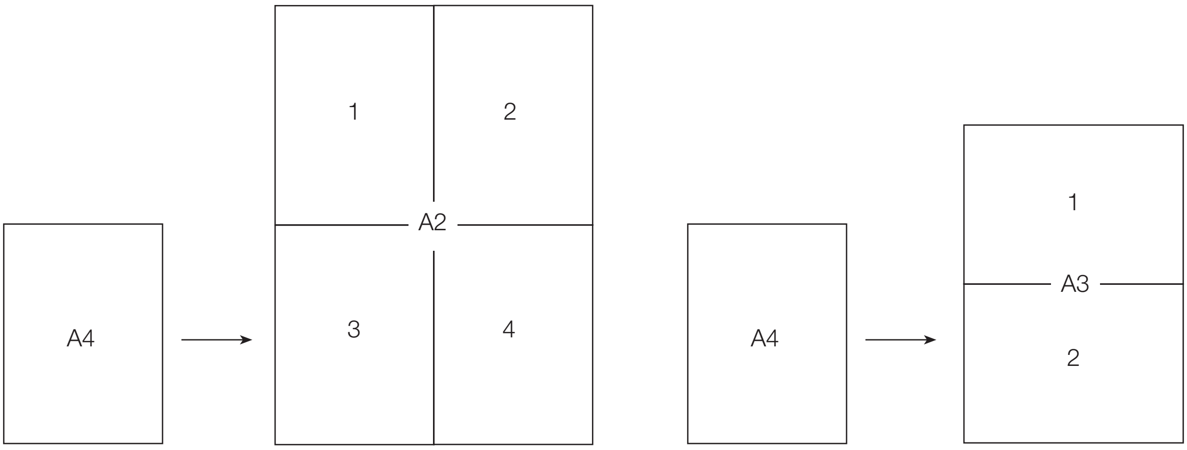
Get your students to continue the pattern for sizes smaller than A4:

Code	A7	A6	A5	A4
Area	0.125	0.25	0.5	1

They should note that as the numerical code for the sheet size gets larger, the area of the sheet gets smaller.

As part of question **1b**, get the students to measure the length and width of each size of paper in millimetres. They can then use a calculator to compare the length with the width and express this as a ratio or as a scale factor (multiplier). In the case of an A4 sheet, length:width = 297:210. $\frac{297}{210}$ = 1.414 (3 d.p.). The scale factor is 1.414, and the ratio is 1.414 : 1. If students check the length:width ratio for different sheet sizes, they will find that it remains the same, regardless of size.

The challenge for students in question **2** is to explain why a copier uses a scale factor or ratio of 141% to enlarge an A4 page to A3 size instead of the 200% that they might have expected, and why it uses a ratio of 71% instead of 50% for an A3 to A4 reduction. The reason is that the ratio used by the copier refers to length, not area:



A 200% enlargement doubles the width and length and increases the area by 4.

A 141% enlargement increases the width and length by 1.41 and doubles the area.

Students could check the above by drawing a rectangle in a computer drawing program and then using the Enlarge function to increase it by a ratio of (i) 200% and (ii) 141%. They could then check the effect of the ratios 50% and 71%.

For questions **3–4**, get the students to make a table of the lengths and widths of paper sizes, looking for connections that will help them to complete the table to A0 size:

Size	A4	A3	A2	A1	A0
Width	210	297	420	594	840
Length	297	420	594	840	1 188

They should see that the length of one becomes the width of the next size up and the width of one doubles to become the length of the next size up. When they check the official dimensions of A1 and A0, they will find that their lengths are out by 1 or 2 mm (due to rounding errors), but this is within the tolerance set down by the ISO standard.

In question **4**, students can use a calculator to work out the area of an A0 sheet. Have them express their answer in square metres (either by changing the lengths from millimetres to metres before calculating the area or by dividing the area in square millimetres by 1 000 000 to convert it to square metres.)

The A0 sheet is the unit on which all other sizes are based. Students can easily check that it has an area of 1 m^2 and find out much more about the ISO system of paper sizes (including series B and C), using the Internet. A search on “ISO paper sizes” will bring up a number of useful sites.

When the students explore folds in question **5**, they should make a chart that shows the number of sheets they get for each number of folds. You could show them how to record this information using exponents. If they do this, they will see that the code for the paper size contains within it the number of folds. (If you fold an A0 sheet 5 times, the resulting sheets will be A5 size.)

Size	A0	A1	A2	A3	A4	A5
Number of sheets	1	2	4	8	16	32
Number as exponent	2^0	2^1	2^2	2^3	2^4	2^5

By reversing and extending the chart they made for questions **3–4**, your students should be able to continue the pattern and answer question **6**:

Size	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Width	840	594	420	297	210	148	105	74	52	37	26
Length	1 188	840	594	420	297	210	148	105	74	52	37

As an extension to this activity, interested students could explore www.iso.org. This is the website of the International Organization for Standardization. Much of its content is expressed in language that is accessible to students.

Pages 22–23: The Right Mix

Achievement Objectives

- write and solve problems which involve whole numbers and decimals and which require a choice of one or more of the four arithmetic operations (Number, level 3)
- solve practical problems which require finding fractions of whole number and decimal amounts (Number, level 3)
- find a given fraction or percentage of a quantity (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- encourage the transition from advanced multiplicative strategies (stage 7) to advanced proportional strategies (stage 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity One

This context may be a little artificial (filling a bathtub using buckets of hot and cold water), but it provides a strong visual model of ratios at work.

Have your students read and discuss the illustrated scenario. Encourage them to link the data in the number story with the equations in the speech bubbles. Good focusing questions could be:

- *What was the data that suggested to Jenna that she should divide by 3?*
- *How did Toni use the same piece of data in her strategy?*

From this discussion, the students will see that the total number of buckets is critical information and that dividing by 3 has the same effect as multiplying by $\frac{1}{3}$.

Use question **1** to get the students to examine the ratios of hot to cold and express these as proportions of the total volume. In question **1a**, for example, 3 buckets of cold to 1 of hot is expressed as the ratio 3:1. This means that the cold is $\frac{3}{4}$ of the total volume and the hot is $\frac{1}{4}$ of the total volume. It is important that students understand both ways of viewing the data.

In question **2a**, ensure that your students think about "the easiest way for me" as well as "the most efficient way" when considering which is their preferred strategy. Toni's strategy is neither easy nor efficient unless the number of buckets is a whole number factor of both the cold and hot temperatures.

Question **2b** is a key question as it seeks to have students see the big idea behind both strategies and express it as a generalisation.

Ask the students to record the full equation for Jenna's strategy: $(12 + 12 + 60) \div 3 = 28$. Do the same for Toni: $(\frac{1}{3} \times 12) + (\frac{1}{3} \times 12) + (\frac{1}{3} \times 60) = 28$. Help the students rewrite this as $\frac{12}{3} + \frac{12}{3} + \frac{60}{3} = 28$.

By comparing the first and last of these three equations, we can make a new equation: $(12 + 12 + 60) \div 3 = \frac{12}{3} + \frac{12}{3} + \frac{60}{3}$. This equation shows that adding the parts and dividing by 3 gives the same result as dividing the parts by 3 and then adding. This is an illustration of the distributive principle at work: the factor 3 is *distributed* over each of the addends inside the brackets. Many mathematical calculations (including all part–whole strategies) and algebraic manipulations make use of this very important principle.

Question **3** gives students further practice using the strategies they explored in question **2**.

The first part of question **4** comes from a new angle, and to answer it, students should reread the conversation in the introduction and see if they can make use of the information there in a different way. The key is to notice that equal amounts of hot and cold water will give a temperature of 36°C. The bath is below this temperature, so there must be more cold than hot. Equal cold and hot would mean 5c + 5h. 6c + 4h gives 31.2°C.

Activity Two

When discussing this activity, highlight the two instructions, "150 litres in it – no more," and "How many more 10 litre buckets ...?" They are essential to the problem. Students will need to have the correct answer for **Activity One**, question **4b** (6 buckets of cold water and 4 of hot) to work from.

The activity deliberately begins with Mary because the bath is already at a temperature that suits her and if that was all that was required, the girls would not need to add any more water. But the bath isn't yet full enough, so the challenge is to bring it up to the desired level while keeping its temperature within the preferred range.

There are at least four approaches that students might follow. You may need to model how to record them:

1. Given that the existing cold:hot ratio is 6:4 or 3:2, $\frac{3}{5}$ must be cold and $\frac{2}{5}$ must be hot. $\frac{3}{5}$ of 15 is 9, so Mary needs 9 buckets of cold water and 6 buckets of hot. This is 3 more buckets of cold water and 2 more of hot.
2. Mary needs to add 5 more buckets to bring the bath up to the desired level. Because the ratio of cold to hot is 3:2, 3 of these extra 5 buckets must be cold and 2 must be hot.
3. Trial and improvement. There are not many ways of splitting 5 into 2 using whole numbers only.
4. Chart all possibilities and their outcomes (temperatures) in a table and then see which suits each of the girls.

Starting with the bath described in **Activity One**, question **4**, it should be clear that Toni will need a little more cold than hot and Jenna will need a little more hot than cold. This will reduce even further the number of possibilities that the students have to consider.

Page 24: Top Shoot

Achievement Objectives

- find fractions equivalent to one given (Number, level 4)
- express quantities as fractions or percentages of a whole (Number, level 4)

AC
EA
AA
AM
AP

Number Framework Links

Use this activity to:

- help students consolidate and apply their knowledge of fractions (stages 7 and 8)
- develop confidence in students who are beginning to use advanced proportional strategies (stage 8).

Activity

Use this high-interest context to highlight the contribution good mathematical thinking can make to everyday situations.

Students who understand how to compare fractions by means of common denominators should be able to attempt this activity independently or in small groups. Others will need to do it in a guided teaching situation.

The key to this activity is found in question **1** and highlighted in the thought bubble. Students should discuss the issue (How can the coach compare ...?) in small problem-solving groups and then report back before going any further.

Students need to understand how the goals:attempts ratio works as a measure of accuracy. Useful questions for discussion could be:

- *Why might the person who scores the most goals not necessarily be the most accurate?*
- *Is it always the fault of the goal shoot if a team does not score many goals?*
- *How can you tell from the goals:attempts ratio which goal shoot is more accurate?*

Make sure that discussion raises and answers the question "Does the number of attempts include the successful shots?" The answer is yes. Students get used to the idea that a ratio shows the parts that make up the whole, but this is clearly not the case in this context.

You could take this opportunity to emphasise that ratios are a kind of mathematical shorthand: they can never be understood without a contextual explanation. Students may realise that a goals:attempts ratio can equally well be written as a goals:misses ratio. 7:12 (goals:attempts) is the same as 7:5 (goals:misses).

Use some simple goals:attempts ratios, for example 10:20 and 5:10, to show how equivalent fractions are a useful strategy for comparing accuracy. Both 10:20 and 5:10 describe situations in which half of the attempts made were successful, because $\frac{10}{20} = \frac{1}{2} = \frac{5}{10}$.

Question **2** presents a challenge because there are four ratios to be ordered by size and, if expressed as fractions, they all have difficult, unrelated denominators. There are at least three ways that the students could approach the problem:

1. Try to find the common denominator, even if it *is* very big. The fractions are $\frac{7}{12}$, $\frac{9}{15}$, $\frac{12}{21}$, and $\frac{9}{13}$, and the denominator is 5 460 (13 x 7 x 5 x 3 x 2 x 2). Students using this method will find that a calculator is indispensable!

2. Avoid comparing the four girls' records simultaneously and focus instead on comparing pairs of girls and finding the best performer in each pair. Here is one line of reasoning:
   - Although Toni makes 6 more attempts than Mere, she scores only 3 more goals. Clearly, Mere has the better result, so Toni is eliminated.
   - Peti scores 9 from 13 attempts, while Mere scores 9 from 15. Peti clearly has the better result, so Mere is eliminated.
   - Peti makes just one more attempt than Rowena but scores 2 more goals. This gives her the better result and eliminates Rowena.

3. Use a variation on the strip diagram theme, as illustrated in the student book. Strip diagrams normally model situations in which there is a small standard unit, and the strips vary in length depending on how many units they represent. In this variation, it is the length of the strip itself that is the unit. The difference is the way in which it is divided up. Diagrams like these can't realistically be created by hand, because the computer program's stretch function is needed to shorten or lengthen one strip to match another. Once this has been done, two or more ratios can be compared at a glance. A diagram of this kind provides an excellent visual model of what is meant by comparing ratios.

All three approaches can be used when answering question **3**, to compare the girls' records at the end of both games. Here is a line of reasoning using pair comparisons:

- If the students combine the results from both halves, they will get the ratios 18:26, 22:33, 20:35, and 15:20. These can be simplified to give the equivalent ratios 9:13, 2:3, 4:7, and 3:4.
- Rowena (18:26) scores the equivalent of 9 goals from 13 attempts while Peti (15:20) scores the equivalent of 9 goals from 12 attempts. This gives Peti the better result and eliminates Rowena.
- Mere (22:33) scores the equivalent of 4 goals from 6 attempts and Toni (20:35) the equivalent of 4 goals from 7 attempts. This gives Mere the better result and eliminates Toni.
- Mere scores the equivalent of 2 goals from 3 (a success rate of $\frac{2}{3}$) while Peti scores the equivalent of 3 goals from 4 (a success rate of $\frac{3}{4}$). $\frac{3}{4}$ is greater than $\frac{2}{3}$ so Peti has the better result and Mere is eliminated.

It may be worth raising with your students the fact that mathematically identical ratios may not be so identical in real-life situations. You could provoke this discussion by asking *"Are the skills of a girl who makes 4 attempts and scores 3 goals in the same league as those of a girl who attempts 40 and scores 30?"* Mathematically, the goals:attempts ratio is identical for each, but in practice, the second girl has proven a level of skill that could in the case of the first be just luck. A player who, in a season, attempts 400 and scores 300 provides even greater proof of skill.

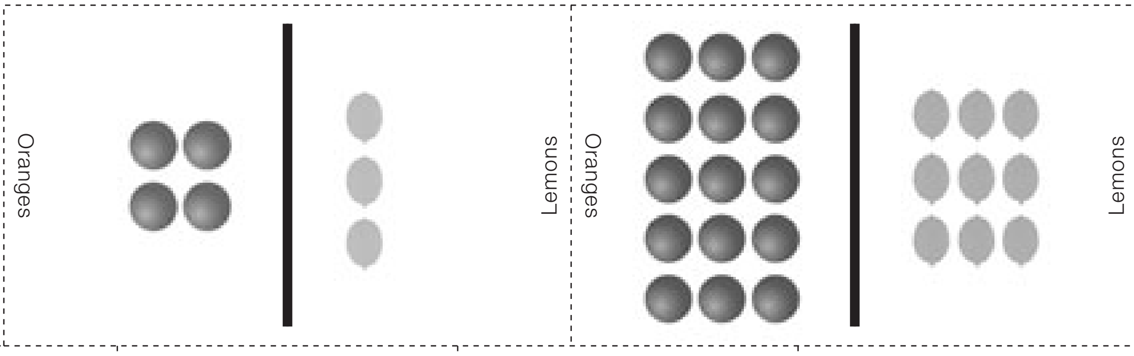
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$
$\frac{7}{8}$	1	

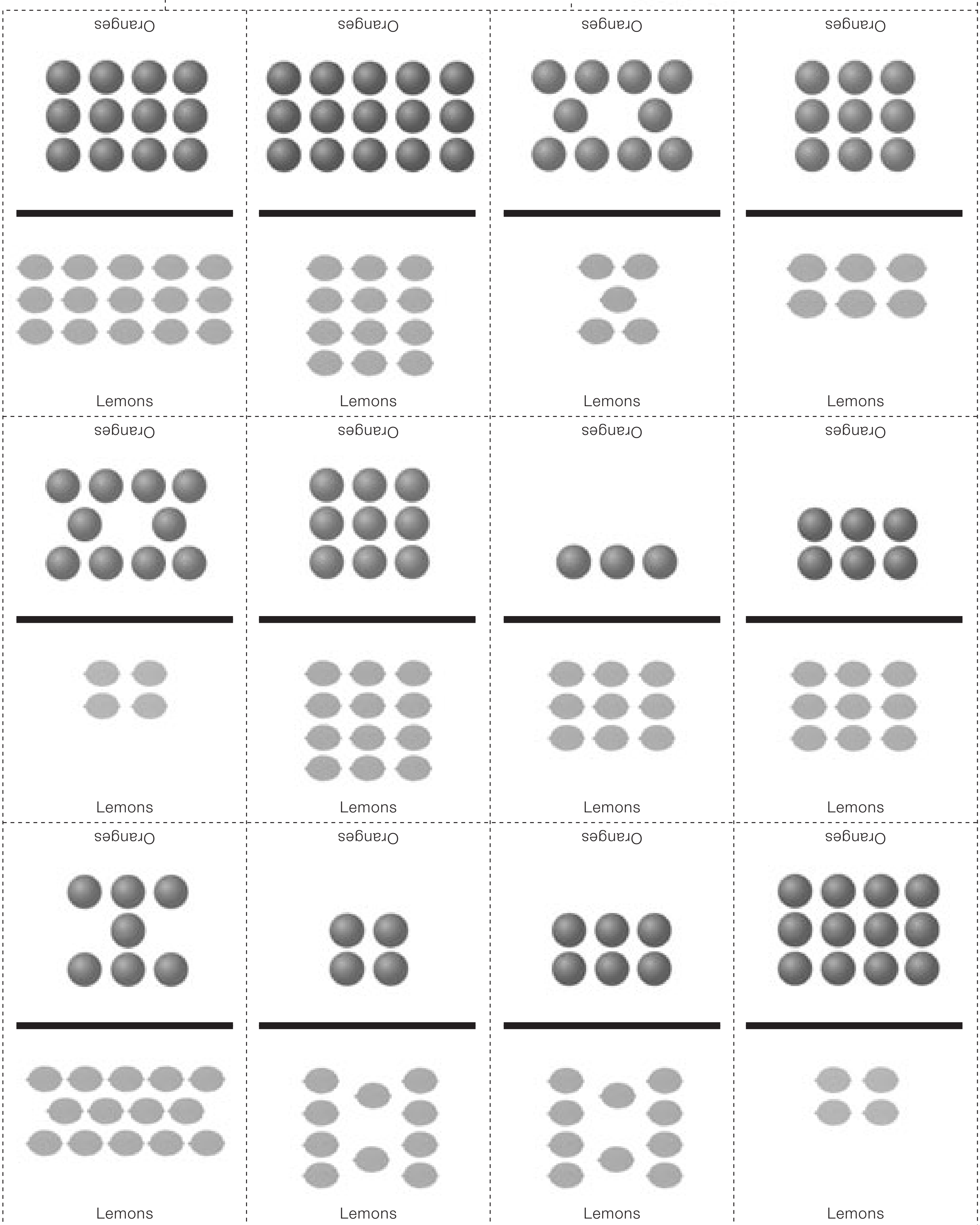
Copymaster: Dot to Dot

$\frac{2}{3}$ of 18	$\frac{1}{2} + \frac{3}{2}$	$\frac{4}{3} + \frac{4}{6}$	$\frac{1}{2} \times \frac{2}{6}$	$\frac{1}{2} \times \frac{4}{8}$	$\frac{1}{8}$ of 104
$\frac{1}{6}$ of $\square = 7$	$\frac{8}{4} + \frac{10}{5}$	$\frac{1}{5}$ of 80	$\frac{1}{2} + \frac{3}{6}$	$\frac{1}{4} \times \frac{1}{2}$	$\frac{1}{3}$ of $\square = 6$
$\frac{1}{7}$ of 91	$\frac{1}{4} + \frac{3}{8}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{2}{3}$ of 42	$\frac{1}{6}$ of 84	$\frac{1}{4}$ of $\square = 8$
$\frac{1}{4} + \frac{1}{2}$	$\frac{2}{9}$ of 72	$\frac{1}{3} + \frac{1}{6}$	$\frac{1}{4} \times \frac{2}{8}$	$\frac{2}{5}$ of 45	$\frac{1}{4}$ of 92

Create your own board

Copymaster:
Fruit Proportions





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