Accelerating Learning in Mathematics

RESOURCE 7: EXPLORING MEASUREMENT

ALiM teachers sometimes focus exclusively on number skills and strategies, believing that until their students have mastered these, there is little point in introducing them to other areas of mathematics. However, helping students to make connections between mathematical ideas (and between strands) helps them to construct a broader, more robust understanding of mathematics.

Exploring measurement is a great place to start. Measurement allows students to quantify and describe the world they live in, and its hands-on nature helps students to make connections between mathematics and real life. Measurement crosses strands and contributes to understanding in all strands: students will encounter fractions and decimals naturally when measuring; area and perimeter can be used to describe geometric figures; measured data forms the basis of many statistical investigations.

This resource suggests ways in which ALiM teachers can capitalise on the connections between mathematical ideas and between measurement and number concepts.

Why is this important?

All mathematical ideas are interconnected, so understanding in one area can depend on and strengthen understanding in another. Effective teaching helps students to make and use these connections.

Measurement and number concepts are intrinsically connected – teachers can draw on students' prior knowledge of number concepts when working on measurement activities, and vice versa. Measurement activities create opportunities for hands-on work and for students to apply their number skills. Emphasising the connections between number and measurement concepts contributes to a broader, more robust understanding of mathematics.

Beliefs underpinning effective teaching of mathematics

- Every student's identity, language, and culture need to be respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

Ten principles of effective teaching of mathematics

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge.

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at www.ibe.unesco.org

CONNECTIONS BETWEEN MEASUREMENT AND NUMBER

There are many ways of demonstrating to students that measurement and number concepts are connected. Some examples are listed below.

- A linear scale is a number line and is divided into groups of equal size – constructing or interpreting a scale involves skip counting.
- Like counting, metric measures are based on groups of 10, providing a real-world model of base-10 numeration. Every time students use a ruler, they see an example of a unit (centimetre) divided into 10 parts. Money (a measure of value) is also based on groups of 10.
- Measurement can help students to recognise that a group of items can be a whole (for example, 100 centimetres is 1 metre), and at the same time a whole is made up of many parts (for example, 1 m represents 100 cm or 1000 mm). This parallels the concept of unitising in number.
- Measuring temperature provides an authentic context for starting to explore negative numbers.
- Learning how to read an analogue clock can reinforce students' understanding of "whole" and "fraction".
- Measurement can help students recognise that a whole is the sum of parts, which is the basis of all part–whole thinking.
- Calculating area and volume provides a reason to use multiplication. Like arrays, calculations of area are based on groups of equal size.
- A measurement is always only an approximation of a quantity (dependent on the accuracy of the measuring tool and the context). For example, when describing a person's height, we measure to the nearest centimetre, when measuring a field, we use the nearest metre. This introduces students to the concepts of rounding and fitness for purpose.
- Measurement tasks often involve estimation (requiring students to calculate using known facts and/or relationships).

Numbers must come from situations that involve counting or situations that involve measurement. Using numbers derived from measurement ups the level of challenge for students at the same time as it develops and reinforces their understanding of number concepts.

- Counting usually involves small quantities, limiting the size of numbers students work with; measurement typically involves much larger numbers.
- Counted data usually involves just whole numbers and simple fractions; measured data is continuous, requiring the use of fractions and decimals.

As they come across large numbers and continuous quantities, students find they must move beyond counting strategies. There is evidence that delaying this transition may not help number understanding.

> *It has been suggested by Sophian (2007) that in many education systems there is an overemphasis on counting. Sophian argues that the focus on discrete quantity (i.e. "how many?") draws attention away from continuous quantity (i.e. "how much?") and could help to explain the difficulties that students worldwide experience with fractional quantity, which requires co-ordination of both "how many?" (e.g. the numerator) and "how much?" (e.g. the denominator). The NDP data analysis shows that students in the early years of school are using counting strategies for longer than is desirable, according to the expectations conveyed in the curriculum and the national standards (Ministry of Education, 2007, 2009) … The strong emphasis on counting in the NDP may send students the wrong message, resulting in some students being reluctant to move away from counting at any stage because of its proven reliability for solving problems with small numbers.*
>
> Young-Loveridge, 2010, page 30

Counting is done by observation – it requires no device (though devices can be used to speed things up for big operations like counting banknotes in banks or votes in elections). Whatever is being counted ($10 notes, people, cars, seats, books, and so on) is automatically the unit.

Measurement always involves a suitable measuring device and unit. The device (and unit) could be, for example: a ruler or tape (centimetre, millimetre, square metre, and so on), thermometer (degree Celsius), clock (hour, minute, second), scales (kilogram, gram, milligram), measuring jug (millilitre), barometer (kilopascal), protractor (degree), or multimeter (amp, milliamp, and so on). Informal devices and units are also sometimes used for measurement, for example: bucket (bucketful), hand (handful), stride (pace).

RECOGNISING SIMILAR BEHAVIOURS

Students need to understand that counting numbers and numbers obtained by measurement behave in the same way (for example: adding two identical lengths gives the same result as doubling one of the lengths; finding the difference in length is the same as subtracting one length from another). This means that students can continue to develop their number skills and knowledge in contexts that involve measurement, and vice versa.

Once students understand that there is nothing you can do with counted numbers that you can't also do with numbers obtained by measurement, they can tackle measurement problems with growing confidence. This understanding is particularly useful when solving word problems.

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Many students find word problems challenging: they need to be able to read and comprehend the text, identify what they are being asked to do, and then convert the challenge into numbers, symbols, and processes. Students who struggle to get over the language hurdle sometimes resort to randomly adding or multiplying numbers without understanding what they need to do or which operation is appropriate.

Numeracy Development Project *Book 5* identifies four types of additive problems: join, separate, combine, and compare. Helping students to identify these four patterns can help them to determine in a given situation:

- what they are being asked to find
- how to organise the information they are given, and which visual models may be useful
- which operation(s) they should use to solve the problem.

Once the students understand the structure of a problem, it doesn't matter whether the context is counting or measuring. The quantities and contexts change, but the pattern is the same.

COMPARE PROBLEMS IN NUMBER AND MEASUREMENT

The following examples show how understanding the *compare* pattern can help students to solve number and measurement problems.

Compare problems involve comparisons (differences). In number, the comparison is between two disjoint sets (groups of objects), in measurement it is between two quantities (for example, lengths).

Comparing sets: Number

There are 18 baboons at the zoo and 25 zookeepers.
How many more zookeepers are there than baboons?

How can students identify that this is a compare problem?

There are two separate groups (baboons and zookeepers), the problem doesn't involve action (the groups are not being joined or separated), and the problem involves "how many *more*" (a comparison).

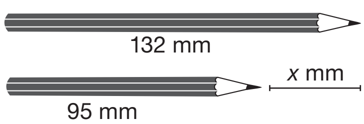
Compare problems can be modelled using two bars:

The model can help students to recognise that there are two ways to find the solution:

- 25 – 18 = ?
- 18 + ? = 25

Comparing quantities: Measurement

Measurements can be (and often are) used to make comparisons. For example, consider the difference in length between these two pencils.

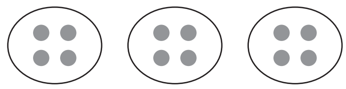


As can be seen from the visual models, this problem is identical in structure to the baboon and zookeeper problem. The difference can be found in two ways:

- measuring each pencil and subtracting one length from the other
- lining the pencils up at one end and working out how much is needed to make the two lengths equivalent.

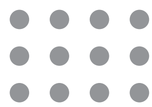
AREA AND MULTIPLICATION

Students are first introduced to multiplication through problems involving groups of equal size:



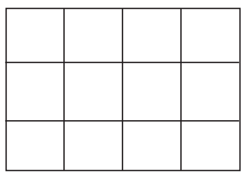
3 x 4 = 12

They then move on to arrays:



3 x 4 = 12

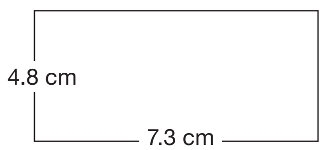
To understand multiplication, students need to recognise that the array represents three groups of four (and also four groups of three). (Students who struggle with mathematics are often unaware of patterns that are obvious to others.) Once the relationship between arrays and multiplication is established, students can apply their understanding to area, for example, by finding the number of squares in a rectangle:



3 x 4 = 12

Once students understand area as length x width, they can generalise this and move beyond small whole numbers to large whole numbers and decimal fractions:

- If the area of a 3 cm x 4 cm rectangle is (3 x 4) cm^2, then the area of a 3000 cm x 4000 cm rectangle is (3000 x 4000) cm^2.
- We can't show 4.8 groups of 7.3 items in array; the counting model has reached its limitations. We can, however, create a rectangle with side 4.8 cm x 7.3 cm and find its area.



Area is 4.8 x 7.3 = 35.0 cm^2

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MEASUREMENT DATA: NEW LEARNING

While measured data functions in exactly the same ways as non-measurement number data, it nevertheless requires students to develop new skills and knowledge. These include:

- using different measuring devices
- reading scales of all kinds
- using units of all kinds
- using the metric system
- estimation as a safeguard against error
- fit-for-purpose rounding.

Each of the above is important and useful in its own right.

WHERE TO NEXT?

Students can practise constructing models of word problems using virtual thinking blocks. See www.thinkingblocks.com for a useful tool and many examples for practice.

Read more about the underlying structure of word problems in *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools (NCEE 2009-4060)*, pages 26–29. This resource can be downloaded from http://ies.ed.gov/ncee/wwc/practiceguides/

See Resource 12 for suggestions of problems that you can explore with ALiM students.

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