Accelerating Learning in Mathematics

RESOURCE 12: CHALLENGING STUDENTS WITH PROBLEMS

The core of mathematics is problem solving: "Without a problem, there is no mathematics" (Holton et al., 1999).

A mathematical problem is any situation that must be resolved using mathematical tools but for which there is no immediately obvious strategy. If the way forward is obvious, it's not a problem – it's a straightforward application.

All mathematical programmes, including those for ALiM students, need to include complex, rich, and/or problematic tasks. The task for the teacher is to set the difficulty level high enough to challenge students, but not so high that they can't succeed. Teachers who get this right create resilient problem solvers who know that with perseverance they can succeed.

Whaia te iti kahuranga ki te tuahu koe me he maunga teitei.
Aim for the highest cloud so that if you miss it, you will hit a lofty mountain.

Why is this important?

Students only learn to handle complex tasks by being exposed to them.

> *Providing students with the opportunity to work on complex tasks – as opposed to a series of simple tasks devolved from a complex task – was crucial for stimulating [the students'] mathematical reasoning and building durable mathematical knowledge.*
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> Francisco and Maher, 2005, quoted in Anthony and Walshaw, 2007, page 118
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> *… if the teacher feels that students in a low-achieving group cannot solve multi-step problems and so does not pose them, the students will not learn how to solve them.*
>
> Sullivan, 2011, page 42

To become powerful learners of mathematics, students need opportunities to:

> *… sort, classify, structure, abstract, generalise, specialise, represent and interpret symbolically and graphically, justify and prove, encode and decode, formulate, communicate, compare, relate, recognise familiar structures, apply and evaluate applications, and automatise.*
>
> Anthony and Walshaw, 2007, page 120

Beliefs underpinning effective teaching of mathematics

- Every student's identity, language, and culture need to be respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

Ten principles of effective teaching of mathematics

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge.

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at www.ibe.unesco.org

It is the problem-solving potential of mathematics that makes it useful and interesting. If students are not exposed to problematic tasks, they are unlikely to see the point of mathematics, show interest in mathematics, or gain satisfaction from mathematics.

Tolerance of difficulty is essential in a problem-solving disposition because being stuck is an inevitable stage in resolving just about any problem. Getting unstuck typically takes time and involves trying a variety of approaches. Students need to learn this experientially.

PROBLEMS THAT ARE "PROBLEMATIC"

Students need to experience a wide range of problems so that they can develop skills and strategies that have real-world relevance. This principle is embedded in the stem to the achievement objectives in the mathematics curriculum.

> *In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to …*

A good problem is "problematic" – it is centred around a genuine problem. Unlike "problems" that can be solved by applying a simple procedure, problematic tasks challenge students' thinking and involve them in testing, proving, explaining, reflecting, and interpreting. Such tasks:

- are accessible and extendable
- allow individuals to make decisions
- promote discussion and communication
- encourage originality and invention
- encourage "what if?" and "what if not?" questions
- contain an element of surprise.

Adapted from Ahmed, 1987

A problematic task offers serious potential for discussion and learning. When students explain their ideas, respond to the ideas of others, and allow their thinking to be challenged, they are "doing maths". For those who think maths is all about "getting it right" and who need the positive reinforcement of long lines of ticks next to "answers" in their books, this can be difficult to accept. But students need to be weaned from this counterproductive orientation.

> *… studies suggest that a classroom orientation that consistently defines task outcomes in terms of the answers rather than the thinking processes entailed in reaching the answers negatively affects the thinking processes and mathematical identities of learners.*

Anthony and Walshaw, 2007, page 122

Where teachers value process over answers, the emphasis shifts from "Did you get it right?" to "Do you understand?" and "What did we learn?" Teachers need to make this emphasis explicit.

Getting real

Word problems are usually attempts to put mathematics into contexts, but putting problems into words doesn't automatically make them real. This can be an issue for students, who get confused by the fact that, simultaneously, they have to pay attention to things that they would normally ignore and ignore things that they would normally assume mattered.

Think about ways in which this word problem is unrealistic:

> *A caterer at a wedding supplies 5 pavlovas. At the end of the wedding, she sees three quarters of all the pavlovas were eaten. How many pavlovas were eaten?*

Most obviously, why would a caterer ever bother with such a calculation? They could hardly assume that, next time around, they should only provide 3 ¾ pavlovas!

The challenge for teachers is to provide students with problems that draw on their experience of reality, rather than asking them to suspend it.

> *Such authentic contexts provide sense-making and experientially real situations for children, rather than simply serve as cover stories for proceduralised and frequently irrelevant tasks.*

English, 2004, page 3

Mathematical problems do not, however, need to be set in real-life contexts. Imaginative contexts can provide engaging and engrossing opportunities for mathematical explorations. And playfulness and humour can increase student engagement and encourage creative thinking.

> *Watson (2004) argues that 'realistic' does not mean that tasks must necessarily involve real contexts … she advocates that tasks should be seen as 'realistic' not because they relate to any particular everyday context, but because they make students think in 'real' ways.*

Anthony and Walshaw, 2007, page 114

For an example of a rich mathematical problem set in a mythical context, see the Icarus and Daedalus problem on the Math Pickle website at http://www.youtube.com/watch?v=R4oINmqHXVY&feature=player_embedded#!

Unpacking word problems

ALiM students are likely to need deliberate support in interpreting word problems. But it is important not to begin by unpacking the problem for them, assuming that they are incapable of getting meaning from the words. Instead of supporting them to learn, this will encourage dependency. Rather, expect the students to do their utmost (working individually or in twos or threes) to make sense of the problem, and then get them to explain to you what they believe it is about. You can then support them to complete the interpretation process.

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Don't be too tidy

The way word problems are constructed can limit student thinking. Real-life problems rarely have a single, straightforward solution, so students need exposure to problems that allow for different solutions. Such problems are important in shifting the focus from the answer to the mathematical thinking involved.

It is also important that students (including ALiM students) are not restricted to problems that have tidy, whole number solutions. If they are, they will be ill-equipped to deal with more realistic problems, which typically involve messier numbers, both as inputs (for example, measurement data) and as solutions. Real-life problems will provide students with the impetus and need to move beyond the set of counting numbers.

WHERE TO LOOK

The following is a small selection of sites and books worth exploring for the ideas, inspiration, and rich problems they offer.

nRich

The nRich website, created as part of the Millenium Mathematics Project (University of Canterbury), provides a wide range of investigations and ways to differentiate tasks. The site uses the term "low threshold, high ceiling" (LTHC) to describe tasks that accommodate a wide range of learners. A LTHC task has relatively easy entry points so that all students can begin it, but has scope for exploration and challenge for students are all levels. See http://nrich.maths.org/7701 for further information and suggested tasks.

The Millenium Mathematics Project has also created a series of sports-related problems. See http://sport.maths.org/content/

Math Pickle

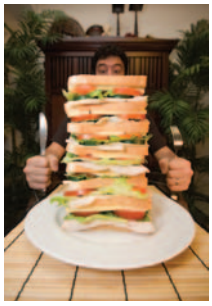
The Math Pickle website (http://mathpickle.com/Videos.html), created by Dr Gordon Hamilton, is an excellent source of creative and challenging problems. The website includes videos, computer-assisted presentations, and downloadable files to support the activities. The problems are arranged by level, beginning with problems for kindergarten students, so there is plenty of scope for easing students into investigative problem solving.

Gordon Hamilton delights in giving students unsolvable tasks ("$1,000,000 problems"). One of his goals is to remove the stigma associated with failure and to "entice teachers to push their students over the cliff-edge of knowledge".

Just wondering

An important shift takes place in students' degree of participation when they become problem posers as well as problem solvers. Annie Fetter, in her talk "Ever Wonder What They'd Notice (if only someone would ask)" (http://www.youtube.com/watch?v=WFvYZDR4OeY), suggests that teachers give students "maths stories" (scenarios) that don't have a defined problem and then ask "What do you notice" and "What do you wonder?"

A simple way to apply this is to give students an interesting image, for example:



Student responses can be used as the basis for investigations.

Dan Meyer

Dan Meyer's short videos (mostly less than one minute) offer a creative catalyst for numerous mathematical investigations. The videos often use fun contexts such as "the world's largest gummi-bear". See http://vimeo.com/ddmeyer/videos

Meyer seldom prescribes an investigative question; sometimes he doesn't even narrate his videos – he simply presents a visual scenario designed to awaken curiosity. In Popcorn Picker (http://vimeo.com/42501010), for example, we see him create two paper cylinders, each made from an A4 sheet of paper, which he fills with popcorn. We find ourselves wondering, "Do the two cylinders hold the same amount?" Students can explore this problem by creating their own cylinders and using popcorn as a measure.

Picture books

Children move with ease between the real and the imagined. One ALiM teacher reported that students in her group had "aha" moments when a concept was introduced using a story.

NZ Maths has examples of picture books, organised by strand and level (see http://nzmaths.co.nz/picture-books-mathematical-content). Two examples are:

Half a Slice of Bread and Butter by Peter Durkin
(ISBN 978-0-3830-3694-0)

In real life, children have many experiences of sharing. In this book, two children learn about the relative size of a half as they share food with more and more children. This book is useful for helping students to understand that as the number of shares increases, the size of the share decreases.

ALiM Resource 12
ISBN 978 0 7903 4205 4

Math Curse by Jon Scieszka (ISBN 0-140-56381-4)

A maths teacher creates a problem for her class when she tells them maths is everywhere and that almost anything can be seen as a maths problem. The main character spends a challenging and humorous week solving real-life maths problems until she finds a way to escape the maths curse.

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