Accelerating Learning in Mathematics

RESOURCE 10: SHIFTING STUDENTS FROM NZC LEVEL 2 TO LEVEL 3 BY DEVELOPING MULTIPLICATIVE STRATEGIES

Multiplicative thinking is characterised by the ability to work flexibly with concepts, strategies, and representations of multiplication and division. Students in years 5–8 who are still at the additive thinking stage urgently need to make the shift to multiplicative thinking, but this may not happen without targeted support. This resource suggests ways to support students to make this shift.

Why is this important?

Many ALiM teachers working with year 5–8 students focus on additive strategies and Stage 5: Early Additive knowledge. However, filling in gaps in a student's understanding of additive strategies is insufficient in terms of preparing them for the demands of the curriculum at levels 3 and 4.

The extent to which students understand multiplication and division will impact on their level of success across all strands. Multiplicative thinking is the gateway to understanding and working with place value and fractions. Students use multiplicative thinking when they calculate area and volume, find the mean of a set of data, work out a useful scale for a graph, and compare rates. Helping students in years 5–8 to make the transition to multiplicative thinking must be viewed as a priority.

Some teachers erroneously believe that students need to have fully mastered addition and subtraction before they are introduced to multiplication and division. Helping students to develop their understanding of multiplication and division can provide opportunities for students to practise their adding and subtracting skills.

Multiplicative thinking is needed for students to develop a robust understanding of place value, and an understanding of place value and partitioning is needed for multiplicative strategies. This reciprocal relationship means that the students' understanding of multiplication and of place value can be developed simultaneously.

Beliefs underpinning effective teaching of mathematics

- Every student's identity, language, and culture need to be respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

Ten principles of effective teaching of mathematics

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge.

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at www.ibe.unesco.org

EMPHASISE THE USEFULNESS OF MULTIPLICATION

Students often learn their multiplication facts without gaining any real understanding of their meaning or usefulness. So, as long as they encounter mainly problems that can be easily solved using skip counting or repeated addition, they fall back on the methods that have served them adequately in the past.

To break this self-reinforcing habit, students need to be challenged with problems that demand the use of multiplicative methods for reasons either of (i) efficiency or (ii) necessity.

A good place to start is helping students to understand the efficiency of multiplication as an alternative to counting or repeated addition. As part of this process, students need to understand that they can use multiplication only when they are working with groups of equal size.

Show the students three cups, each containing a different number of counters. Discuss why you need to use counting or adding rather than multiplication to find the total.

Show the student three cups, each containing the same number of counters. Discuss that counting, adding, or multiplying could be used to find the total. Have the student confirm that all three methods give the same result.

Next, discuss how you could solve a similar problem with cups containing the same number of counters but that this time there are 100 cups. Can you still use all three methods? Which method here is the most efficient?

In many situations, multiplicative strategies are not only more efficient (quicker and/or easier) than addition, they may be necessary to solve a problem. This is true when the numbers involved are large and when they are not whole numbers.

Demonstrate the need for multiplicative strategies through the use of appropriate examples. For example, skip counting can be used to find 16 x 3 or 5 x 25, but hardly to find 116 x 3 or 40 x 26. The latter requires the use of a multiplicative strategy such as doubling and halving (40 x 26 = 20 x 52 = 10 x 104 = 1040) or partitioning (40 x 20 = 800 and 40 x 6 = 240 so 40 x 26 = 800 + 240 = 1040), or a combination of the two. Many solution strategies will involve both multiplication and addition.

Asked to find the area of a rectangular room where the units are whole numbers (for example, 12 metres x 9 metres), students may count squares or even skip count. But to find the area of a room 12.5 metres x 8.9 metres (NZC level 4), they will need to use a multiplicative strategy.

MAKE LINKS WITH PLACE VALUE

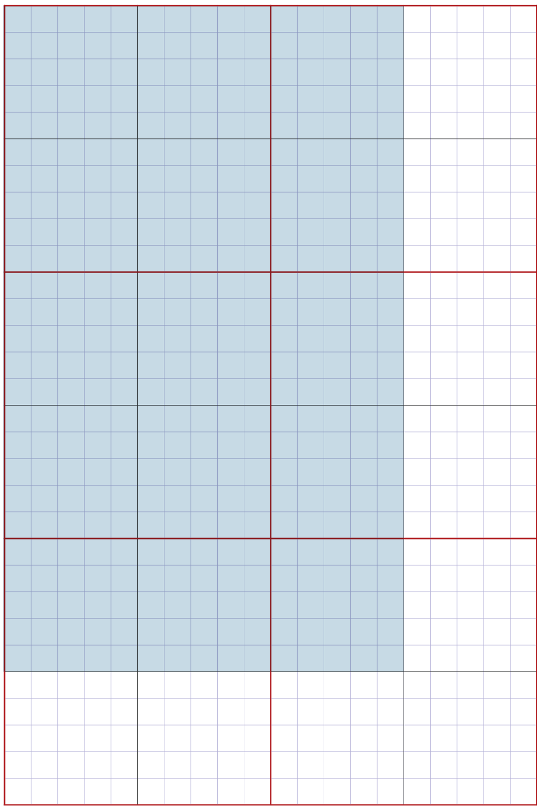
Place value and multiplicative thinking are intricately linked. Central to both is the idea that a group of items can be a single entity (for example, one "hundred") and that at the same time the group contains a number of objects (for example, one hundred represents ten 10s or 100 ones). Like multiplication, place value is built on the principle of groups of equal size. For example, 340 represents three groups of 100, and 4 groups of 10.

Place-value equipment can be used to strengthen this understanding of equal groups. For example, if a student needs to solve 3 x 26, they could use place-value blocks to make three groups of "two tens and six ones" then trade 10 ones to make an additional ten (3 x 26 makes 6 tens and 18 ones, which is 7 tens and 8 ones = 78).

Using arrays to solve two-digit multiplication problems provides a visual model of 10 tens making one hundred. The following example is based on the grid in "*Fostering Multiplicative Thinking Using Array-based Materials*" (Young-Loveridge, 2005).

The grid is structured in blocks of 10 by 10 (red lines), each divided into 5 by 5 sub-blocks (grey lines).

Multi-digit multiplication problems up to can be shown on the grid. For example, here is 25 x 15:



The base-ten structure of the grid makes it easy to see the two blocks of 100, the three blocks of 50 (5 x 10) that together make another 150, and a block of 5 x 5 that completes the array. Summing the partial products (200 + 150 + 25) gives a total of 375, the product of 25 by 15.

Nzmaths provides a number of activities that make links between place value and multiplication:

http://nzmaths.co.nz/resource/multiplying-tens

http://nzmaths.co.nz/resource/bean-counters

See also:

http://illuminations.nctm.org/LessonDetail.aspx?id=L858

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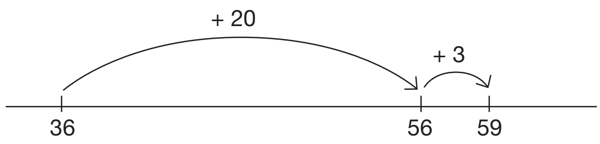
MODELS OF MULTIPLICATION

Low achievers in mathematics often struggle to identify structures and patterns. Making these structures explicit can help shift their conceptual understanding. Diagrams are a useful way for students to analyse the structure of problems. However, check that the students understand the relationship between the diagram and the problem.

Additive and multiplicative problems are structurally very different, which is reflected in the tools that can be used to solve them.

Additive problems are one-dimensional, which makes a number line a useful tool. Students can move forwards and backwards along a sequential line of numbers to find an answer. For example:

> *Kayla has 36 stickers. Her sister gives her another 23 stickers. How many stickers does she have now?*

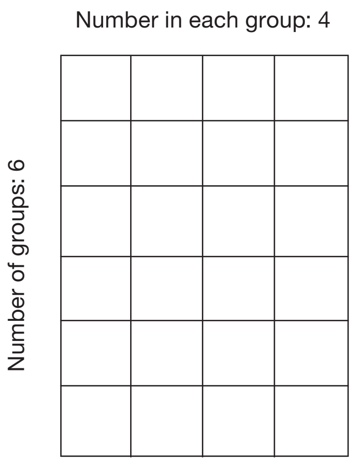


Simple multiplicative problems can be represented on a number line if the student uses skip-counting. This is, however, essentially a counting strategy. Some students skip-count without understanding the concept of groups of equal size.

Multiplicative problems always have at least two dimensions: the number in each group and the number of groups. It has been suggested that "the most flexible and robust interpretation of multiplication is based on a rectangle" (Davis, 2008, page 88). This makes an array a perfect model with which to explore multiplication.

Work with students to identify these two dimensions across a variety of problems. Once students can identify the two dimensions, have them display them in an array. For example:

> *There are 6 tables in the room. Each table has 4 chairs around it. How many chairs are there altogether?*



The NDP *Book 6: Teaching Multiplication and Division*, page 5, provides examples of different types of multiplicative problems. Consider how models could be used to represent each problem in the table.

Students can practise constructing different models at www.thinkingblocks.com/tb_addition/addition.html

Work through the problems and models with the student.

DEVELOP LANGUAGE

Brickwedde (2011) found that language played an important role in helping primary-school students make the shift to multiplicative thinking. Students who could express multiplicative ideas in speech and who could correct or identify mistakes in other students' explanations achieved a more sophisticated level of multiplicative understanding.

Support students to know and recognize language related to multiplicative problems, for example: equal groups, equal shares, doubling, halving, trebling, thirding, factors.

OPEN UP PROBLEMS

Using open problems can help students to recognise that a number can often be partitioned in different ways. For example, 24 is 24 ones, 2 twelves, 3 eights, 4 sixes, 6 fours, 8 threes, etc. The table below provides examples of open questions that involve multiplicative thinking.

Multiplication	**Division**
Equal grouping: *There are 6 tables in the classroom. Each table has the same number of chairs. How many chairs might there be altogether?* A student could work on this problem using a drawing or using counters. Help the student to recognise that because the problem involves equal groups, they could also display it in an array. Discuss with the student what a reasonable upper limit might be.	Equal grouping: *I have 24 chairs, and I want to arrange them in groups of equal size. How many chairs could I put in each group?* Give the student 24 counters. A student might begin to solve this problem using trial and error. Work with them to show them that if the counters can be arranged in an array, then they can use the array to see the number of groups and how many chairs would be in each group. Rotate the page to show the student that if there can be 8 groups of 3 chairs, there can also be 3 groups of 8 chairs.

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Rate: *Misha's mum went to the shop to buy some cards. Each card cost $4. How many cards might she have bought if she spent less than $20?*	Rate: *I spent $24 on cards. If each card cost the same amount, how many cards might I have bought?*
Comparison: *Shona has 4 times as many toy cars as John. John has fewer than 10 cars. How many cars might Shona have?*	Comparison: *I have 4 times as many toy cars as John. If I have fewer than 20 cars, how many cars might John have?*

USE DIGITAL LEARNING OBJECTS TO CONSOLIDATE LEARNING

Digital representations of the array model can be used to support work students have done with concrete materials. The following digital learning objects are available through Digistore:

- L616: The Multiplier
- L106: The Array
- L2008: The Divider
- L2056: Pobble Arrays
- L384: Finding the area of a rectangle

See also:

http://nlvm.usu.edu/en/nav/frames_asid_192_g_2_t_1.html?from=grade_g_2.html

REFERENCES AND FURTHER READING

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