Accelerating Learning in Mathematics

DRAFT

Learning to: use place value and part–whole thinking to solve problems

Target group: students in years 4–8

Focusing on:

- using part–whole strategies to solve simple +/– problems involving one- and two-digit numbers
- recalling basic facts relating to addition to 10, doubles, and halves
- making estimates before solving.

Beliefs underpinning effective teaching of mathematics:

- Every student's identity, language, and culture is respected and valued.
- Every student has the right to access effective mathematics education.
- Every student can become a successful learner of mathematics.

Ten principles of effective teaching of mathematics:

1. An ethic of care
2. Arranging for learning
3. Building on students' thinking
4. Worthwhile mathematical tasks
5. Making connections
6. Assessment for learning
7. Mathematical communication
8. Mathematical language
9. Tools and representations
10. Teacher knowledge

See *Effective Pedagogy in Mathematics* by G. Anthony and M. Walshaw, Educational Practices Series 19, International Bureau of Education, available at: www.ibe.unesco.org

TEACHER OBSERVATION OVER A RANGE OF ACTIVITIES

The student has been assessed as being at advanced counting. They rely on counting strategies to solve problems and may give up when the size or nature of the numbers they are working with makes these strategies impractical. Students who have been advanced counters for several years can often use these strategies with speed.

POSSIBLE BARRIERS TO THE STUDENT'S PROGRESS

1	Limited understanding of place value
2	Limited knowledge of basic facts
3	Poor estimating skills

EXPECTATIONS FOR NUMBER

AFTER 1 YEAR AT SCHOOL	AFTER 2 YEARS AT SCHOOL	AFTER 3 YEARS AT SCHOOL	BY THE END OF YEAR 4	BY THE END OF YEAR 5	BY THE END OF YEAR 6	BY THE END OF YEAR 7	BY THE END OF YEAR 8
COUNTING FROM ONE	ADVANCED COUNTING	EARLY PART–WHOLE THINKING	EARLY ADDITIVE	EARLY ADVANCED ADDITIVE	ADVANCED ADDITIVE – EARLY MULTIPLICATIVE	EARLY ADVANCED MULTIPLICATIVE	ADVANCED MULTIPLICATIVE – EARLY PROPORTIONAL
NZC EARLY LEVEL 1 / NUMERACY STAGE 2 OR 3	NZC LEVEL 1 / NUMERACY STAGE 4	NZC EARLY LEVEL 2 / NUMERACY EARLY STAGE 5	NZC LEVEL 2 / NUMERACY STAGE 5	NZC EARLY LEVEL 3 / NUMERACY EARLY STAGE 6	NZC LEVEL 3 / NUMERACY STAGE 6	NZC EARLY LEVEL 4 / NUMERACY EARLY STAGE 7	NZC LEVEL 4 / NUMERACY STAGE 7

Aligned to *The New Zealand Curriculum* and *Mathematics Standards for years 1–8*

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BARRIER BEING ADDRESSED **1** LIMITED UNDERSTANDING OF PLACE VALUE

DIAGNOSTIC QUESTIONS

As you ask each question, write out the corresponding equation. After the student has recorded their answer, ask them how they worked it out.

1. Jason has 7 trading cards. He gets a new pack with 10 more.
   How many cards does he have now?
   Answer: 17
2. Anna has 43 trading cards. She gets a new pack with 10 more.
   How many cards does she have now?
   Answer: 53
3. Leon has 30 trading cards and gets 27 new ones.
   How many cards does he have now?
   Answer: 57
4. Leon has 57 cards and wants to have 100.
   How many more cards does he need?
   Answer: 43

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student solve these questions using:

- counting on?
- groups of ten?
- place value partitioning and basic facts?

For example, a student using place value may add 30 + 10 + 10 + 7, or 30 + 20 + 7 (recognising 27 as an element that can be split and reassembled).

Does the student use a strategy beyond counting-on to solve 57 + ☐ = 100?

For example, "3 more makes a new ten, and 4 more tens make 100".

DELIBERATE ACTS OF TEACHING

Understanding that a group of 10 is a unit is fundamental to understanding place value.

Focus on the early additive concept of making tens, and the concept of working with tens and ones separately, when solving problems. Revisit the AC-EA learning experiences *(Book 5: Teaching Addition, Subtraction, and Place Value),* giving the student guided practice and an opportunity to work with equipment.

Use a place value flip chart to show how tens and ones can be added separately.

Make a Ten
Use the Make a Ten lesson outline on www.nzmaths.co.nz (lessons, Digital Learning Object links, and other activities).

Construct models using tens frames and track movements on a number line.

Allow the student to construct their own materials to use when they solve simple problems. For example, the student could place groups of 10 bears in transparent bags. Ask the student how many bears they have if they have 2 bags and 4 individual bears.

MATERIALS/LINKS

Place value flip chart

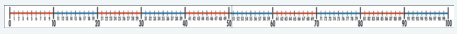
Tens frames

Number lines

Make a Ten
(Number and Algebra, level 2, www.nzmaths.co.nz)

Plastic animals or counters

Transparent bags



WHAT TO DO NEXT IF THE STUDENT IS STUCK

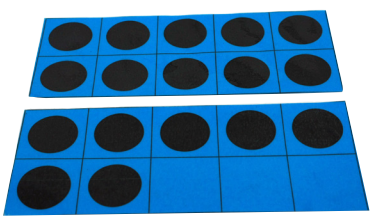
Revisit working within ten until the student can visualise both a full tens frame and one with empty spaces. Gradually progress to "ten and ☐" to make a teen number.

INITIATING HOME-BASED ACTIVITIES

Use the games listed on the Number Facts Activities for Advanced Counting Children page on www.nzmaths.co.nz. The student can explain their reasoning to their parents and suggest other ways to work out answers. Adjust the size of the numbers to suit the student.

NEXT TEACHING STEPS BACK IN THE CLASSROOM

The student should work in a small group to solve problems involving two-digit numbers. Encourage the student to look for opportunities to make a ten. Praise attempts at part–whole thinking.



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BARRIER BEING ADDRESSED **2** LIMITED KNOWLEDGE OF BASIC FACTS

DIAGNOSTIC QUESTIONS

Assess the student's recall of addition and subtraction of basic facts. Each question should be both read aloud and given to the student in writing. If necessary, put questions into a story context and observe the methods the student uses to solve them.

Doubles/halves to 10: For example, 3 + 3 and half of 8 …
Doubles/halves to 20: For example, 6 + 6 and half of 14 …
Doubles/halves to 100: For example, 30 + 30 and half of 40 …

If the student can recall the doubles, try near doubles, for example, 8 + 7, 29 + 30, …

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Can the student recall basic facts with ease?

Does the student use strategies such as counting, nodding, or using their fingers?

DELIBERATE ACTS OF TEACHING

Knowing the doubles to 20 and the double-digit doubles to 100 is useful for the development of part–whole thinking and can be used as a building block for the first set of multiplication facts (the 2 times tables). The inverse of these facts, the halves, can be taught at the same time.

Teaching a basic facts set begins with teaching the student strategies and then providing opportunities for practice until the fact becomes known. Use a concrete model to explore what each set means. Make links between known and new facts.

To teach doubles, create two egg-carton "tens frames" (two rows of five cups).

Doubles to 10: Start with one egg-carton frame. Working with single sticks, demonstrate the doubles by placing the sticks side by side down the frame, emphasising the symmetry. Continue until the student has fast and fluent recall. Give a set of flashcards to the student for additional practice.

Doubles to 20: Place the second egg-carton frame in front of the student, slightly below the first. Fill the first frame one fact at a time. When you get to 6 + 6, emphasise that you are moving to a new frame. Support the student to "see" 10 + 2 ("5 + 5 makes 10 plus 2 more"). Repeat with each new double. Discourage "counting all" or "counting on in ones". For example, for 7 + 7 = 14, explain that you can see the 10 and the 4 without counting. Make a second set of flashcards using paper of a different colour.

Easy doubles to 100: Reuse the egg-carton frames but replace the single stick with bundles of ten sticks. Ensure that the student can count in tens to 200 and can record and read multiples of 10 to 200. When building this set of basic facts, move back to the early doubles or known facts. When you reach 60 + 60, remind the student that a full 50 + 50 frame makes 100, so the 60 + 60 is 100 + 20. Make a third set of flashcards using a different colour.

- 1 + 1 = 2, 1 ten + 1 ten = 2 tens, 10 + 10 = 20
- 6 + 6 = 12, 6 tens + 6 tens = 12 tens, 60 + 60 = 120

Seeing Nearly Double
The above sequence can be used to also teach the "near doubles", for example,

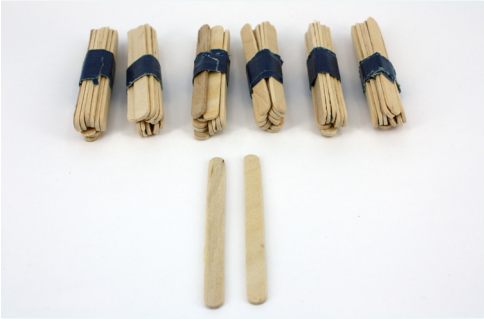
- 8 + 8 = 16, 8 + 7 = 15
- 30 + 30 = 60, 29 + 30 = 59

Use the Double Somersaults game for practice.

MATERIALS/LINKS

Sticks and bundles

Double Somersaults
(Material master 4-33)



WHAT TO DO NEXT IF THE STUDENT IS STUCK

Reduce the number of basic facts. Move back to working on smaller chunks (for example, doubles to 6) until the student can recall them with confidence.

Emphasise links between prior knowledge and each new basic facts set.

INITIATING HOME-BASED ACTIVITIES

Give the student flashcards to take home for extra practice. Explain to parents the importance of the student working out the answer without counting.

Digital Learning Object: Addition and Subtraction Basic Facts provides additional practice.

NEXT TEACHING STEPS BACK IN THE CLASSROOM

Incorporate doubles into number and measurement work, encouraging the student to identify and use their knowledge of doubles to solve problems. Use real contexts so that practice is not limited to flashcards. Emphasise that recognising and recalling doubles are useful strategies when solving problems.

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BARRIER BEING ADDRESSED **3** **POOR ESTIMATING SKILLS**

DIAGNOSTIC QUESTIONS

Explain that estimating means to use a simple calculation to get a rough idea of an answer. Tell the student that you are going to give them some problems. Their job is to estimate the answer to each problem.

1. Show the student collections of counters. Ask the student to show you an easy way to work out roughly how many counters there are in each pile.

   a. 24 counters b. about 50 counters c. about 100 counters.

2. Jackson eats two bananas every morning. How many bananas would he eat in a year? (Provide the student with a calendar to help them make an estimate.)

3. Six friends win $1,000 in a raffle. They want to share the money evenly. About how much money should each person get? (Provide the student with 10 x $100 paper money notes.)

WHAT TO NOTICE IN THE STUDENT'S RESPONSE

Does the student:

- avoid the question by saying "I don't know"?
- give a random response with little or no evidence of reasoning?
- try to find an exact answer?
- come up with a sensible estimate?

DELIBERATE ACTS OF TEACHING

Estimation and approximation strategies rely on a good sense of the number system. Knowing how to estimate allows students to check whether their answers to mathematical problems are likely to be correct.

Estimating an amount

One way to estimate an amount is to compare a known quantity to an unknown quantity.

Show the students a chain made of 10 paper clips and a longer chain of about 32 paper clips. Discuss with student how you could use the small chain to estimate the number of paper clips in the long chain.

Ask the student to estimate how long a chain of 100 paper clips would be and get them to explain their reasoning.

Estimating an answer

Give the student a problem involving multiplying. For example, 4 children each have 23 toys. Approximately how many toys are there altogether? If the student finds it difficult to estimate the answer, give them a 200 chart. Tell the student to work from 1 and to cross out lines of numbers that they are certain are not the answer. Once they are no longer certain, get them to work backwards from 200, crossing out lines of numbers that they think are too large. Ask the student to explain how they made their decisions.

Discuss with the student how many groups of ten there are in 23. Show them how they can use this to make an estimate (4 x 2 tens = 8 tens = 80).

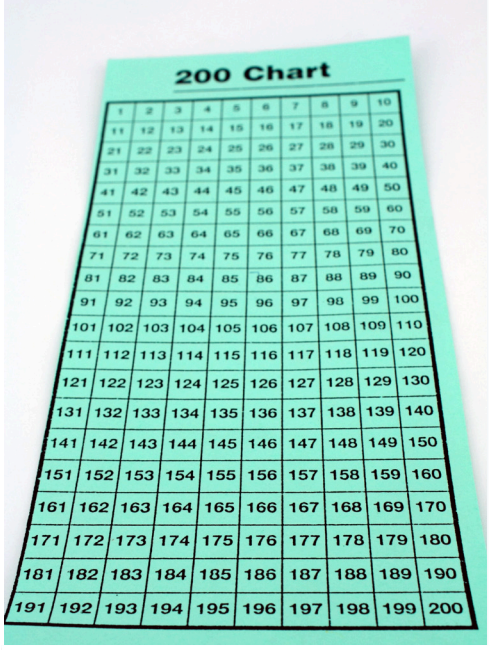
The "unit" does not have to be a group of 10. For example, if you have 50 lollies to share between 8 people, you can break the 50 into ten groups of 5. Each person gets one group of 5 and there will be a few lollies left over. Some students may recognise that the remaining 2 groups of 5 can be separated into 10 ones, which can then be shared out among the 8 people (with 2 left over). If the student doesn't make this connection, don't point it out. Keep the emphasis on using groups to find an approximate answer.

MATERIALS/LINKS

Paper clips

Counters

200 Chart



WHAT TO DO NEXT IF THE STUDENT IS STUCK

Continue working with materials. Make links between using a unit to estimate an amount and using groups of 10 to estimate the answer to a problem.

INITIATING HOME-BASED ACTIVITIES

Encourage parents to discuss how and when they need to estimate a value, for example, at the grocery store or when cooking.

NEXT TEACHING STEPS BACK IN THE CLASSROOM

Model the use of estimation when introducing a new problem. Ask the student what size they expect an answer to be. Ask a range of students to volunteer the method they used to make their estimate.

Using a specific task, explain the process you would use to estimate the answer and ask the student to repeat it back to you.

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